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with Supersymmetry and Leptogenesis

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Abstract

We study the cosmological interplay between supersymmetry, thermal leptogenesis as the origin of matter and the Peccei-Quinn mechanism as solution to the strong CP problem.

We investigate to what extent the decay of the lightest ordinary supersymmetric particle, which usually spoils primordial nucleosynthesis in scenarios with gravitino dark matter, can become harmless due to entropy production. We study this possibility for a general neutralino. We find that strong constraints on the entropy-producing particle exclude generic thermal relics as source of sufficient entropy. However, the Peccei-Quinn supermultiplet (axion, saxion, axino) may not only be part of the particle spectrum, but the saxion can also produce a suitable amount of entropy. Exploiting cosmological perturbation theory we show that the corresponding expansion history can be falsified by future observations of the gravitational wave background from inflation, if polarisation measurements of the cosmic microwave background are combined with very sensitive gravitational wave probes.

Since the same problem can also be solved by a small breaking of R-parity, we investigate the impact of broken R-parity on the Peccei-Quinn supermultiplet. We find that naturally expected spectra become allowed. Bounds from late particle decays become weaker than those from non-thermal axion production. Thus the strong CP problem serves as an additional motivation for broken R-parity.

We show that, if the gravitino problem is solved by a light axino, "dark radiation" emerges naturally after primordial nucleosynthesis but before photon decoupling. Current observations of the cosmic microwave background could confirm an increase in the radiation energy density. The other way around, this solution to the gravitino problem implies that thermal leptogenesis might predict such an increase. The Large Hadron Collider could endorse this opportunity. In the same parameter range, axion and axino can naturally form the observed dark matter.

Zusammenfassung

Wir untersuchen das kosmologische Zusammenspiel von Supersymmetrie, thermaler Leptogenese als Ursprung der Materie und dem Peccei-Quinn-Mechanismus als Lösung des starken CP-Problems.

Der Zerfall des Leichtesten der gewöhnlichen supersymmetrischen Teilchen lässt, falls das Gravitino die Dunkle Materie bildet, üblicherweise die primordiale Nukleosynthese scheitern. Wir untersuchen inwiefern Entropieproduktion dies verhindern kann. Starke Schranken an das entropieproduzierende Teilchen schließen ein generisches thermisches Relikt als Quelle hinreichender Entropie aus. Allerdings ist das Peccei-Quinn-Supermultiplett (Axion, Saxion, Axino) nicht nur im Teilchenspektrum zulässig, sondern das Saxion kann auch eine geeignete Menge an Entropie erzeugen. Als Beispiel dient ein allgemeines Neutralino. Mittels kosmologischer Störungstheorie zeigen wir, dass die dazugehörige Expansionsgeschichte des Universums durch zukünftige Beobachtungen des Gravitationswellenhintergrunds der Inflation ausgeschlossen werden könnte, falls Polarisationsmessungen des kosmischen Mikrowellenhintergrunds mit sehr sensitiven Gravitationswellenbeobachtungen kombiniert werden.

Da das selbe Problem auch durch eine kleine Verletzung der R-Parität gelöst werden kann, untersuchen wir die Auswirkungen einer R-Paritätsverletzung auf das Peccei-Quinn-Supermultiplett. Natürliche Spektren werden zulässig. Schranken von spätem Teilchenzerfall werden schwächer als solche von nicht-thermaler Axionproduktion. Somit dient das starke CP-Problem als zusätzliche Motivation für verletzte R-Parität.

Falls ein leichtes Axino das Gravitinoproblem löst, entsteht "Dunkle Strahlung" natürlicherweise nach primordialer Nukleosynthese und vor Photonenentkopplung. Beobachtungen des kosmischen Mikrowellenhintergrunds könnten einen Anstieg der Strahlungsenergiedichte bestätigen. Diese Lösung des Gravitinoproblems impliziert, dass thermische Leptogenese solch einen Anstieg vorhersagen könnte. Der Large Hadron Collider könnte dies bekräftigen. Im selben Parameterbereich können das Axion und das Axino natürlicherweise die beobachtete Dunkle Materie bilden.

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Chapter 1

Introduction

The existence of dark matter in the Universe is firmly established on the basis of astrophysical and cosmological observations. Current observational data strongly disfavour non-luminous astrophysical objects or modifications of the theory of gravity as explanation for the observed gravitational effects. This makes the hypothesis of particle dark matter the best candidate.[1] However, no evidence for dark matter particles has been found on microscopic scales so far. Thus little is known about their properties. The nature, identity and origin of dark matter, that actually makes up more than 80% of the matter in the Universe, is one of the biggest mysteries of science.

The Standard Model of particle physics provides an astonishingly successful description of the elementary particles and their interactions. However, it is known to be incomplete. Cosmologically it does neither provide any dark matter candidate nor an origin of baryonic matter composed of ordinary atoms. Indeed, it is still a mystery why the matter humans are made of is there. In conflict with microscopic observations, the Standard Model does not provide neutrino masses. Since charge-parity symmetry (CP) cannot be invoked to be exactly conserved, the observed smallness of its violation in the strongly interacting sector is not understood. And maybe most pressing, the scale of the Higgs boson mass in the electroweak sector and its stability are unexplained.

The underlying motivation for this thesis is the idea that the fundamental theory of Nature should lead to a consistent cosmology. The ultimate consistent cosmology

[1] For a recent review of relevant observations see [1].

does not only account for all cosmological observations but also enables solutions to all problems of the Standard Model. Since we are far from this utopia, we need to focus on a subset of problems in particle physics and cosmology.

Any modern, physical theory should be as simple as possible and as comprehensive as necessary to explain all phenomena. An extension of the Standard Model by right-handed Majorana neutrinos not only elegantly explains the tiny neutrino masses, but without further ingredients also provides an origin of matter: thermal leptogenesis [2]. The standard solution to the strong CP problem is the Peccei-Quinn mechanism [3, 4], while all other proposed solutions seem to fail. It implies the existence of another elementary particle: the axion. If softly broken supersymmetry[2] is realised in Nature at a low scale, there is a superpartner for every Standard Model particle. This stabilises the Higgs boson mass to a calculable value at the required scale. The considered physical processes span various energy scales and reside in different sectors of our description of Nature.

In this thesis we focus on the cosmological interplay between these longstanding, well-studied solutions and explanations. Since every supersymmetric theory containing gravity predicts inevitably and uniquely the gravitino, there is most famously the cosmological gravitino problem. But also the supersymmetrised axion is known to be afflicted with cosmological problems. Intriguingly, these problems provide also chances like perfect dark matter candidates, in case the gravitino or the superpartner of the axion are the lightest supersymmetric particle (LSP). Since thermal leptogenesis requires a large initial temperature of the hot early universe, cosmological problems become particularly severe. The prime example might be the apparent, mutual exclusion of supersymmetry and leptogenesis. We argue that the definite clash between both notions is finally the decay of the lightest ordinary superparticle (LOSP) spoiling primordial nucleosynthesis, i.e., the LOSP decay problem. Naively, one might expect that further ideas like the Peccei-Quinn mechanism either cannot be implemented or simply coexist without any interplay. In contrast to the naive expectation, we find non-trivial connections between the different notions. Problems often even turn out as fortunes and physical processes at various different scales become connected. This provides further insights, especially, when observable consequences are found.

This thesis is organised as follows: Chapter 2 introduces briefly the most relevant

[2]A review of supersymmetry is provided in [5].

cosmological observations and notions, while Chapter 3 specifies the elementary particle content considered in this thesis and outlines the arising set of cosmological problems. This is where we relate our work to the current state of research. We investigate to what extent supersymmetry and leptogenesis can be reconciled by entropy production using the axion multiplet in Chapter 4 and show that this solution can be tested by future observations of the gravitational wave background from inflation. In Chapter 5 we investigate the impact of R-parity violating solutions to the LOSP decay problem on the cosmological constraints from and on the axion multiplet. We present the axino solution to the gravitino problem of thermal leptogenesis in Chapter 6 and show that the scenario is tested by the Planck satellite mission and the Large Hadron Collider (LHC). After summarising our results we conclude and provide an outlook in the last chapter. In Appendix A we provide details of the calculation of the impact of entropy production on the gravitational wave background.

The content of this thesis has been published in parts in [6, 7, 8, 9, 10, 11].

Chapter 2

Dark and Visible Matter in Cosmology

In this chapter we introduce briefly the most relevant cosmological observations and notions with a strong focus on our needs.[1]

2.1 Big Bang Cosmology

Einstein's equations of general relativity,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_{\rm N} T_{\mu\nu} \, , \tag{2.1}$$

determine the geometry of space-time by the matter and energy content of the Universe. $R_{\mu\nu}$ and R are the Ricci tensor and Ricci scalar, respectively, while $g_{\mu\nu}$ is the space-time metric. The right-hand side of (2.1) consists of the energy-momentum tensor $T_{\mu\nu}$, while $G_{\rm N}$ denotes Newton's gravitational constant. Eq. (2.1) is a set of ten coupled equations. To solve them analytically, we ought to assume symmetries. Fortunately, measurements of the cosmic microwave background (CMB) show that the Universe is highly isotropic, cf. Sec. 2.4, and galaxy surveys indicate that the Universe is also homogeneous on large scales of $\mathcal{O}\,(100\ \text{Mpc})$. The most general space-time metric compatible with isotropy and homogeneity is the

[1] For a pedagogical introduction see [12]. Parts are following previous work of the author [11].

Friedmann-Lemaitre-Robertson-Walker metric,

$$g_{\mu\nu}dx^\mu dx^\nu = ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\Theta^2 + \sin^2\Theta d\Phi^2)\right). \tag{2.2}$$

r, Θ and Φ are comoving spatial coordinates. So the scale factor $a(t)$ completely describes the evolution with time t. The constant $k = -1, 0, +1$ characterises the spatial curvature, where $k = -1$ corresponds to an open, $k = 0$ to a flat and $k = +1$ to a closed universe.

