Soft-gluon resummation for squark- and gluino hadroproduction - towards NNLL accuracy

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DESY, Hamburg

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based on

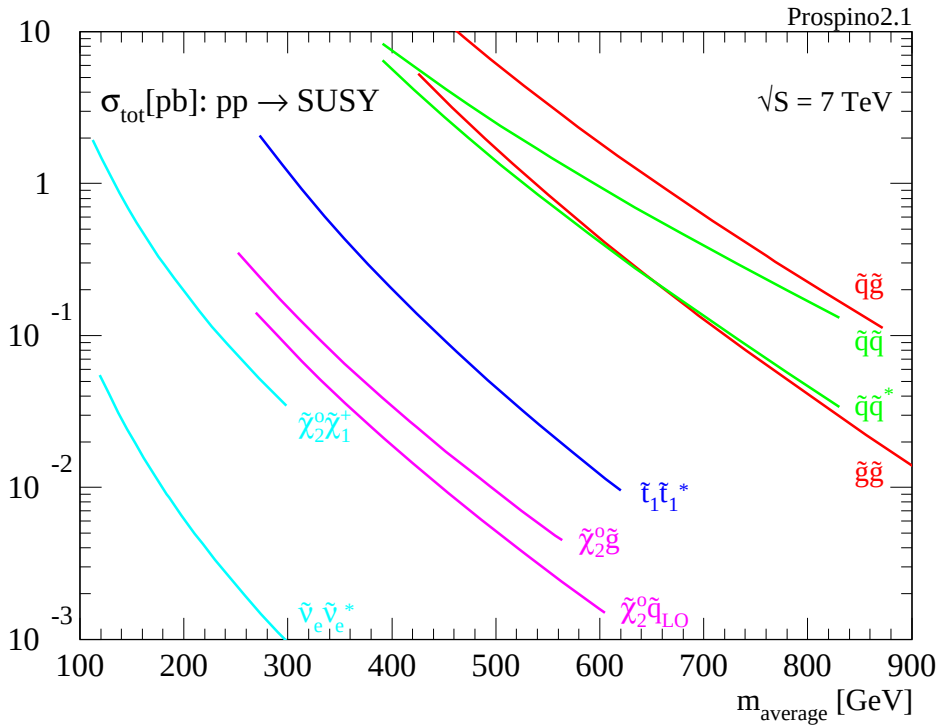
W. Beenakker, SB , M. Krämer, A. Kulesza, E. Laenen, I. Niessen [JHEP 1201 (2012) 076]
and work in preparation

Production of SUSY particles at hadron colliders

Framework: MSSM with R-parity conservation

LHC:

[Beenakker,Höpker,Krämer,Plehn,Spira,Zerwas'96-'98]

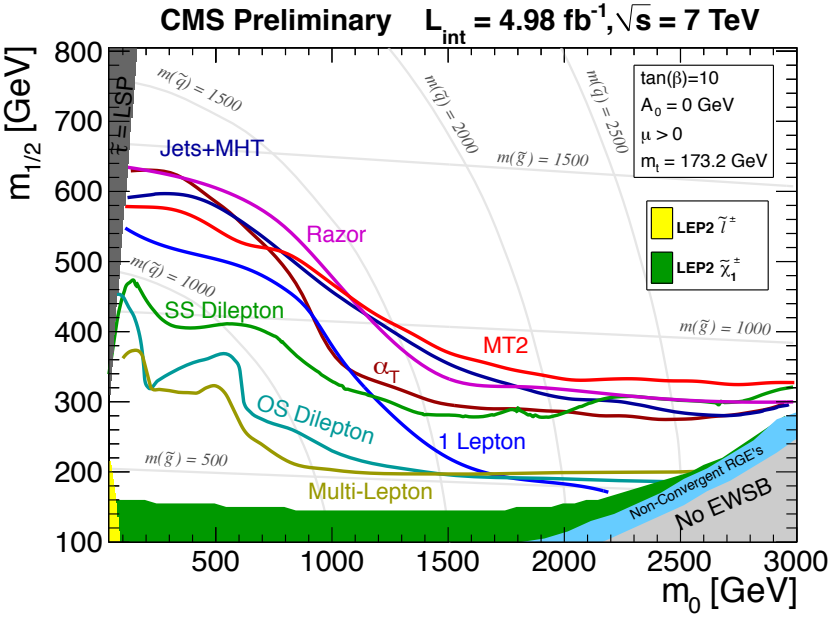


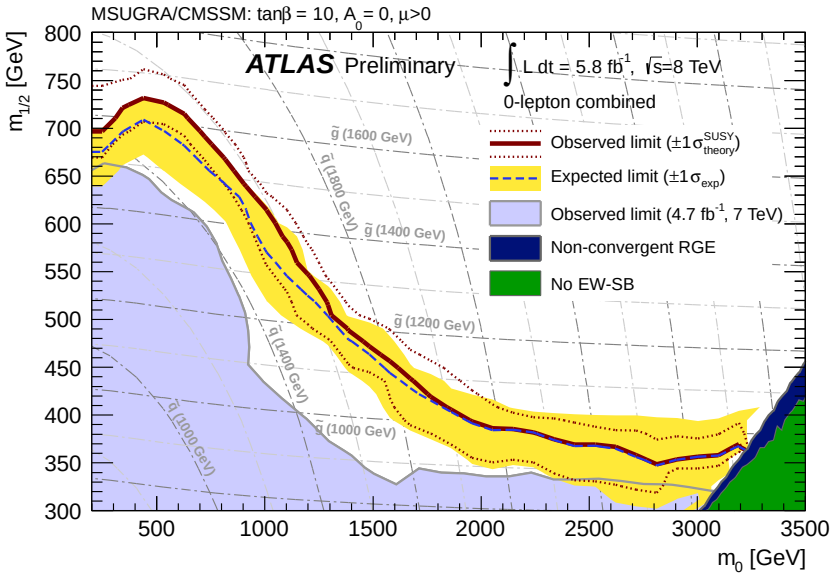
- dominated by processes involving coloured particles in the final state: $\tilde{q}\bar{\tilde{q}}, \tilde{q}\tilde{q}, \tilde{g}\tilde{g}, \tilde{q}\tilde{g}$ and $\tilde{t}\bar{\tilde{t}}$
- Squarks and gluinos are produced with high-production rates

→ offer strongest sensitivity for SUSY searches

Search for squarks and gluinos at the LHC

Total cross section are used to derive exclusion limits

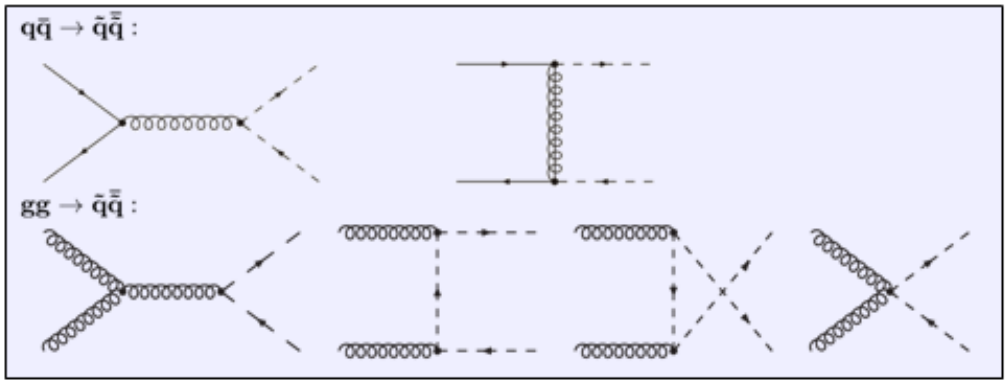




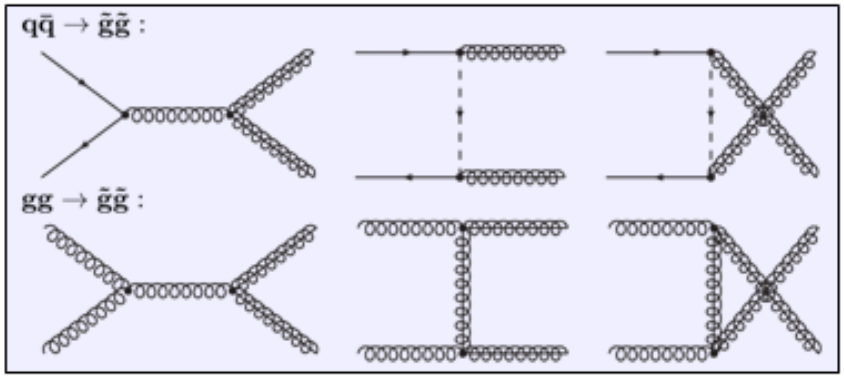
Classification of processes

Processes at LO: [Kane, Leveille '82; Harrison, Llewellyn, Smith '83; Eichten, Dawson, Quigg '85]

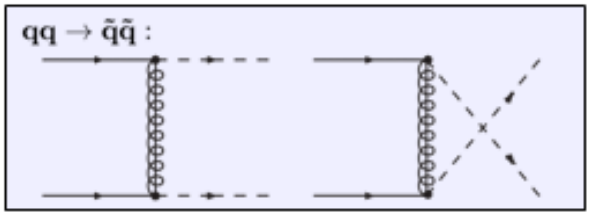
Squark-antisquark:



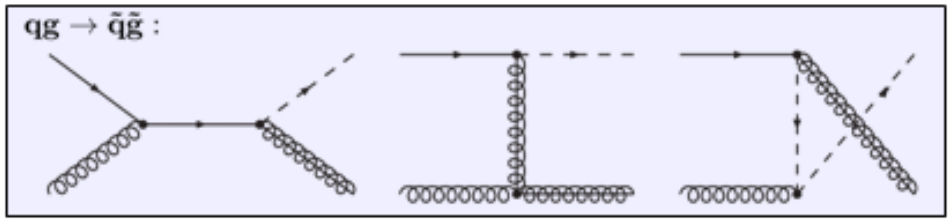
Gluino-gluino:



Squark-squark:



Squark-gluino:



Assume all squarks $\tilde{q} = (\tilde{q}_L, \tilde{q}_R)$ with $\tilde{q} \neq \tilde{t}$ mass degenerate

NLO SUSY-QCD calculation

NLO SUSY-QCD corrections [Beenakker et al. '96]

- Large positive corrections, depending in detail on squark- and gluino mass
- Significant part can be attributed to the threshold region $\hat{s} \approx 4m^2$

NLO partonic cross section near threshold $\beta = \sqrt{1 - 4m^2/\hat{s}} \to 0$:

$$\hat{\sigma}^{(\mathrm{NLO})} = \hat{\sigma}^{(0)} \left[\alpha_s \{a \log^2(\beta^2) + b \log(\beta^2) + c \log(\beta^2) \log(\mu^2/m^2) + d\,(1/\beta)\}\right]$$

Soft-gluon corrections Coulomb corrections

Generic form of higher-order corrections near threshold:

$$\hat{\sigma} = \hat{\sigma}^{(0)} \times \left[1 + \alpha_s(L^2 + L + ...) + ... + \alpha_s^n(L^{2n} + L^{2n-1} + ...)\right] \qquad L = \log(\beta^2)$$

- Logarithmic terms become large near threshold
- Spoil convergent behaviour of perturbative series in α_s