A convenient simplifying assumption is to describe the matter and energy of the Universe by a perfect fluid, thereby respecting isotropy and homogeneity. The energy-momentum tensor of a perfect fluid in its rest frame is

$$T^\mu_\nu = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}, \tag{2.3}$$

where ρ and p denote energy density and pressure, respectively. The assumptions lead from Einstein's equations (2.1) to the Friedmann equations,

$$\ddot{a} = -\frac{4\pi G_{\rm N}}{3} a \sum_i (\rho_i + 3p_i)\,, \tag{2.4}$$

that is also known as acceleration equation, and

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_{\rm N}}{3}\sum_i \rho_i - \frac{k}{a^2}\,, \tag{2.5}$$

where an overdot indicates the derivative with respect to time t. These equations describe the dynamics of the Universe. We introduced the Hubble parameter H that gives the expansion rate $\dot{a}/a$ of the Universe. The sums account for several forms of energy, which are characterised by their equation of state,

$$p_i = w_i \rho_i\,. \tag{2.6}$$

There are radiation and relativistic particles with $w_r = 1/3$ and non-relativistic matter with $w_m = 0$. The cosmological constant Λ can be described by an energy component with $w_\Lambda = -1$. The energy conservation equation in an isotropic, homogeneous universe reads

$$\dot{\rho} = -3H(\rho + p)\,. \tag{2.7}$$

This can also be derived from the Friedmann eq. (2.4) and (2.5). Therefore, the combination of (2.5) and either (2.4) or energy conservation (2.7), supplemented by the equation of state (2.6), forms a complete system of equations that determines the two unknown functions $a(t)$ and $\rho(t)$.

Today, we observe a spatially flat ($k = 0$), expanding ($H > 0$) universe. The energy of photons and other relativistic particles decreases during their propagation in an expanding universe. In other words, their wavelength λ grows with time. So the redshift parameter

$$z = \frac{\lambda_{\mathrm{obs}} - \lambda_{\mathrm{em}}}{\lambda_{\mathrm{em}}} = \frac{a(t_{\mathrm{obs}})}{a(t_{\mathrm{em}})} - 1 \tag{2.8}$$

grows with time. λ_{obs} denotes the observed wavelength of an originally with wavelength λ_{em} emitted photon, while $a(t_{\mathrm{obs}})$ and $a(t_{\mathrm{em}})$ denote the corresponding values of the scale factor. Obviously, there is a one-to-one correspondence between z and the time of emission t_{em}. In this way z is used as a measure of time. It is convenient to rewrite the Friedmann equations using the density parameter

$$\Omega_i(z) = \frac{\rho_i(z)}{\rho_{\mathrm{c}}(z)}\,, \tag{2.9}$$

where $\rho_{\mathrm{c}}(z) = 3H^2/(8\pi G_{\mathrm{N}})$ is the critical density corresponding to a spatially flat universe. The present day critical density is given by [13]

$$\rho_{\mathrm{c}}(0) = \rho_0 = \frac{3H_0^2}{8\pi G_{\mathrm{N}}} \simeq 1.05 \times 10^{-5} h^2 \ \mathrm{GeV\ cm^{-3}}\,, \tag{2.10}$$

where H_0 is the present day Hubble parameter. In the usual parametrisation

$$H_0 = 100\, h \ \mathrm{km\ s^2\ Mpc^{-1}} \tag{2.11}$$

with $h \simeq 0.7$. Already the simple extrapolation of this expansion back in time leads to a singularity in the past of the Universe. So the history of the Universe began in a very dense and hot phase, the “big bang”. Since that time the Universe has expanded and due to its expansion has cooled down. Doing the extrapolation back in time more carefully gives us the age of the Universe as $\simeq 13.7 \times 10^9$ y $\simeq 4.3 \times 10^{17}$ s. Using the density parameter (2.9) the Friedmann equation (2.5) becomes

$$1 = \sum_i \Omega_i - \frac{k}{a^2 H^2} \equiv \Omega_{\mathrm{tot}} - \frac{k}{a^2 H^2}\,, \tag{2.12}$$

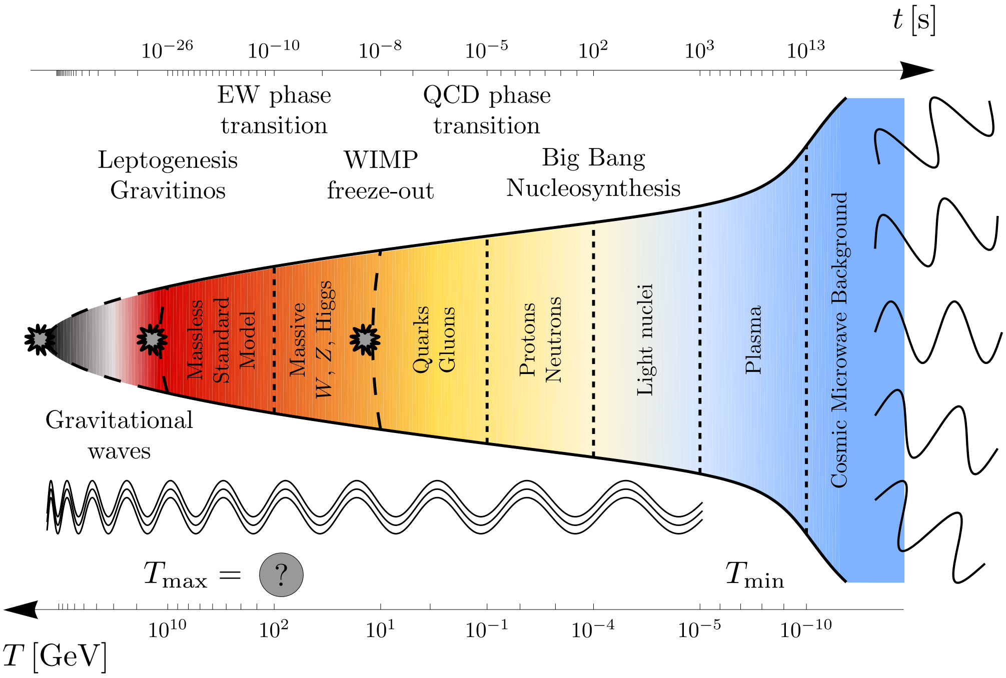


Figure 2.1: Timeline of the thermal early universe starting at the end of inflation. Various stages in the evolution of the Universe are highlighted at the corresponding physical scales. Note that these span more than 20 orders of magnitude. In this thesis non-trivial connections between most of these scales will be identified. This figure might provide orientation. Courtesy of Kai Schmitz.

where we have defined the total energy density parameter Ω_{tot} as the sum of the components Ω_i. The acceleration equation (2.4) using (2.6) rewritten in Ω_i reads

$$\frac{\ddot{a}}{aH^2} = -\frac{1}{2}\sum_i \Omega_i(1+3w_i) \equiv -\frac{1}{2}\Omega_{\text{tot}}(1+3w_{\text{eff}}) \,, \tag{2.13}$$

where we have defined an effective $w = w_{\text{eff}} = \sum_i w_i\Omega_i/\Omega_{\text{tot}}$.

The thermal early universe is filled with relativistic particles (radiation) in local thermal equilibrium. The radiation energy density is given by

$$\rho_{\text{rad}} = \frac{\pi^2}{30}\, g_*(T)\, T^4 \,, \tag{2.14}$$

where g_* counts the effective relativistic degrees of freedom in the Universe. Energies are often given according to the temperature T of this thermal plasma. Since the Universe expands, the cosmic (plasma) temperature decreases as

$$T = T_0(1+z) \,, \tag{2.15}$$

where T_0 denotes the present day CMB radiation temperature and we assume that no particles drop out of equilibrium, i.e., $g_* =$ constant. Since z is a measure of time, (2.15) shows that the temperature is a measure of time as well. In Fig. 2.1 a timeline of the thermal early universe is shown starting at the end of inflation. It is indicated how the Universe expands with time and thereby cools down.

With the expansion of space the entropy density

$$s \equiv \frac{S}{V} = \frac{2\pi^2}{45} g_{*s} T^3 \,, \tag{2.16}$$

where S denotes entropy per comoving volume and V the physical volume, decreases. At times before electron-positron freeze-out around 0.5 MeV, $g_* = g_{*s}$. Afterwards g_* and the effective number of degrees of freedom determining the entropy density g_{*s} decouple and $g_{*s} = 3.91$ becomes constant. The energy density of some species i is often given as

$$\rho_i = m_i n_i = m_i Y_i s \,, \tag{2.17}$$

where n_i denotes as in the following the number density of a particle i. As long as no entropy is produced, the particle yield $Y \equiv n/s$ stays constant while the Universe expands.

Inflation Standard big bang theory has two major problems. First, cosmic microwave background observations indicate that the Universe was highly isotropic at $z \simeq 1100$. Indeed, the observed CMB sky is many orders of magnitude larger than the causal horizon at that time. If different parts of the CMB sky have been causally disconnected, this isotropy cannot be achieved by physical interactions. Instead, it must be arranged by fine-tuning of initial conditions. Following the standard theory the observed CMB must have consisted of around 10^5 causally disconnected patches, which would require a tremendous amount of fine-tuning. This is known as the horizon problem.

Secondly, there is the flatness problem. By differentiation of (2.12) with respect to the time and using (2.13), we obtain

$$\frac{d(\Omega_{\rm tot} - 1)}{dt} = (1 + 3w_{\rm eff}) H \Omega_{\rm tot} (\Omega_{\rm tot} - 1) \,. \tag{2.18}$$

Since $\Omega_{\rm tot}$ is positive and the Hubble parameter is always positive in an expanding universe, we see that $\Omega_{\rm tot}$ departs from 1 in a universe consisting of matter and radiation unless it is exactly 1 in the beginning. Thus, in order to obtain the present day value of $\Omega_{\rm tot} \simeq 1$ the initial value must be extremely fine-tuned again. Together these are two different problems of fine-tuning that both are resolved by an inflationary phase in the very early universe.

Inflation means exponential expansion driven by $w_{\rm eff} \simeq -1$. This is indicated in Fig. 2.1 at the earliest time. If in the inflationary phase the scale factor grows by a factor of $> e^{60}$, the entire observed universe had been a small causally connected region before the tremendous expansion during inflation. In this way, inflation resolves the horizon problem. From (2.12) we see that $\Omega_{\rm tot} \rightarrow 1$, if the scale factor grows exponentially with time, i.e., $a \propto e^{Ht}$ with constant H. Therefore, the Universe may arrive at $\Omega_{\rm tot} \simeq 1$ regardless of the initial conditions and then stays close to that value until today. In this way, inflation resolves also the flatness problem.

Obviously, at the end of inflation the density of all particles, that have been in the Universe before, is extremely diluted. The inflationary phase can be realised by a scalar field, the so-called inflaton. Then, the decay of the inflaton at the end of inflation transfers its energy into a hot thermal plasma of elementary particles. This process is known as reheating. Thus the initial temperature of the plasma is called reheating temperature. The reheating temperature is unknown and might well be higher than any temperature that will ever be reached experimentally. After reheating the evolution of the Universe is described by standard thermal cosmology.

As we will see in Sec. 3.3, for our purpose we have to assume an inflationary phase also from overclosure considerations. The reheating temperature will play a crucial role. The quantum fluctuations of the inflaton generate a (nearly) scale-invariant spectrum of scalar and tensor fluctuations. The scalar fluctuations are imprinted as temperature fluctuations in the CMB and serve as seeds for the formation of structures like galaxy clusters. In Sec. 4.5 we will use and discuss the detection prospects of generated tensor fluctuations that are also known as the gravitational wave background from inflation.

2.2 Leptogenesis

The observed baryon-to-photon ratio $\eta \equiv \frac{n_\mathrm{b}}{n_\gamma} \simeq \frac{n_\mathrm{b}-n_{\bar{b}}}{n_\gamma} \gg 10^{-20}$ shows that there has been more matter (b) than antimatter ($\bar{b}$) when matter and antimatter particles dropped out of thermal equilibrium in the early universe.

In this thesis we consider baryogenesis via the simplest form of *thermal leptogenesis* [2]. A cosmic lepton asymmetry is generated by CP-violating out-of-equilibrium decays of heavy Majorana neutrinos ν_R^i. Since heavy Majorana neutrinos can explain very light neutrinos via the seesaw mechanism, this is closely related to the observation of non-vanishing neutrino masses in the last years, see Chapter 3. No further ingredients are required. Thus the observation of non-vanishing neutrino masses delivers support for the mechanism of thermal leptogenesis. Non-perturbative sphaleron processes [14, 15] convert the lepton asymmetry into a baryon asymmetry. In the case of hierarchical masses the maximal resulting baryon-to-photon ratio of the Universe can be given as [16, 17]

$$\eta_\mathrm{b}^\mathrm{max} \simeq 9.6 \times 10^{-10}\, \Delta^{-1} \left(\frac{M_{\nu_\mathrm{R}^1}}{2\times 10^9\ \mathrm{GeV}}\right) \left(\frac{m_{\nu_\mathrm{L}^3}}{0.05\ \mathrm{eV}}\right) \left(\frac{\kappa_0}{0.18}\right) \tag{2.19}$$

for the minimal supersymmetric standard model (MSSM) introduced in Sec. 3.1.1 and weak washout. The observed value lies in the range $5.89 \times 10^{-10} < \eta_\mathrm{b}^\mathrm{obs} < 6.49 \times 10^{-10}$ (95% CL) [18]. $M_{\nu_\mathrm{R}^1}$ denotes the lightest Majorana mass of the right-handed neutrinos. The given baryon asymmetry is maximal in the sense that the CP violation in the decays is chosen to be maximal [19]. Since thermal leptogenesis strongly favours hierarchical light neutrino masses [20], the mass $m_{\nu_\mathrm{L}^3}$ of the heaviest left-handed neutrino has to be close to $\sqrt{\Delta m_{31}^2} \simeq 0.050$ eV, using the best-fit value from neutrino data [21] and assuming a normal mass ordering. The efficiency factor κ_0 should be computed case-by-case by solving the relevant Boltzmann equations [22, 23, 24]. For zero initial ν_R^1 abundance in the small $M_{\nu_\mathrm{R}^1}$ regime [16], i.e. for $M_{\nu_\mathrm{R}^1} \lesssim 4 \times 10^{13}$ GeV, the maximal value is $\kappa_0^\mathrm{peak} \simeq 0.18$ [25]. This value is reached for

$$\widetilde{m}_{\nu_L^1} \simeq m_* = \frac{8\pi^2 \sqrt{g_*}}{3\sqrt{10}} \frac{v^2}{M_\mathrm{pl}} \simeq 1.6 \times 10^{-3}\ \mathrm{eV}\,, \tag{2.20}$$

where m_* is known as the equilibrium neutrino mass, $v \simeq 174$ GeV, and $M_\mathrm{pl} \simeq 2.44 \times 10^{18}$ GeV denotes as in the following the reduced Planck mass. The effective neutrino mass

$$\widetilde{m}_{\nu_L^1} = \frac{\left(m_{\rm D}^\dagger m_{\rm D}\right)_{11}}{M_{\nu_{\rm R}^1}} \tag{2.21}$$

equals the mass of the lightest neutrino if the Dirac mass matrix $m_{\rm D}$ is diagonal. Its natural range is $m_{\nu_{\rm L}^1} < \widetilde{m}_{\nu_{\rm L}^1} < m_{\nu_{\rm L}^3}$. The parameter Δ denotes the dilution factor by entropy production after the decay of the right-handed neutrinos. It equals one in standard cosmology, while we will consider the general case $\Delta \geq 1$ later.

There are some uncertainties entering (2.19). Possible spectator field uncertainties [26] and flavour effects [27, 28] are neglected, and the naive sphaleron conversion factor [29, 30] is used. We have assumed the particle content of the MSSM with $g_* = 228.75$ for the number of effective massless degrees of freedom at high temperatures. To be conservative we consider the effects of the MSSM by a factor $2\sqrt{2}$ relative to the SM, which is valid for weak washout [17]. For strong washout this factor reduces to $\sqrt{2}$.

We see from (2.19) that leptogenesis in its minimal version as described above can generate the observed baryon-to-photon ratio of the Universe, because $\eta_{\rm b}^{\rm max}$ can exceed $\eta_{\rm b}^{\rm obs}$. On the other hand, it is clear that there is a lower bound $M_{\nu_{\rm R}^1} \gtrsim 2 \times 10^9$ GeV.

It is especially appealing for the considered neutrino mass range that leptogenesis can emerge as the unique source of the cosmological baryon asymmetry [20]. Washout processes may reduce a pre-existing asymmetry by two to three orders of magnitude for the situation of (2.19). Stronger washout decreases the efficiency factor and thus requires a larger right-handed neutrino mass to keep $\eta_{\rm b}^{\rm max} \geq \eta_{\rm b}^{\rm obs}$. For thermal leptogenesis, the bound on the lightest right-handed neutrino mass can be translated into a lower bound on the reheating temperature after inflation, $T_{\rm R} \gtrsim M_{\nu_{\rm R}^1}$ and thus

$$T_{\rm R} \geq T_{\rm L}^{\rm min} \simeq 2 \times 10^9 \text{ GeV}\,. \tag{2.22}$$

In the strong washout regime, i.e. for $\widetilde{m}_{\nu_L^1} > m_*$, this changes to $T_{\rm R} \gtrsim 0.1\, M_{\nu_{\rm R}^1}$ [25], but we cannot relax the bound on the absolute value of $T_{\rm R}$, since in this case the efficiency factor decreases as well, requiring a larger $M_{\nu_{\rm R}^1}$. The crucial experimental input to determine the minimal temperature (2.22) are future measurements of the light neutrino masses.

To soften the lower bound on the reheating temperature (2.22) more involved variants of thermal leptogenesis have been investigated. It has been found that it is possible to reconcile thermal leptogenesis with a smaller reheating temperature, if there is a resonant enhancement of the generated asymmetry [31, 32, 33], some fine-tuning that violates the naturalness assumptions entering into the lower bound on the reheating temperature [34], or a violation of R-parity [35].

2.2.1 WIMP Freeze-out

At high temperatures ($T \gg m_\chi$) a particle χ in the plasma exists in thermal equilibrium and thus with its equilibrium abundance. The equilibrium abundance is maintained by annihilation of the particle with its antiparticle $\bar{\chi}$ into lighter particles l ($\chi\bar{\chi} \to l\bar{l}$) and vice versa ($l\bar{l} \to \chi\bar{\chi}$). As the universe cools to a temperature less than the mass of the particle ($T < m_\chi$), the equilibrium abundance drops exponentially until the rate Γ for the annihilation reaction $\chi\bar{\chi} \to l\bar{l}$ falls below the expansion rate H, i.e. $\Gamma < H$. At this point, the χs cease to annihilate, since collisions become unlikely. The interactions which have maintained thermal equilibrium "freeze out" and a relic cosmological abundance remains.

This picture is described quantitatively by the Boltzmann equation, that describes the time evolution of the number density $n_\chi(t)$,

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma_A v\rangle[(n_\chi)^2 - (n_\chi^{eq})^2]\,, \tag{2.23}$$

where $\langle\sigma_A v\rangle$ is the thermally averaged total cross section for annihilation of $\chi\bar{\chi}$ into lighter particles times the relative velocity v. Here, n_χ^{eq} denotes the number density of χ in thermal equilibrium. The second term on the left-hand side accounts for the expansion of the universe, while the right-hand side represents number-changing interactions. The first term in brackets of (2.23) accounts for depletion due to annihilation, while the second term stems from creation due to the inverse reaction.

There is no closed-form analytic solution to the Boltzmann equation (2.23). However, computer codes like MicrOMEGAs [36] and DarkSUSY [37] solve it numerically taking into account many different effects. In general, if there are additional particles with a mass within 10% of m_χ that share a quantum number with χ, *coannihilation* will occur. For instance, χ could annihilate readily with such a particle, in which case this reaction could determine the relic abundance. If an annihilation process via a particle A is allowed, it can happen resonantly when the mass of the annihilating particle is $m_\chi \approx m_A/2$. At *resonant annihilation* the particle χ annihilates readily via this channel and thus its relic abundance might be lowered. MicrOMEGAs computes the relic density of the lightest supersymmetric particle in the MSSM. It is conventional to give the mass density $m_\chi n_\chi$ in units of the present day critical density (2.10) as

$$\Omega_\chi h^2 = m_\chi n_\chi / \rho_0 \,. \tag{2.24}$$

A weakly-interacting massive particle (WIMP) freezes out at a temperature $T^{\mathrm{fo}} \simeq m_\chi/25$ as indicated in Fig. 2.1 for a suggestive mass of $\mathcal{O}\,(100 \text{ GeV})$ around the electroweak scale.