→ Requires all-order summation

→ Soft-gluon resummation

Theoretical Status

- NLO SUSY-QCD corrections [Beenakker et al.'96]
- NLL-resummed corrections [Kulesza, Motyka '08,'09] [Beenakker, SB, Krämer, Kulesza, Laenen, Niessen '09]
- NNLL-resummed corrections ($\tilde{q}\bar{\tilde{q}}$) [Beenakker, SB, Krämer, Kulesza, Laenen, Niessen '11]
- resummation of leading Coulomb-corrections ($\tilde{q}\bar{\tilde{q}}, \tilde{g}\tilde{g}$) [Kulesza, Motyka '09]
- combined (soft-gluon & Coulomb) NLL-resummed corrections [Beneke, Falgari, Schwinn '09,'10] [Falgari, Schwinn, Wever '12]

- approximated NNLO results ($\tilde{q}\bar{\tilde{q}}, \tilde{g}\tilde{g}$) [Langenfeld, Moch '09][Langenfeld, Moch, Pfoh '12]
- bound state effects in $\tilde{g}\tilde{g}$ and $\tilde{q}\tilde{g}$ [Hagiwara, Yokoya '09] [Kauth et al. '09,'11][Kauth, Kress, Kühn '11]

- NLO EW corrections [Beccaria et al. '07,'08][Hollik, Kollar, Trenkel '07][Hollik, Mirabella '08] [Hollik, Mirabella, Trenkel '08][Mirabella '09][Germer et al. '09, '11]
- LO EW and QCD-EW interference [Bozzi, Fuks, Klasen '05][Alan, Cankocak, Demir '07] [Bornhauser et al '07][Hollik, Kollar, Trenkel '07][Hollik, Mirabella '08][Hollik, Mirabella, Trenkel '08][Germer et al. '09,'11]

Public Code: NLL-fast

- NLO*+NLL resummed predictions for total cross sections for all possible squark and gluino pair-production processes (Mellin-space approach)
  - significant enhancement of NLO cross section predictions (e.g. LHC@8TeV at m=1.5TeV up to $\sim$ 30%)
  - most pronounced for processes involving large colour charges
  - significant reduction of theoretical error due to scale variation
  - pdf and α_s uncertainties included
  - used in experimental analysis (CMS, ATLAS)

Available at: (LPCC BSM Cross Section Working Group)

http://web.physik.rwth-aachen.de/service/wiki/bin/view/Main/BSMCrossSectionWorkingGroup

*:calculated using PROSPINO

Soft-gluon resummation

[Contopanagos, Kidonakis, Laenen, Oderda, Sterman, Bonciani, Catani, Mangano, Nason '96 – '98]

- Perform resummation of soft-gluon contributions using approach in Mellin-space

$$\sigma_{h_A h_B \to kl}\left(N, \{m^2\}\right) \equiv \int_0^1 d\rho\, \rho^{N-1}\, \sigma_{h_A h_B \to kl}\left(\rho, \{m^2\}\right)$$

- Hadronic cross section for the production of two massive coloured sparticles *k,l*

$$\sigma_{h_A h_B \to kl}\left(\rho, \{m^2\}\right) = \sum_{i,j} \int dx_1\, dx_2\, dz\; \delta\left(z - \frac{\rho}{x_1 x_2}\right)\; f_{i/h_A}(x_1, \mu^2)\, f_{j/h_B}(x_2, \mu^2)\, \hat{\sigma}_{ij \to kl}\left(z, \{m^2\}, \mu^2\right)$$

with $\rho = \dfrac{(m_k + m_l)^2}{S}$ and $z = \dfrac{(m_k + m_l)^2}{\hat{s}}$

$$\rightarrow \quad \sigma_{h_A h_B \to kl}\left(N, \{m^2\}\right) = \sum_{i,j} f_{i/h_A}(N{+}1, \mu^2)\, f_{j/h_B}(N{+}1, \mu^2)\, \hat{\sigma}_{ij \to kl}\left(N, \{m^2\}, \mu^2\right)$$

- Form of soft-gluon corrections $\alpha_s^n \log^m(N) \quad m \leq 2n \qquad N \to \infty$

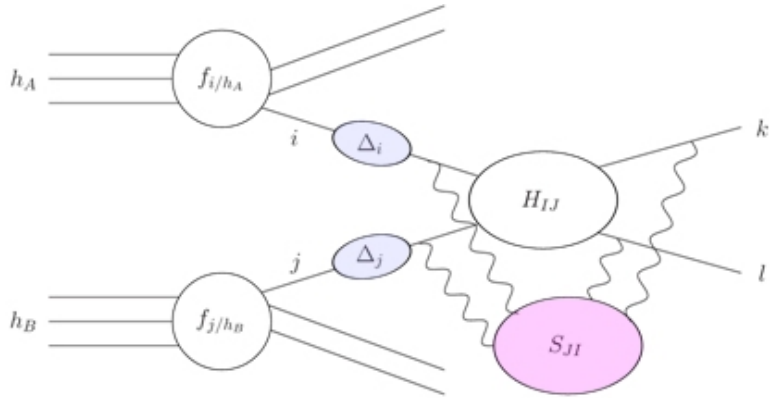
- Resummation: $\hat{\sigma}^{(\mathrm{res})}_{ij \to kl}(N) = \exp\left[\underbrace{L g_1(\alpha_s L)}_{\mathrm{LL}} + \underbrace{g_2(\alpha_s L)}_{\mathrm{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\mathrm{NNLL}} + \ldots\right] \times P(\alpha_s) \qquad L = \log(N)$

Soft-gluon resummation for coloured heavy (s)particles

[Contopanagos, Kidonakis, Laenen, Oderda, Sterman '96-'98; Bonciani, Catani, Mangano, Nason '98]

- Based on near threshold factorisation of the cross section

$$\hat{\sigma}_{ij\to kl}(N) = \underbrace{\Delta_i\,\Delta_j}_{\text{soft-collinear}} \sum_{IJ} H_{ij\to kl,JI}\, \underbrace{S_{ij\to kl,IJ}}_{\text{wide-angle soft}}$$