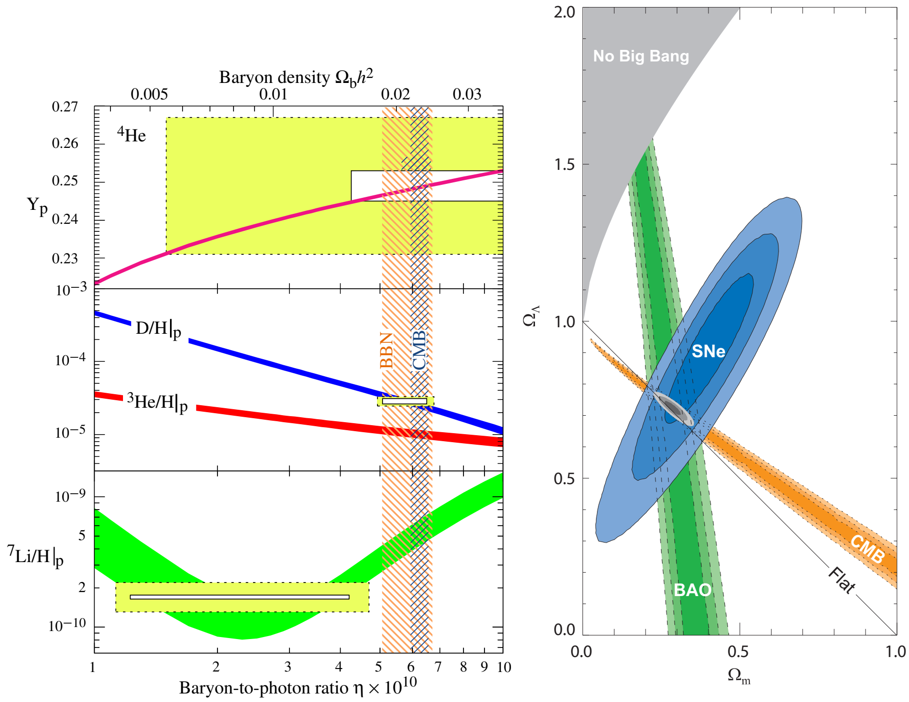
QCD phase transition Around the confinement scale of quantum chromodynamics $\Lambda_{\mathrm{QCD}} \sim 220$ MeV quarks form bound states like neutrons and protons. This is a prerequisite for the subsequent formation of light nuclei. While the transition itself seems to give no contraint on, for example, the equation of state of the Universe at the transition, axion physics, that is bound to the same scale, might do so as discussed below.

2.3 Nucleosynthesis in Cosmology

The main processes responsible for the chemical equilibrium in the thermal plasma between protons p and neutrons n are the weak reactions:

$$n + \nu_e \rightleftarrows p + e^- \,, \qquad n + e^+ \rightleftarrows p + \bar{\nu}_e \,. \tag{2.25}$$

While the universe cools down, at a temperature below a few MeV, which corresponds to an age of the universe around 0.5 s, the neutron-to-proton ratio n_n/n_p freezes out. At these comparatively low temperatures, the universe obeys well understood Standard Model physics. The weak interactions become inefficient to maintain the equilibrium, which leads to a neutron-to-proton ratio of $n_n/n_p \approx 1/6$. Due to neutron decay, $n \to pe^-\bar{\nu}_e$, this ratio further decreases to about 1/7 before neutrons are stabilised in bound states, e.g. deuterium. Regardless of the exact process, nearly all neutrons fuse to helium ^{4}He. Therefore, we can estimate the relative abundance by weight $Y_p = \rho_{^4\mathrm{He}}/(\rho_n + \rho_p)$ of ^{4}He. With n_N denoting the



(a) BBN predictions (b) Determination of Ω_Λ and Ω_m

Figure 2.2: (a): The abundances of ^{4}He, D, ^{3}He, and ^{7}Li as predicted by the standard model of big bang nucleosynthesis. Bands show the 95% CL range. Boxes indicate the observed light element abundances (smaller boxes: $\pm 2\sigma$ statistical errors; larger boxes: $\pm 2\sigma$ statistical and systematic errors). The narrow vertical band indicates the CMB measure of the cosmic baryon density, while the wider band indicates the BBN ΛCDM range (both at 95% CL). Taken from [13].**(b):** Shown is the preferred region in the $\Omega_m - \Omega_\Lambda$ plane from the compilation of observations from the CMB, supernovae and baryonic acoustic oscillations. Taken from [38].

total number of nucleons, it is

$$Y_p \approx X_{^4\text{He}} := \frac{4n_{^4\text{He}}}{n_N} = \frac{2n_n}{n_p + n_n} = \frac{2n_n/n_p}{1 + n_n/n_p} \approx 25\% \,. \tag{2.26}$$

This and other processes have a strong sensitivity to the baryon-to-photon ratio η_b [13]. For instance, deuterium is destroyed by photons with an energy larger than the binding energy of deuterium, i.e. $\gamma + D \to n + p$. Anyway, a few minutes after the big bang, nuclear interactions become effective in building light elements. This procedure lasting roughly 20 minutes is known as big bang nucleosynthesis (BBN) or primordial nucleosynthesis. The computation of light element abundances like deuterium D, helium ^{3}He or ^{4}He and lithium ^{7}Li involves all the details of nuclear interactions. There are computer codes [39] predicting the abundances in good agreement with data from astrophysical observations, see Figure 2.2. In doing so, BBN predicts a present day baryon abundance,

$$0.018 < \Omega_b h^2 < 0.023\,. \tag{2.27}$$

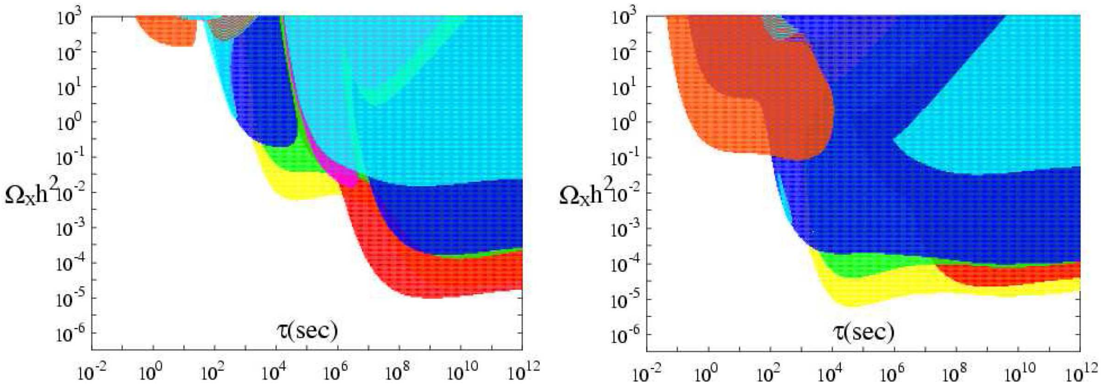
We point out that BBN gives us the deepest insight into the early universe. We gain testable information about physics happening just a second after the big bang. If we want to maintain the success of BBN, we obtain constraints on physics beyond the Standard Model.

Big bang nucleosynthesis constrains, in particular, the relic energy density of late-decaying massive particles. If their energy density dominates the energy density of the Universe at their decay, significant entropy is produced, when the decay products thermalise. Supergravity theories include many fields with only gravitational interactions. Typical entropy producing particles may reside in such a "hidden sector", cf. Sec. 3.2. Investigations of the thermalisation of neutrinos produced in decays or subsequent thermalisation processes lead to lower limits on the temperature of the Universe after the entropy production T^{after} [40]. Neutrinos, which can thermalise through weak interactions only, are most important. All other SM particles thermalise much faster due to their stronger interactions. The bounds found are in the range

$$T_\phi^{\mathrm{dec}} > T_{\mathrm{min}}^{\mathrm{after}} \simeq (0.7 \ldots 4)\ \mathrm{MeV} \tag{2.28}$$

where weaker bounds come from BBN calculations [41, 42] and stronger bounds rely on the neutrino energy density [43, 44] exploiting overall best fits for cosmological parameters. Note that $T = 4$ MeV corresponds to a cosmic time as early as $t_{\mathrm{BBN}} \sim 0.05$ s.

Even if its energy density is much smaller and the late-decaying particle never dominates, its energy density is constrained by BBN. If a WIMP χ with relic density Ω_χ given by (2.24) has to decay into, for instance, the gravitino, its decay occurs late, cf. Sec. 3.3. If the lifetime of χ is longer than ~ 0.1 s, its decay may cause non-thermal nuclear reactions during or after BBN, altering the predictions of the standard BBN scenario. The constraints differ for radiative and hadronic decays, since these decay processes cause different types of reactions. One simple



(a) $B^{\chi}_{had} = 10^{-3}$ (b) $B^{\chi}_{had} = 1$

Figure 2.3: (b): BBN constraints on the abundance of relic decaying neutral particles $\Omega_\chi h^2$ (if they would not have decayed) as a function of their lifetime for a $M_\chi = 1$ TeV particle with hadronic branching ratio $B_{had} = 1$. The coloured regions are excluded and correspond to the constraints imposed by the observationally inferred upper limit on ^{4}He –orange–, upper limit on ^{2}H –blue–, upper limit on ^{3}He/^{2}H –red–, and lower limit on ^{7}Li –light blue–. Constraints derived from ^{6}Li/^{7}Li are shown by the green region. The region indicated by yellow violates the less conservative ^{6}Li/^{7}Li constraint but should not be considered ruled out. **(a):** The same as right but for hadronic branching ratio $B_{had} = 10^{-3}$. The region excluded by the lower limit on ^{2}H/H is indicated by the colour magenta. Figures are taken from [39].

example is the destruction of a previously built light element by an energetic photon. Examples for this photo dissociation are

$$\gamma + D \rightarrow n + p\ , \quad \gamma + {}^3He \rightarrow p + D \quad \text{or} \quad \gamma + {}^4He \rightarrow n + {}^3He\,. \tag{2.29}$$

For sure, also hadronic decays lead to distortions. For instance, emitted hadrons could lose energy during scattering processes by the emission of photons and thus induce photo dissociation as well. But there are also many other processes. To name another example, antinucleons released in hadronic decays tend to increase the neutron-to-proton n_n/n_p ratio as they are more likely to annihilate with protons. A BBN calculation with decaying particles requires the detailed study of the thermalisation of the decay products in the plasma. Actual calculations include this and consider many processes of photo dissociation and the impact of hadronic decay products [39, 45, 46, 47]. Maintaining the predictions of standard big bang nucleosynthesis, they offer constraints on the relic abundance of decaying relic particles $\Omega_\chi h^2$ as a function of their lifetime τ. Such constraints are shown in Figure 2.3. As we can see, the physics of BBN varies with time and different abundances and relative abundances lead to constraints at different times. Therefore, the upper bounds on $\Omega_\chi h^2$ in Figure 2.3 are given as a function of the lifetime of χ.

2.3.1 Radiation Energy Density during BBN

During BBN the energy density of the Universe has to be dominated by radiation. Actually, the process is so well understood that the expansion rate H is tightly constrained and so is the radiation energy density during BBN. Bounds on the radiation energy density are usually given in terms of the effective number of neutrino species N_{eff} defined by

$$\rho_{\text{rad}} = \left(1 + N_{\text{eff}} \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4\right)\rho_\gamma \,, \tag{2.30}$$

where the radiation energy density ρ_{rad} (2.14) after electron-positron freeze-out is given as a sum of the energy density in photons $\rho_\gamma = (\pi^2/15)T^4$, the energy density in the neutrinos of the standard model with $N_{\text{eff}}^{\text{SM}} = 3.046$ [48] and $T_\nu/T_\gamma = (4/11)^{1/3}$ and any departure from the standard scenario parametrised as $N_{\text{eff}} = N_{\text{eff}}^{\text{SM}} + \Delta N_{\text{eff}}$. Since the radiation energy density affects the expansion rate of the Universe, ΔN_{eff} can be constrained.