- Evolution equations
  e.g.: $$\mu\frac{d}{d\mu}S_{JI} = -\Gamma^{\dagger}_{JK}\,S_{KI} - S_{JK}\,\Gamma_{KI}$$
- Solving evolution equations $\longrightarrow$ resummed expressions

$$\tilde{\sigma}^{(\mathrm{res})}_{ij\to kl}(N,\{m^2\},\mu^2) = \sum_I \tilde{\sigma}^{(0)}_{ij\to kl,I}(N,\{m^2\},\mu^2)\,\mathcal{C}_{ij\to kl,I}(N,\{m^2\},\mu^2)$$
$$\times\ \Delta_i(N+1,Q^2,\mu^2)\,\Delta_j(N+1,Q^2,\mu^2)\,\Delta^{(\mathrm{s})}_{ij\to kl,I}(N+1,Q^2,\mu^2)$$

- $\mathcal{C}_{ij\to kl,I}$: matching coefficients
- Δ_i,Δ_j: resums soft and collinear gluon radiation
- $\Delta^{(\mathrm{s})}_{ij\to kl,I}$: resums wide-angle soft gluon radiation

Soft-radiative factors at NNLL

Soft-radiative factors $\Delta_i \Delta_j \Delta^{(s)}_{ij\to kl,I}$:

$$\Delta_i = \int_0^1 \frac{z^{N-1}-1}{1-z} \int_{\mu^2}^{Q^2(1-z)^2} \frac{dq^2}{q^2} A_i(\alpha_s(q^2))$$

$$\Delta^{(s)}_{ij\to kl,I} = \int_0^1 \frac{z^{N-1}-1}{1-z} D_{ij\to kl,I}(\alpha_s((1-z)^2 Q^2))$$

$A_i, D_{ij\to kl,I}$: power series in α_s

In terms of g_n : ($LL(g_1)$, NLL(g_2), NNLL(g_3))

$$\Delta_i \Delta_j \Delta^{(s)}_{ij\to kl,I} = \exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right]$$

For NNLL accuracy: Three-loop $A^{(3)}$ [Moch, Vermaseren, Vogt '05]

Two-loop $D^{(2)}_{(I)}$ [Contapanogos, Laenen, Sterman '96, Catani, de Florian, Grazzini '01]
[Czakon, Mitov, Sterman '09, Beneke, Falgari, Schwinn '09, ...]

Matching coefficients for NNLL

Soft-Coulomb factorisation [Bonciani et al. '98, Beneke, Falgari, Schwinn '10]

$$C_I^{\mathrm{NNLL}} = \left(1 + \frac{\alpha_s}{\pi}\, C_I^{(1)}(\{m^2\}, \mu^2)\right) \left(1 + \frac{\alpha_s}{\pi}\, C_I^{\mathrm{Coul},(1)}(N, \{m^2\}, \mu^2)\right)$$

$C_I^{(1)}$: NLO hard matching coefficients

obtained by taking Mellin transform of $\mathcal{O}(\beta)$ term of NLO cross section and omitting leading Coulomb corrections and $\log(N)$ terms

$C_I^{\mathrm{Coul},(1)}$: leading Coulomb corrections

obtained by taking Mellin transform of

$$\sigma_{ij\to kl,I}^{\mathrm{Coul},(1)} = -\frac{\pi\alpha_s}{4\beta}\sqrt{\frac{m_k m_l}{m_{av}}}\,\sigma_{ij\to kl,I}^{(0)}(C_2(R_I) - T_k^2 - T_l^2)$$

NLO hard matching coefficients

- calculated for all squark- and gluino production processes

[Beenakker, Janssen ,Krämer, Kulesza, Laenen, Lepoeter, Niessen, ST, Van Daal in prep.]

- e.g.: $q\bar{q} \to \tilde{q}\bar{\tilde{q}}$ [Beenakker, SB, Krämer, Kulesza, Laenen, Niessen '11]

$$
\begin{aligned}
\mathcal{C}^{(1)}_{q\bar{q}\to\tilde{q}\bar{\tilde{q}},I} = \mathrm{Re}\Bigg\{ & \frac{2C_F}{3}\pi^2 + \gamma_g \log\left(\frac{\mu_R^2}{m_{\tilde{q}}^2}\right) - \gamma_q \log\left(\frac{\mu_F^2}{m_{\tilde{q}}^2}\right) + F_0(m_{\tilde{q}}, m_{\tilde{g}}, m_t) + \frac{19N_c}{24} \\
& + \frac{23}{8N_c} + \frac{1-3N_c^2}{N_c}\log\left(\frac{m_+^2}{m_{\tilde{q}}^2}\right) - \frac{2}{N_c}\log(2) + \left(\frac{7N_c}{6} + \frac{2m_{\tilde{g}}^2}{m_+^2}C_F\right)\log\left(\frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2}\right) \\
& - \frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2}\left(\frac{m_-^2}{m_{\tilde{q}}^2}\log\left(\frac{m_-^2}{m_{\tilde{g}}^2}\right) + 1\right)C_F + \frac{m_{\tilde{g}}^2}{2m_-^2}\left(\frac{m_{\tilde{g}}^2}{m_-^2}\log\left(\frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2}\right) - 1\right)C_F \\
& - \frac{1}{2N_c}\left(\frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2} - 3\right)F_1\left(m_{\tilde{q}}, m_{\tilde{g}}\right) + \left(\frac{m_+^2}{2m_{\tilde{q}}^2}C_F + \frac{1}{N_c}\right)F_2\left(m_{\tilde{q}}, m_{\tilde{g}}\right) \\
& + 2C_F\left(\gamma_E^2 - 2\gamma_E\log(2) + \gamma_E\log\left(\frac{\mu_F^2}{m_{\tilde{q}2}}\right)\right) \\
& + \Bigg[- \frac{\pi^2}{4} + \log\left(\frac{m_+^2}{m_{\tilde{q}}^2}\right) - \log(2) - \frac{m_{\tilde{g}}^2}{m_+^2}\log\left(\frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2}\right) + 2 + \gamma_E \\
& - \frac{1}{4}\left(\frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2} - 3\right)\left(F_1\left(m_{\tilde{q}}, m_{\tilde{g}}\right) + F_2.ft(m_{\tilde{q}}, m_{\tilde{g}})\right]C_2(I)\Bigg\}
\end{aligned}
$$