Big bang nucleosynthesis provides a constraint on N_{eff} consistent with the standard model [49]

$$N_{\text{eff}}^{\text{BBN}} = 2.4 \pm 0.4 \quad (68\%\ \text{CL}) \tag{2.31}$$

and thus severe for any new physics. This constraint seems to be consistent with earlier studies and will be used later. A recent study [50] suggests an increased N_{eff} at BBN

$$N_{\text{eff}}^{\text{BBN}} = 3.8 \pm 0.35 \quad (68\%\ \text{CL})\,. \tag{2.32}$$

The large central value arises partly from an adopted shorter neutron lifetime. Furthermore, the uncertainty is found to be essentially larger in an independent, simultaneous study [51].

However, these uncertainties open up the possibility of an increased N_{eff} during BBN, i.e., $\Delta N_{\text{eff}}^{\text{BBN}} > 0$. This is typically achieved by stable additional degrees of freedom that have once been in thermal equilibrium giving rise to $\Delta N_{\text{eff}} = 1$ or the emergence of "dark radiation" from particle decays before BBN. We elaborate on the second possibility in Sec. 5.1.

2.4 Cosmic Microwave Background

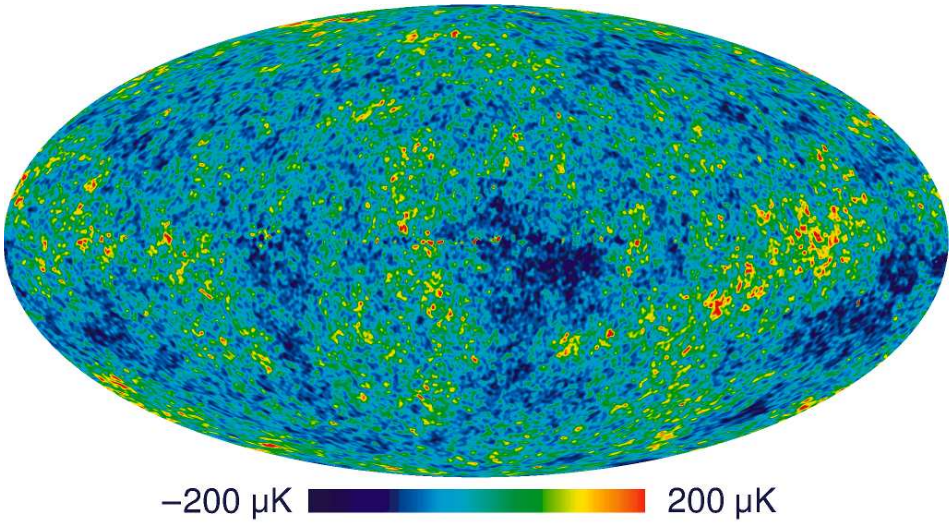


Figure 2.4: CMB temperature fluctuations from the 7-year WMAP data seen over the full sky. The colours (red/blue) represent temperature fluctuations of about $\pm 0.0002°$. Image from http://map.gfsc.nasa.gov/.

As the Universe cools down at $z \simeq 1100$, which corresponds to a time of $\sim 10^{12}$ s after the big bang, the temperature of the photons drops below the energy to ionise hydrogen, i.e., $T \lesssim 0.25$ eV. Around this time nearly all free electrons and protons recombine and form neutral hydrogen H. At the time of *recombination*, the Universe becomes transparent to the photon background radiation. The present-day cosmic microwave background is the redshifted relic of this radiation. The Cosmic Background Explorer (COBE) satellite mission found that the CMB spectrum corresponds to an almost perfect black body with a temperature of $T_0 \simeq 2.7$ K $\simeq 2.3 \times 10^{-4}$ eV [13]. This is what we expect from standard thermal big bang theory, since a black body by definition emits the spectrum that would be present in an environment in thermal equilibrium. Furthermore, COBE found the CMB highly isotropic, i.e., temperature anisotropies are of $\mathcal{O}(10^{-5})$.

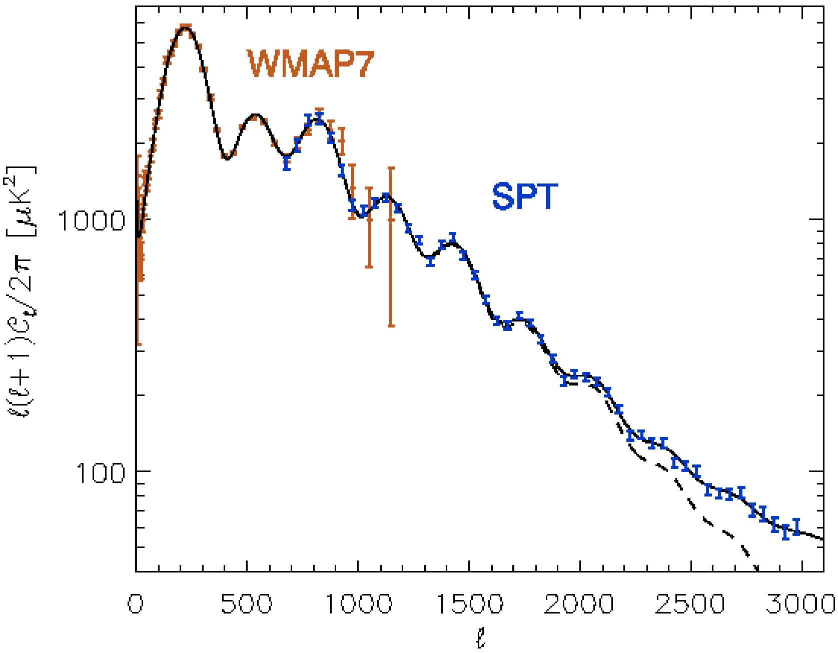


Figure 2.5: CMB power spectrum as measured from 7-year WMAP data and the South Pole Telescope (SPT) [52]. Figure taken from [53].

Temperature fluctuations are induced by the slightly inhomogeneous matter distribution at recombination. Figure 2.4 shows the microwave sky as observed by the Wilkinson Microwave Anisotropy Probe (WMAP) [18]. WMAP investigated the temperature anisotropies found by COBE in detail. The expansion of the temperature anisotropies $\delta T/T$ of Figure 2.4 in spherical harmonics Y_{lm},

$$\frac{\delta T}{T}(\Theta, \Phi) = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\Theta, \Phi) \,, \tag{2.33}$$

yields the CMB power spectrum $l(l+1)C_l/2\pi$ (see Figure 2.5) in terms of the multipole moment l (roughly corresponding to an angular size $\Theta \sim 180°/l$) with

$$C_l \equiv \langle |a_{lm}|^2 \rangle = \frac{1}{2l+1} \sum_{m=-l}^{l} |a_{lm}|^2 \,. \tag{2.34}$$

The analysis of the power spectrum provides particular information on the Universe at the time of recombination. For instance, the position of the first peak implies that the universe is spatially flat, i.e., $\Omega_{\text{tot}} \simeq 1$. As one can see in Figure 2.2 the analysis of the CMB power spectrum implies also $\Omega_\Lambda \simeq 0.75$ and $\Omega_m \simeq 0.25$.

The amount of baryonic matter can be inferred from the difference in magnitude between the first and second peak, whereas the difference in magnitude between the second and third peak gives the amount of dark matter. This information is extracted by fits of cosmological models. As can be seen in Figure 2.2, the ΛCDM model, which assumes a cosmological constant Λ and cold dark matter (CDM $\equiv$ particles that are non-relativistic before structure formation), fits successfully.

Late particle decays are also constrained by the CMB. They can lead to unacceptable spectral distortions. However, for most cases, especially those considered in this thesis, these constraints have been found to be less constraining than bounds by big bang nucleosynthesis [54, 55, 56].

Chapter 3

Cosmological Particle Content

In this chapter we introduce briefly the elementary particle content under consideration in this thesis.[1] Emphasis is put on the cosmology of the gravitino and the supersymmetrised axion, where we relate our work to the current state of research.

See-Saw Mechanism The existence of neutrino oscillations is observationally established [60, 61, 62, 63, 64].[2] This requires the neutrinos to have non-vanishing masses, which is not possible within the Standard Model. An extension of the Standard Model by right-handed neutrinos in order to accommodate neutrino masses is straightforward. If neutrinos are Dirac or Majorana particles is unknown. However, for Majorana neutrinos the tiny masses of the observed neutrinos can be elegantly explained by the see-saw mechanism [66, 67, 68, 69, 70]. Therefore, we extend the particle content of the Standard Model by three right-handed Majorana neutrinos with a large mass term. The relevant implications for cosmology have been outlined in Sec. 2.2.

3.1 Local Supersymmetry

Supersymmetry (SUSY) is a space-time symmetry, i.e., an extension of the symmetries of translations, rotations and boosts. SUSY turns bosonic states into

[1] Useful reviews on supersymmetry and supergravity can be found in [57, 58, 59].

[2] For a recent review of neutrino physics see [65].

fermionic states, and vice versa. The operator Q_α that generates such transformations is an anticommuting spinor with

$$Q_\alpha|\text{Boson}\rangle \simeq |\text{Fermion}\rangle\,, \qquad Q_\alpha|\text{Fermion}\rangle \simeq |\text{Boson}\rangle\,. \tag{3.1}$$

Translations, rotations and boosts together form the Poincaré symmetry. Under general assumptions including the existence of a mass gap the Poincaré symmetry is the maximal space-time symmetry of identical particles [71], i.e., leaving the particle spin unchanged. SUSY, in turn, is the maximal possible extension of Poincaré symmetry [72]. Particles are transformed into particles with spin differing by 1/2. If SUSY is discovered, all mathematically consistent space-time symmetries will have been realised in Nature.

Supersymmetry might explain the hierarchy of the gauge couplings by the unification of all gauge interactions at some higher scale. More importantly, there are many reasons to expect SUSY to be accessible to the LHC [73]: It could have been seen already in electroweak precision observables. It stabilises the mass-squared of the Higgs boson to a light value. In case of such a light Higgs boson it stabilises the electroweak vacuum. Various dark matter candidates are offered. Actually, we elaborate on supersymmetric dark matter.

In general relativity, which is the current theory of gravity, space-time symmetries are local, i.e., the transformation parameters ξ depend on the space-time coordinate, $\xi \to \xi(x)$. Local supersymmetry combines the principles of supersymmetry and general relativity. Therefore, it is referred to as supergravity (SUGRA). Defining the parameter of SUSY to be space-time dependent corresponds to gauging supersymmetry. Since the gravitino is the gauge field of local supersymmetry transformations, it is a unique and inevitable prediction of local supersymmetry. For a field-theoretical treatment of the massive gravitino and its interactions we refer to [11, 74, 75, 76].