Matching with fixed-order calculation

NLO (NNLO-calculation not available) and NNLL-resummed are combined through a matching procedure:

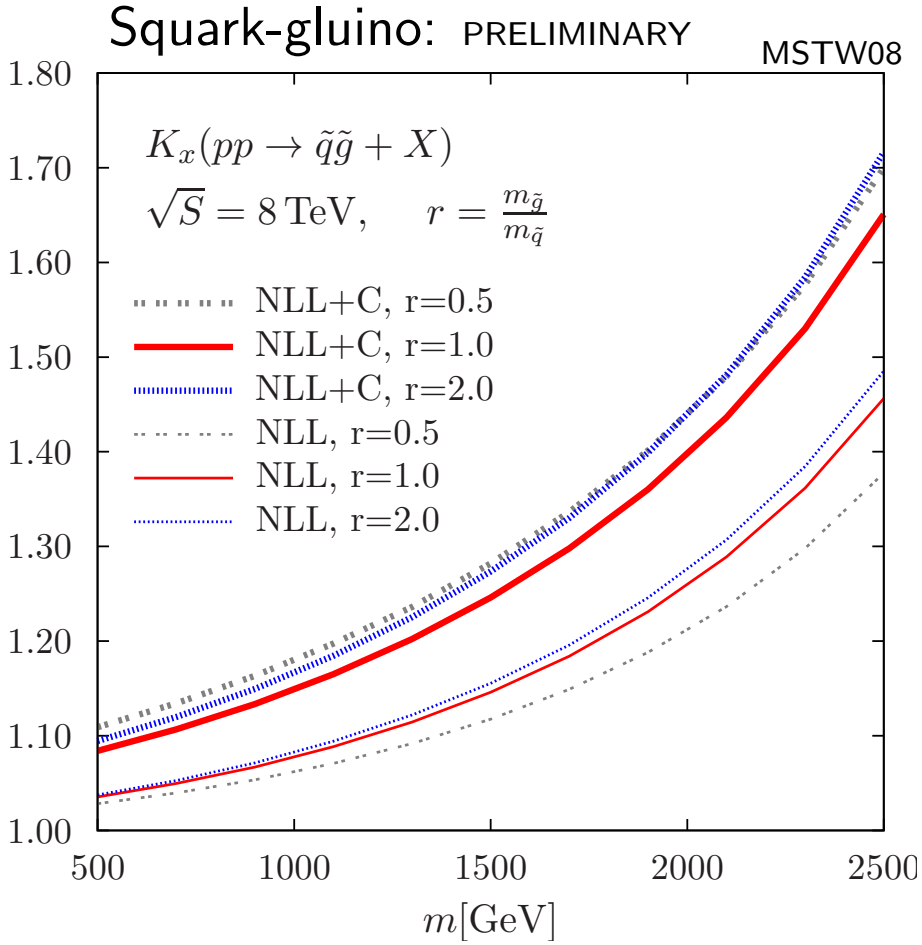
$$\begin{aligned}
\sigma^{\mathrm{NLO+NNLL}}_{h_A h_B \to kl}(\rho, \{m^2\}, \mu^2) \quad &= \quad \sum_{i,j} \int_{C_{MP}-i\infty}^{C_{MP}+i\infty} \frac{dN}{2\pi i}\, \rho^{-N}\, f_{i/h_A}(N+1, \mu^2)\, f_{j/h_B}(N+1, \mu^2) \\
&\times \quad \left[\hat{\sigma}^{\mathrm{res,NNLL}}_{ij \to kl}(N, \{m^2\}, \mu^2) \;-\; \hat{\sigma}^{\mathrm{res,NNLL}}_{ij \to kl}(N, \{m^2\}, \mu^2) \Big|_{(\mathrm{NLO})} \right] \\
&+ \quad \sigma^{\mathrm{NLO}}_{h_A h_B \to kl}(\rho, \{m^2\}, \mu^2)
\end{aligned}$$