Supergravity extends the field content of any theory unambiguously by the gravity supermultiplet (Table 3.1). In some sense the gravity multiplet is the minimal field content of any SUGRA model. It consists of the spin-2 graviton, that mediates gravity, and the spin-3/2 gravitino. Both are neutral with respect to the SM gauge group (3.2).

SUGRA is non-renormalisable. In short, quantum corrections diverge and for SUGRA it is not known how to deal with them. To this day there is no con-

Name	Bosons	Fermions	$(\mathrm{SU}(3)_\mathrm{C}, \mathrm{SU}(2)_\mathrm{L})_Y$
Graviton, gravitino	$g_{\mu\nu}$	$\Psi_{3/2}$	$(\mathbf{1}, \mathbf{1})_0$

Table 3.1: Gravity supermultiplet

sistent quantum theory of gravity. We assume that SUGRA is an appropriate low-energy approximation of a more general theory. Actually, supersymmetry is an essential ingredient in string theory, the most-developed candidate for unifying all particle interactions including gravity. As effective theory the requirement of renormalisability disappears (cf. Fermi theory of weak interaction).

3.1.1 The Minimal Supersymmetric Standard Model

The Minimal Supersymmetric Standard Model (MSSM) is the minimal phenomenologically viable supersymmetric extension of the Standard Model of particle physics (SM). Due to supersymmetry there is a supersymmetric partner particle to each SM fermion and SM gauge boson. Global SUSY is assumed. So to begin with the MSSM does not contain gravity. The spin-1/2 fermions, i.e. three families of quarks and leptons, reside in chiral supermultiplets with their spin-0 scalar boson partners. The spin-1 gauge bosons, i.e. gluons, W and B bosons, reside in gauge supermultiplets with their spin-1/2 fermionic partners. There are two chiral Higgs supermultiplets. Each consists of one spin-0 Higgs SU(2)-doublet and its spin-1/2 fermionic superpartner. The chiral supermultiplets in Table 3.2 and the gauge supermultiplets in Table 3.3 make up the field content of the MSSM. They are summarised according to their transformation properties under the Standard Model gauge group,

$$G_\mathrm{SM} = G_\mathrm{MSSM} = \prod_{\alpha=1}^{3} G_\alpha = \mathrm{U}(1)_\mathrm{Y} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{C}\,. \tag{3.2}$$

Particles in the same supermultiplet transform in identical gauge group representations. Superpartners are denoted by a tilde. See, for instance, the gauginos $\lambda^{(\alpha)a}$ in Table 3.3. They are the fermionic partners of the SM gauge bosons $A_\mu^{(\alpha)a}$, where a labels the gauge group generators.

Name	Scalar Bosons ϕ^i	Fermions χ^i_L	$\left(\mathrm{SU}(3)_\mathrm{C}, \mathrm{SU}(2)_\mathrm{L}\right)_Y$
Sleptons, leptons $I = 1, 2, 3$	$\widetilde{L}^I = \begin{pmatrix} \widetilde{\nu}^I_L \\ \widetilde{e}^{-\,I}_L \end{pmatrix}$	$L^I = \begin{pmatrix} \nu^I_L \\ e^{-\,I}_L \end{pmatrix}$	$(\,\mathbf{1}\,,\mathbf{2}\,)_{-1}$
	$\widetilde{E}^{*I} = \widetilde{e}^{-\,*\,I}_R$	$E^{\dagger\,I} = e^{-\,\dagger\,I}_R$	$(\,\mathbf{1}\,,\mathbf{1}\,)_{+2}$
Squarks, quarks $I = 1, 2, 3$ ($\times$ 3 colours)	$\widetilde{Q}^I = \begin{pmatrix} \widetilde{u}^I_L \\ \widetilde{d}^I_L \end{pmatrix}$	$Q^I = \begin{pmatrix} u^I_L \\ d^I_L \end{pmatrix}$	$(\,\mathbf{3}\,,\mathbf{2}\,)_{+\frac{1}{3}}$
	$\widetilde{U}^{*I} = \widetilde{u}^{*I}_R$	$U^{\dagger\,I} = u^{\dagger\,I}_R$	$(\,\overline{\mathbf{3}}\,,\mathbf{1}\,)_{-\frac{4}{3}}$
	$\widetilde{D}^{*I} = \widetilde{d}^{*I}_R$	$D^{\dagger\,I} = d^{\dagger\,I}_R$	$(\,\overline{\mathbf{3}}\,,\mathbf{1}\,)_{+\frac{2}{3}}$
Higgs, Higgsinos	$H_d = \begin{pmatrix} H^0_d \\ H^-_d \end{pmatrix}$	$\widetilde{H}_d = \begin{pmatrix} \widetilde{H}^0_d \\ \widetilde{H}^-_d \end{pmatrix}$	$(\,\mathbf{1}\,,\mathbf{2}\,)_{-1}$
	$H_u = \begin{pmatrix} H^+_u \\ H^0_u \end{pmatrix}$	$\widetilde{H}_u = \begin{pmatrix} \widetilde{H}^+_u \\ \widetilde{H}^0_u \end{pmatrix}$	$(\,\mathbf{1}\,,\mathbf{2}\,)_{+1}$

Table 3.2: Matter fields of the MSSM

All matter fields of Table 3.2 are written in terms of left-handed Weyl spinors χ_L since they stem from left-chiral supermultiplets. Therefore, we enter the left-handed hermitian conjugate $(\cdot)^\dagger$ of a right-handed field instead of the field itself. Strongly interacting particles reside in colour triplets $\mathbf{3}$, i.e., quarks and squarks. Colour indices are not written out. In contrast, $\mathrm{SU}(2)_\mathrm{L}$ doublets $\mathbf{2}\,(\mathbf{2} = \overline{\mathbf{2}})$ are explicitly given in Table 3.2. Gauge singlets are denoted by $\mathbf{1}$ or in the case of $U(1)_Y$ carry hypercharge $Y = 0$. Note that the normalisation of the hypercharges is such that the electric charge Q is given by $Q = T_3 + Y/2$, where T_3 denotes the weak isospin eigenvalue $\pm 1/2$ for upper/lower entries in the $\mathrm{SU}(2)_\mathrm{L}$ doublets and accordingly $T_3 = 0$ for $SU(2)_L$ singlets. The SM contains three families of quarks and leptons, so the family index I in Table 3.2 counts $I = 1, 2, 3$. It is understood that there is an antiparticle for each particle of Table 3.2.

The Lagrangian of the MSSM is determined by (3.7) and the superpotential,

$$W_\mathrm{MSSM} = \widetilde{U}^*\,\mathbf{y}_u \widetilde{Q}\cdot H_u - \widetilde{D}^*\,\mathbf{y}_d \widetilde{Q}\cdot H_d - \widetilde{E}^*\,\mathbf{y}_e \widetilde{L}\cdot H_d + \mu H_u \cdot H_d\,, \tag{3.3}$$

which is a holomorphic function of the scalars of the supermultiplets. The dou-

Name	Gauge bosons $A_\mu^{(\alpha)\,a}$	Gauginos $\lambda^{(\alpha)\,a}$	$\big(\mathrm{SU}(3)_\mathrm{C}, \mathrm{SU}(2)_\mathrm{L}\big)_Y$
B-boson, bino	$A_\mu^{(1)\,a} = B_\mu\,\delta^{a1}$	$\lambda^{(1)\,a} = \widetilde{B}\,\delta^{a1}$	$(\mathbf{1}\,,\mathbf{1}\,)_0$
W-bosons, winos	$A_\mu^{(2)\,a} = W_\mu^a$	$\lambda^{(2)\,a} = \widetilde{W}^a$	$(\mathbf{1}\,,\mathbf{3}\,)_0$
gluon, gluino	$A_\mu^{(3)\,a} = G_\mu^a$	$\lambda^{(3)\,a} = \widetilde{g}^a$	$(\mathbf{8}\,,\mathbf{1}\,)_0$

Table 3.3: Gauge fields of the MSSM

blet structure is tied together as $\widetilde{Q}\cdot H_u = \varepsilon^{ij}\widetilde{Q}_i H_{u\,j}$, with ε^{ij} as understood. Furthermore, $\widetilde{U}^*\,\mathbf{y}_u\widetilde{Q}$ is meant to be a matrix multiplication in family space, $\widetilde{U}^*\,\mathbf{y}_u\widetilde{Q} = \widetilde{U}^{*\,I}\,y_u^{IJ}\widetilde{Q}^J$.

R-parity Accidentally, the Standard Model conserves baryon number B and lepton number L. So the proton as lightest baryon with $B = 1$ is absolutely stable, since there is no baryon to decay into and decays into something else would violate baryon number conservation. W_MSSM is the most general superpotential that respects gauge invariance and all SM conservation laws. The MSSM contains by definition only renormalisable interactions and only terms that respect baryon and lepton number conservation. But the most general renormalisable superpotential which respects gauge invariance contains also B and L violating terms,

$$W_{\not P_R} = \mu_i H_u\cdot\widetilde{L} + \frac{1}{2}\lambda_{ijk}\widetilde{L}_i\cdot\widetilde{L}_j\widetilde{E}^*_k + \lambda'_{ijk}\widetilde{L}_i\cdot\widetilde{Q}_j\widetilde{D}^*_k + \frac{1}{2}\lambda''_{ijk}\epsilon^{abc}\widetilde{U}^*_{i\,a}\widetilde{D}^*_{j\,b}\widetilde{D}^*_{k\,c}\,, \tag{3.4}$$

where in the last summand $a, b, c = 1, 2, 3$ are $\mathrm{SU}(3)_\mathrm{C}$ indices. Some of these terms lead to proton decay, whereas no proton decay has been observed.

Usually, the presence of all baryon and lepton number violating terms is forbidden by requiring the conservation of an additional global symmetry, so called R-parity, defined for each particle as

$$P_R = (-1)^{3(B-L)+2s}\,, \tag{3.5}$$

where s denotes the spin of the particle. In the MSSM with only renormalisable interactions and conserved R-parity the proton is absolutely stable.

From (3.5) it is easy to check that Standard Model particles and the Higgs bosons have even R-parity ($P_R = +1$), while all supersymmetric partners have odd R-parity ($P_R = -1$). This implies that sparticles must be produced in pairs, that heavier sparticles must decay into lighter ones and that the lightest supersymmetric particle (LSP) must be absolutely stable, since it has no allowed decay mode. Thus the LSP becomes a "natural" candidate for particle dark matter.

Again, the gauge group (3.2), the field content summarised in Tables 3.2, 3.3, the superpotential (3.3) and the soft breaking terms (3.7) define the Minimal Supersymmetric Standard Model. The whole MSSM Lagrangian can, for instance, be found in [77].

Throughout this thesis we assume for simplicity the particle content of the MSSM when we determine, for example, the effective relativistic degrees of freedom (2.14) with $g_*^{\mathrm{MSSM}} = 228.75$ at high temperatures in the early universe. The numerical changes from adding, for instance, the axion multiplet are tiny as we can see in all estimates.

3.2 Symmetry breaking

Tables 3.3, 3.2 and 3.1 list gauge eigenstates. In general, gauge eigenstates differ from physical mass eigenstates. Gauge eigenstates can mix, so that the mass eigenstates become linear combinations of gauge eigenstates. Equally, one can give the gauge eigenstates as linear combinations of mass eigenstates.

The mass operator P^2 commutes with the SUSY operators,

$$[P^2, Q_\alpha] = [P^2, \bar{Q}_{\dot{\alpha}}] = 0\,. \tag{3.6}$$

Therefore, particles within the same supermultiplet are degenerate in mass. Since no sparticles have been observed yet, supersymmetry has to be a broken symmetry, if realised in Nature.

In the MSSM, supersymmetry breaking is parametrised by additional terms in the

Lagrangian,

$$\begin{aligned}\mathcal{L}_{\text{soft}}^{MSSM} &= -\frac{1}{2}(M_3\widetilde{g}^a\widetilde{g}^a + M_2\widetilde{W}^a\widetilde{W}^a + M_1\widetilde{B}\widetilde{B} + h.c.)\\ &-(\widetilde{U}^*\,\mathbf{a_u}\widetilde{Q}\cdot H_u - \widetilde{D}^*\,\mathbf{a_d}\widetilde{Q}\cdot H_d - \widetilde{E}^*\,\mathbf{a_e}\widetilde{L}\cdot H_d + h.c.)\\ &-\widetilde{Q}^\dagger\,\mathbf{m_Q^2}\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m_L^2}\widetilde{L} - \widetilde{U}^\dagger\,\mathbf{m_U^2}\widetilde{U} - \widetilde{D}^\dagger\,\mathbf{m_D^2}\widetilde{D} - \widetilde{E}^\dagger\,\mathbf{m_E^2}\widetilde{E}\\ &-m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (bH_uH_d + h.c.)\,. \end{aligned} \tag{3.7}$$

In (3.7), M_3, M_2 and M_1 are gluino, wino and bino mass terms. Here, we suppress all gauge indices. In the second line each of $\mathbf{a_u}$, $\mathbf{a_d}$, $\mathbf{a_e}$ is a complex 3×3 matrix in family space. They are in direct correspondence with the Yukawa couplings of W_{MSSM} (3.3). The third line of (3.7) consists of squark an slepton mass terms. Each of $\mathbf{m_Q^2}$, $\mathbf{m_L^2}$, $\mathbf{m_U^2}$, $\mathbf{m_D^2}$, $\mathbf{m_E^2}$ is a hermitian 3×3 matrix in family space. In the last line we have SUSY breaking contributions to the Higgs potential.

The supersymmetry-breaking couplings in $\mathcal{L}_{\text{soft}}^{\text{MSSM}}$ are soft (of positive mass dimension) in order to naturally maintain a hierarchy between the electroweak scale and the Planck scale, i.e., in order not to reintroduce quadratically divergent contributions to scalar masses. Eq. (3.7) is the most general renormalisable soft SUSY breaking Lagrangian that respects gauge invariance and R-parity in the MSSM.

In a viable model soft mass terms make sparticles heavy enough to be in accord with experiment. In principle, due to off-diagonal entries $*$ in the family matrices of $\mathcal{L}_{\text{soft}}^{\text{MSSM}}$,

$$\mathbf{a_u},\ldots,\mathbf{m_E^2} \sim \begin{pmatrix} \cdot & * & * \\ * & \cdot & * \\ * & * & \cdot \end{pmatrix}\,, \tag{3.8}$$

any scalars with the same charges and R-parity can mix with each other. For the squarks and sleptons of the MSSM large mixings are forbidden as they would lead to flavour-changing neutral currents, that have not been observed. The general hypothesis of *flavour-blind soft parameters* predicts that most of these mixings are very small. Since these would have even smaller impact on our results, as an appropriate simplification, we take these mixings to be zero. So all family matrices of $\mathcal{L}_{\text{soft}}^{\text{MSSM}}$ are assumed to be diagonal, i.e.

$$\mathbf{a_u} \propto \ldots \propto \mathbf{m_E^2} \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\,. \tag{3.9}$$

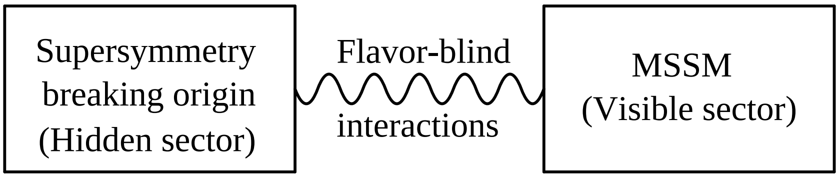


Figure 3.1: The presumed schematic structure for supersymmetry breaking. Figure taken from [57].

However, the soft parameters cannot be arbitrary.

In order to understand how patterns like (3.9) can emerge, it is necessary to consider models of SUSY breaking. The condition for the *spontaneous breakdown* of a symmetry is a ground state that (in contrast to the underlying Lagrangian) does not respect this symmetry. Typically, scalars, i.e. the auxiliary fields of some chiral supermultiplets (*F-term breaking*), acquire non-zero vacuum expectation values $\langle F \rangle$ and thus break SUSY. In order not to break conserved gauge symmetries, these scalars are neutral under those. SUSY breaking is expected to occur in a "hidden sector" of particles without direct couplings to the "visible sector", consisting of the supermultiplets of the MSSM. However, the two sectors have some common interactions that mediate SUSY breaking from the hidden sector to the visible sector, see Figure 3.1, which results in the MSSM soft terms (3.7). Although SUSY breaking is strongly constrained, it is not known how it should be done and there are many different SUSY breaking scenarios. In order to be independent of a particular SUSY breaking model, we do not consider any particular pattern. However, we would like to stress that SUSY breaking is unavoidably mediated by gravity (*gravity mediation*). If supersymmetry is broken in the hidden sector by a VEV $\langle F \rangle$, then the order of the gravity-mediated soft terms m_{soft} should be by dimensional analysis

$$m_{\text{soft}} \sim \langle F \rangle / M_{\text{pl}} \,. \tag{3.10}$$

In supergravity, an analogy to the Higgs mechanism of electroweak-symmetry breaking exists, i.e., the *super Higgs mechanism*. Spontaneous breaking always implies a Goldstone particle with the same quantum numbers as the broken symmetry generator. Since Q_α is fermionic, in the case of SUSY breaking the Goldstone particle ought to be a fermion. This goldstino is absorbed by the gravitino, which acquires thereby its $\pm 1/2$ spin components and a mass

$$m_{3/2} \sim \langle F \rangle / M_{\mathrm{pl}} \,. \tag{3.11}$$

Without the assumption of space-time dimensions beyond the known four, (3.10) cannot be suppressed with respect to (3.11). Thus we see that the gravitino mass $m_{3/2}$ is comparable to the masses of the MSSM sparticles,

$$m_{3/2} \sim m_{\mathrm{soft}} \sim \langle F \rangle / M_{\mathrm{pl}} \sim \mathcal{O}\left(100 \text{ GeV}\right) \,, \tag{3.12}$$

if there is no additional mediation mechanism. Therefore, the gravitino can be the LSP with $m_{3/2}$ expected to be at least of $\mathcal{O}\left(100 \text{ GeV}\right)$. Models of SUSY breaking depending on the mediation mechanism provide boundary conditions for the soft terms at some high energy. The low energy MSSM masses can be obtained by solving renormalisation group equations. In a given theory renormalisation group equations describe the evolution of physical quantities like masses and couplings with the energy scale.

If SUSY breaking is mediated by gauge interactions of an additional intermediate scale particle sector (*gauge mediation* [78]), the gravity mediated contribution to the soft terms can be subleading. Then the gravitino is always the LSP typically with a mass $m_{3/2} \ll m_{\mathrm{soft}}$. Alternatively, the mediation strength of gravity can be suppressed, if the SUSY breaking sector is spatially separated from the visible sector in some additional hidden space dimension beyond the known three (*anomaly mediation* [79, 80]). Then the gravitino might be heavier than the MSSM sparticles, for instance, with a mass gap of one or two orders of magnitude.

It is understood that the electroweak symmetry is broken down to electromagnetism,

$$\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}} \to \mathrm{U}(1)_{\mathrm{em}} \,. \tag{3.13}$$

In the MSSM this can be achieved dynamically after supersymmetry breaking by radiative corrections to the soft Higgs masses m_{H_u}, m_{H_d}. One of the Higgs masses squared evolves to a negative value at the electroweak scale and thus breaks electroweak symmetry, because the neutral Higgs fields H_u^0 and H_d^0 acquire non-zero vacuum expectation values that break the electroweak symmetry.[3]

Table 3.4 comprises the physical particles as used in this thesis. Spin and R-parity values are provided. Thus it is easy to identify scalars (spin$=0$), fermions

[3] Electroweak symmetry breaking is sketched by the author in [11].

(spin= $1/2, 3/2$) and gauge bosons (spin= 1). And it is easy to see whether a particle is a Standard Model field or Higgs boson, since they have $P_R = +1$, in contrast to the sparticles with $P_R = -1$. We list explicitly gauge and mass eigenstates. In the case of squarks, sleptons, quarks and leptons, the identification of gauge and mass eigenstates is an assumption and approximation, respectively. The quark mixing matrix (CKM matrix) is known by experiment, see [13]. Mixing with the third generation is small ($\mathcal{O}\left(10^{-3}\right)$). First and second generation mixing is larger ($\mathcal{O}\left(10^{-1}\right)$). However, since we expect the consideration of quark mixing would give a negligible correction only, we take, for simplicity, the quark mixing matrix diagonal like (3.9).

3.3 Gravitino Cosmology

Requiring a consistent cosmology the existence of the gravitino in the particle spectrum implies several generic problems. By *cosmological gravitino problem* we denote the whole network of cosmological problems emerging due to the gravitino.[4] In this section, we point out the network of problems arising in our cosmological setting.