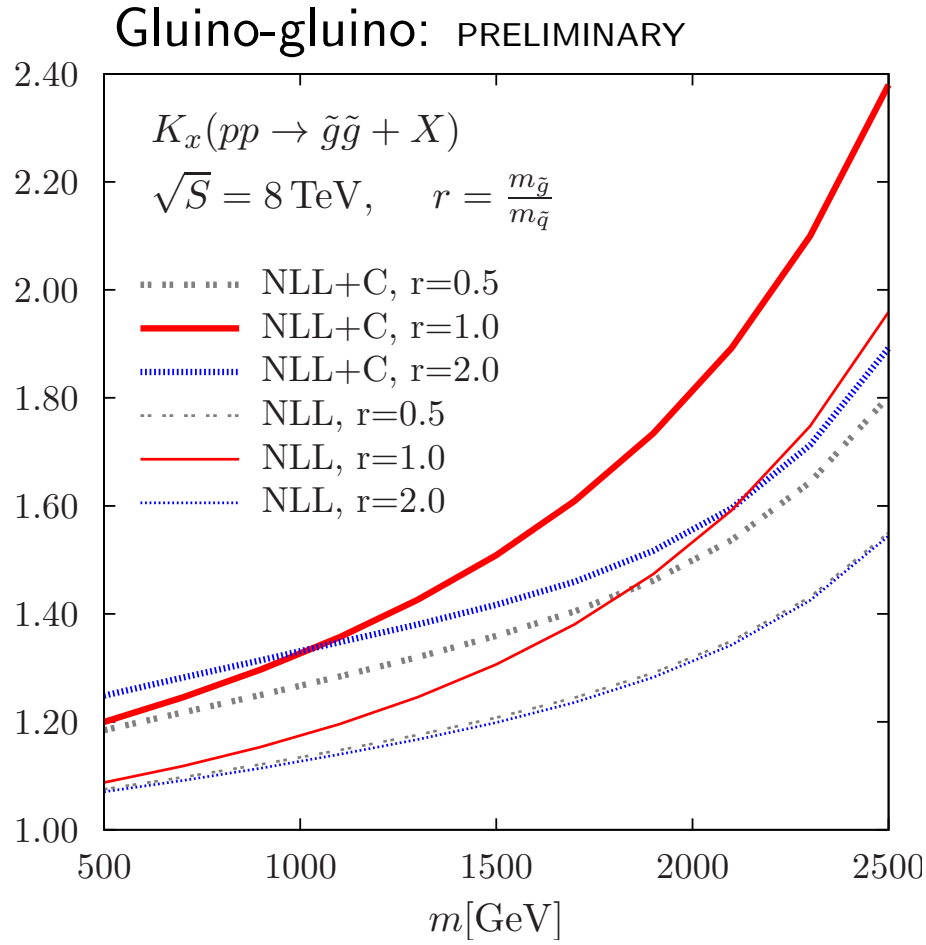
- Avoids double counting of logarithmic terms
- Using "minimal prescription" for the contour of the inverse Mellin transform
  [Catani et al., '96]
- NLO cross section calculated with PROSPINO
  [Beenakker, Höpker, Krämer, Plehn, Spira, Zerwas, '96-'98]

Numerical impact of hard matching coefficients

$K_{NLL(+C)} = \sigma_{NLO+NLL(+C)}/\sigma_{NLO}$, LHC@8TeV

[Beenakker, Janssen, Lepoeter, Krämer, Kulesza, Laenen, Niessen, ST, Van Daal in prep.]





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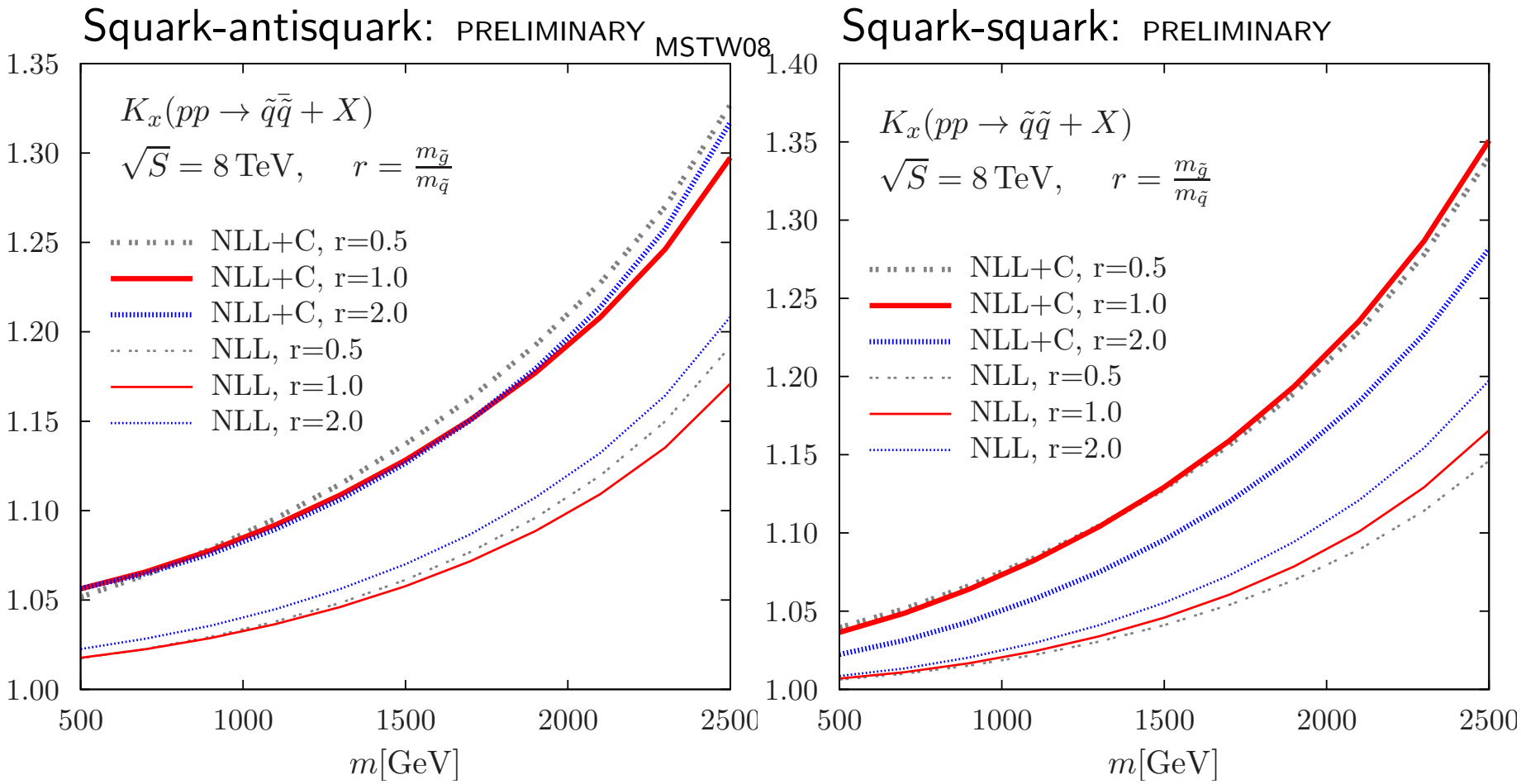
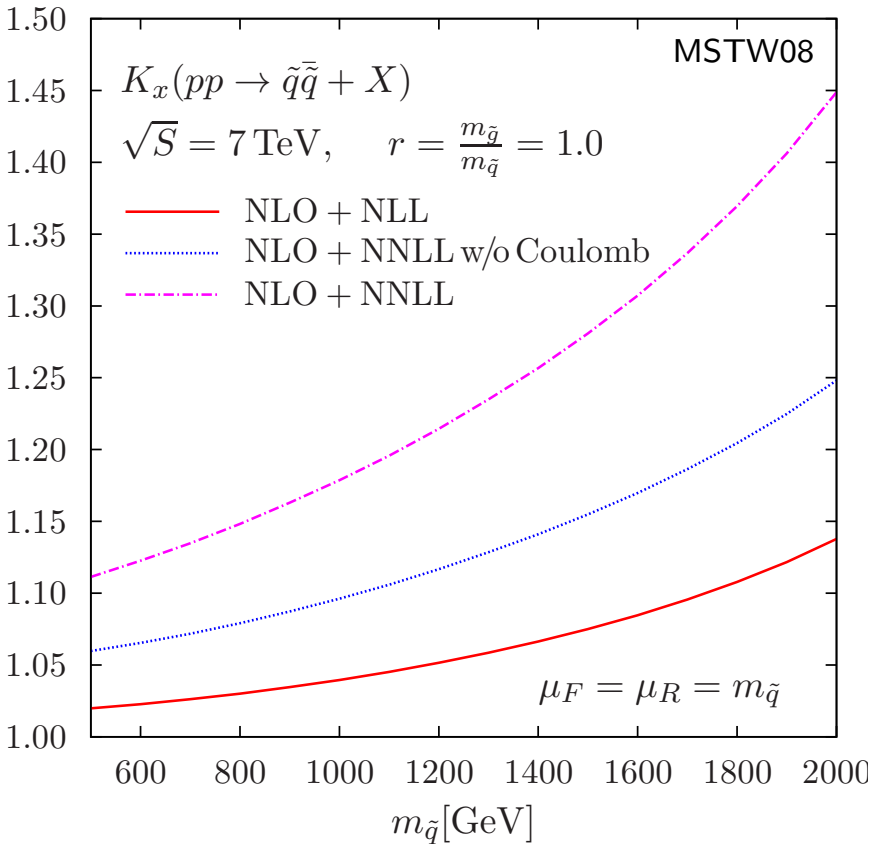


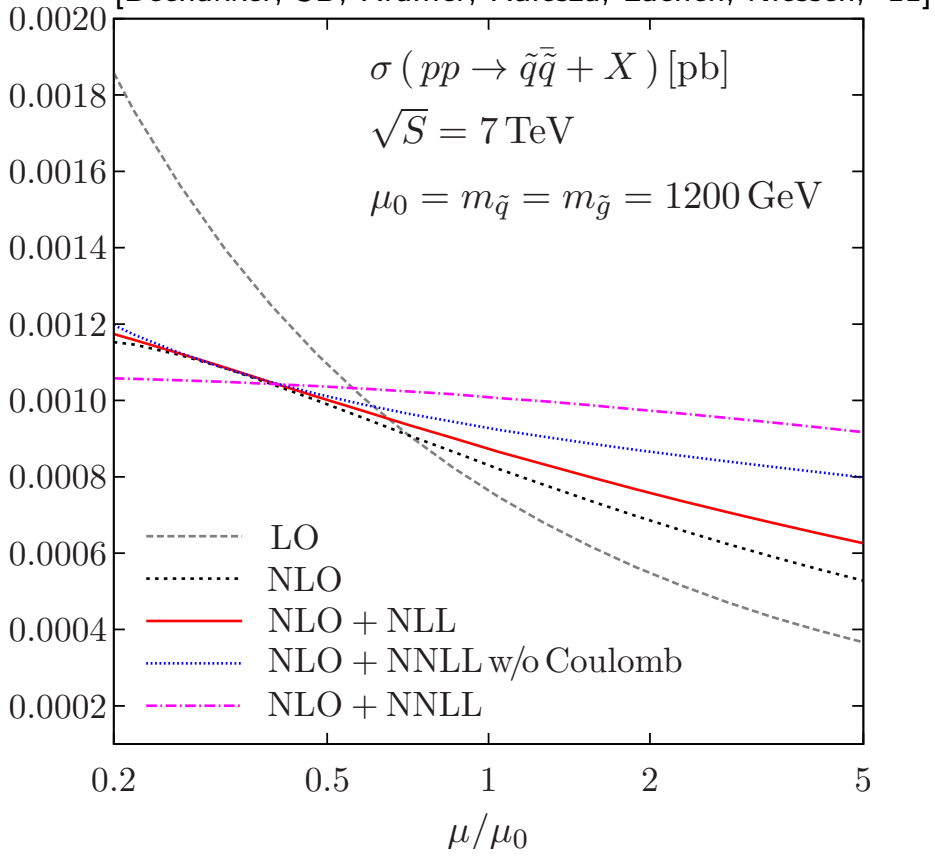
NNLL resummation for squark-antisquark production,

LHC@7TeV

$K_{\mathrm{NNLL}} = \sigma_{\mathrm{NLO+NNLL}}/\sigma_{\mathrm{NLO}}$



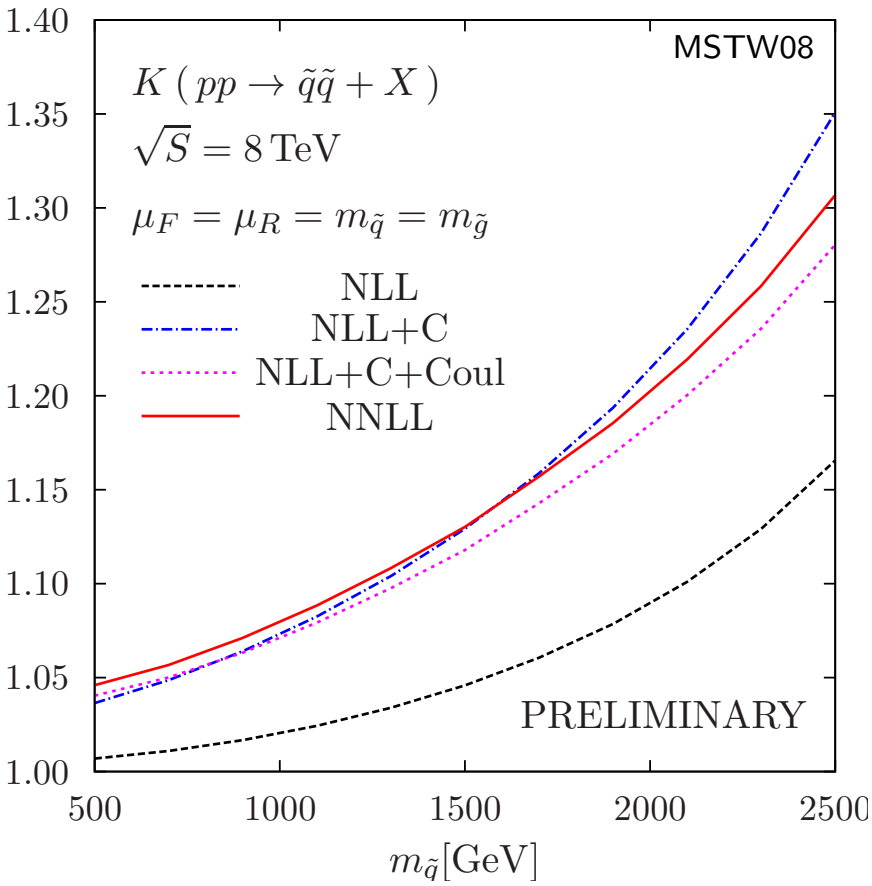
[Beenakker, SB, Krämer, Kulesza, Laenen, Niessen, '11]



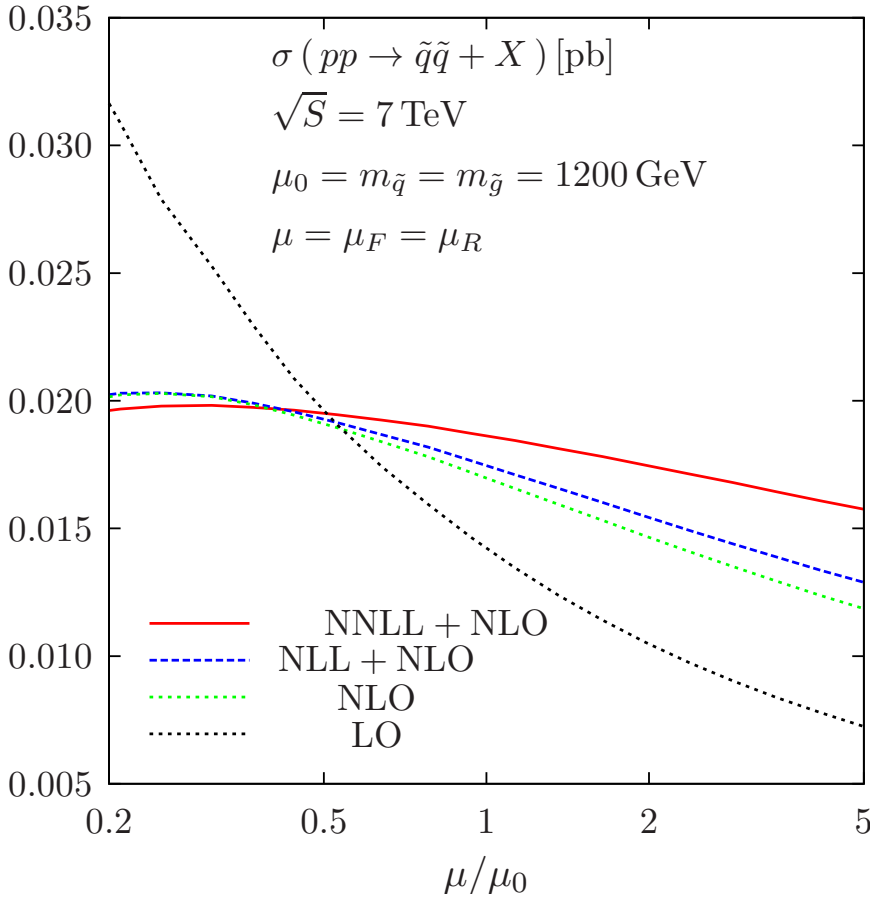
- significant corrections at NLO+NNLL
- reduced scale sensitivity

NNLL resummation for squark-squark production, LHC@8TeV

$K_{\mathrm{NNLL}} = \sigma_{\mathrm{NLO+NNLL}}/\sigma_{\mathrm{NLO}}$



[Beenakker, Krämer, Kulesza, Laenen, Niessen, ST in prep.]



- significant corrections at NLO+NNLL
- reduced scale sensitivity

Summary

- NLO+NLL resummed predictions for total cross sections for all possible squark and gluino pair-production processes: Public code `NLL-fast`
- Analytical results for hard matching coefficents at one-loop for all possible squark- and gluino pair-production processes
  $\rightarrow$ allows for soft-gluon resummation to NNLL accuracy
- Numerical impact of hard matching coefficients significant
- Numerical results for NNLL resummed matched to NLO
  - for squark-antisquark and squark-squark production
    $\rightarrow$ significant enhancement of NLO+NLL cross section predictions
    $\rightarrow$ further reduction of scale dependence