If the gravitino reaches thermal equilibrium in the early universe, its yield $Y_{3/2}$ reaches its equilibrium value [12]

$$Y_{3/2}^{\mathrm{eq}} = \frac{45}{2\pi^2 g_{*s}(T_{3/2}^{fo})} \frac{\zeta(3)}{\pi^2} \frac{3}{4} g_{3/2} \simeq 1.8 \times 10^{-3} \,, \tag{3.14}$$

where $T_{3/2}^{\mathrm{fo}}$ denotes the freeze-out temperature of the gravitinos and $g_{3/2}$ is given by the internal goldstino degrees of freedom. Thus $g_{3/2} = 2$. ζ denotes the zeta function. If the gravitino energy density $\rho_{3/2} = m_{3/2} Y_{3/2} s$ became larger than the critical energy density, the Universe would have re-collapsed or—less dramatic—the observed Hubble rate today H_0 would be inconsistent with the theory. This problem is resolved by inflation, because any initial yield of gravitinos is diluted away, see Sec. 2.1. Alternatively, if the gravitino mass is suppressed, so that its energy density becomes negligible for the energy budget of the Universe, $m_{3/2} \lesssim \mathcal{O}\left(\mathrm{eV}\right)$, any gravitino overproduction problem is circumvented. However, at the

[4]Often any subproblem depending on the particular cosmological setting under consideration is called "the cosmological gravitino problem".

same time all other sparticles would become unstable, such that none of them could form dark matter.

Even if gravitinos do not enter thermal equilibrium after inflation, they are, nevertheless, regenerated by thermal scatterings and decays in the thermal plasma like $g + g \to \widetilde{g} + \Psi_{3/2}$. This thermally produced gravitino density is given by [5]

$$\Omega_{3/2}^{\mathrm{tp}} h^2 = m_{3/2}\, Y_{3/2}^{\mathrm{tp}}(T_0)\, \frac{s(T_0)\, h^2}{\rho_0}\,, \tag{3.15}$$

where $s(T_0)$ refers to today's entropy density of the Universe and we obtain $s(T_0)h^2/\rho_0 \simeq 2.8 \times 10^8\ \mathrm{GeV}^{-1}$. The thermal gravitino yield for low temperatures $T_{\mathrm{low}} \ll T_{\mathrm{R}}$ is given by [81, 82]

$$Y_{3/2}^{\mathrm{tp}}(T_{\mathrm{low}}) \simeq \sum_{i=1}^{3} y_i g_i^2(T_{\mathrm{R}}) \left(1 + \frac{M_i^2(T_{\mathrm{R}})}{3m_{3/2}^2}\right) \ln\left(\frac{k_i}{g_i(T_{\mathrm{R}})}\right) \left(\frac{T_{\mathrm{R}}}{10^{10}\ \mathrm{GeV}}\right), \tag{3.16}$$

where the gauge couplings $g_i = (g', g, g_s)$, the gaugino mass parameters M_i as well as the constants $k_i = (1.266, 1.312, 1.271)$ and $y_i/10^{-12} = (0.653, 1.604, 4.276)$ are associated with the gauge groups $\mathrm{U}(1)_{\mathrm{Y}}$, $\mathrm{SU}(2)_{\mathrm{L}}$ and $\mathrm{SU}(3)_{\mathrm{C}}$, respectively. Depending on the reheating temperature T_{R} the thermally produced gravitino density can be significant with dramatic impact on cosmology. Thus there is a cosmological gravitino problem after inflation. The arising constraints are crucially different whether the gravitino is stable or unstable.

Unstable If the gravitino is heavier than other sparticles, it decays into a sparticle and the corresponding SM particle with a lifetime roughly given by [83]

$$\tau_{3/2} \approx \frac{8\pi}{\alpha_{3/2}} \frac{M_{\mathrm{pl}}^2}{m_{3/2}^3}\,, \tag{3.17}$$

where $\alpha_{3/2}$ is a dimensionless number of at most $\mathcal{O}(1)$. Thus we have an estimate of the gravitino lifetime

$$\tau_{3/2} \gtrsim 3\ \mathrm{years} \left(\frac{100\ \mathrm{GeV}}{m_{3/2}}\right)^3, \tag{3.18}$$

[5] This production mechanism is thermal, because the produced yield depends directly on the reheating temperature. The gravitinos inherit a thermal distribution from the scatterings of the thermally distributed particles in the bath.

that is surely larger than the time of nucleosynthesis happening seconds after the big bang, see Section 2.3. The requirement of a small enough (decaying) density yields an upper bound on the reheating temperature $\lesssim 10^6$ GeV [84]. This bound excludes leptogenesis not only as described in Sec. 2.2 but also many variants.

As can be seen from (3.18) the gravitino decays early enough, if it is heavier than $\mathcal{O}(10\text{ TeV})$. Via (3.11) and (3.10), which does not always hold (cf. anomaly mediation), this would imply an unattractive high scale $\langle F\rangle$ of spontaneous supersymmetry breaking [83, 85, 86]. If the gravitino decays after the LSP is frozen out, each decay yields due to R-parity finally an LSP, i.e., $Y_{\text{lsp}} = Y_{\text{lsp}}^{\text{fo}} + Y_{\text{lsp}}^{\text{ntp}} = Y_{\text{lsp}}^{\text{fo}} + Y_{3/2}^{\text{tp}}$. Due to the non-thermal contribution $Y_{\text{lsp}}^{\text{ntp}}$ from gravitino decays, there is another upper bound on the reheating temperature in scenarios of anomaly mediation potentially excluding leptogenesis. If $Y_{\text{lsp}}^{\text{fo}} \ll Y_{\text{DM}}$, this bound scales as those in scenarios with gravitino dark matter as found below. But it is by a factor $m_{\text{DM}}/m_{3/2} \sim 100\text{ GeV}/10\text{ TeV} \sim 10^{-2}$ milder. A gravitino lifetime shorter than the time of LSP freeze-out, while $m_{\text{soft}} \lesssim$ TeV, would require a much larger mass splitting between $m_{3/2}$ and m_{soft}.

An alternative solution in the unstable gravitino case is a sparticle mass spectrum that allows decays only into particles that are decoupled from the thermal bath. Since they are decoupled from the bath, they do not thermalise. Neither formerly built nuclei are dissociated nor entropy is produced. A testable model like this, that features at the same time dark and visible matter, is presented in Chapter 6.

Stable If the gravitino is the LSP with conserved R-parity, it is stable. Therefore, there are no dangerous gravitino decays. The gravitino is a gauge singlet and thus a perfect dark matter candidate. However, its relic density may not be larger than the observed dark matter density, $\Omega_{3/2} \leq \Omega_{\text{DM}}$. The arising constraints depend crucially on the order of the gravitino mass. From (3.15) and (3.16) we see that if the gravitino is the LSP, for fixed gaugino masses the relic gravitino density decreases for increasing gravitino mass. Assuming universal gaugino masses at the GUT scale, we can approximate

$$\Omega_{3/2}^{\text{tp}} h^2 \simeq 0.11 \left(\frac{T_{\text{R}}}{3\times 10^8\text{ GeV}}\right)\left(\frac{M_{\tilde{g}}(m_{\text{Z}})}{10^3\text{ GeV}}\right)^2\left(\frac{10\text{ GeV}}{m_{3/2}}\right). \tag{3.19}$$

Exploiting the requirement $\Omega_{3/2}h^2 \leq \Omega_{\text{DM}}h^2 = 0.112 \pm 0.007$ (2σ) [18], we obtain an upper bound on the reheating temperature. If $m_{3/2} \ll m_{\text{soft}} \sim \mathcal{O}(100\text{ GeV})$, this bound excludes leptogenesis not only as described in Sec. 2.2, but also many variants. The case of a light gravitino with a mass of $\mathcal{O}\,(\mathrm{keV})$ entering thermal equilibrium and thus allowing arbitrary reheating temperatures is excluded by warm dark matter constraints [87].

If $m_{3/2} \sim m_{\mathrm{soft}} \sim \mathcal{O}\,(100\ \mathrm{GeV})$ as in (3.12), we see from (3.19) that the gravitino forms the observed dark matter for reheating temperatures of $\mathcal{O}\,(10^{10}\ \mathrm{GeV})$. Gravitino dark matter turns the gravitino problem as discussed into a virtue. However, a consistent cosmology requires at the same time an explanation for the amount of visible matter in the Universe. As discussed in Sec. 2.2 successful thermal leptogenesis requires a large reheating temperature $T_{\mathrm{R}} \gtrsim 2 \times 10^9$ GeV. Following the previous discussion—with the exception of the approach pursued in Chapter 6—none of the solutions is able to provide visible matter from thermal leptogenesis and dark matter. The apparent, mutual exclusion of both notions—supergravity and thermal leptogenesis—is manifest in the *gravitino problem of thermal leptogenesis*. Following the discussion so far gravitino dark matter is left as the best motivated solution. For given reheating temperature and gaugino masses, we obtain a lower bound on the gravitino mass exploiting the requirement $\Omega_{3/2} \leq \Omega_{\mathrm{DM}}$. We combine (2.19) and (3.19) using the best-case relation $T_{\mathrm{R}} \simeq M_{\nu_{\mathrm{R}}^1}$ to eliminate the large Majorana neutrino mass and obtain

$$\eta_{\mathrm{b}}^{\max} \simeq 1.4 \times 10^{-10} \left(\frac{\Omega_{3/2}^{\mathrm{tp}} h^2}{0.11}\right) \left(\frac{10^3\ \mathrm{GeV}}{M_{\tilde{g}}(m_{\mathrm{Z}})}\right)^2 \left(\frac{m_{3/2}}{10\ \mathrm{GeV}}\right) \left(\frac{m_{\nu_{\mathrm{L}}^3}}{0.05\ \mathrm{eV}}\right) \left(\frac{\kappa_0}{0.18}\right). \tag{3.20}$$

Since in (3.20) $\eta_{\mathrm{b}}^{\max}$ can be larger than η^{obs}, dark matter might be formed by thermally produced gravitinos, when the visible matter originated from standard thermal leptogenesis. However, recalling the discussion after (2.19), $m_{\nu_{\mathrm{L}}^3}$ cannot be raised without lowering κ_0. Thus, even for the most optimistic scenario with $T_{\mathrm{R}} = T_{\mathrm{L}}^{\min} = 2 \times 10^9$ GeV the gravitino mass is restricted to a value $\gtrsim 40$ GeV. Note that the gravitino shall be the LSP and we want to maintain sparticle masses in the TeV range. The parameter space of successful thermal leptogenesis is drastically reduced, if dark matter is formed by gravitinos with a mass in the TeV range.

Even worse, the *LOSP decay problem*[6] is a definite clash between both notions,

[6] The lightest ordinary superparticle (LOSP) is the lightest superparticle of the minimal supersymmetric standard model (MSSM). In this sense, gravitino and axino are extraordinary superparticles.

because gravitino LSP masses larger than about 10 GeV are excluded in most cases. In the MSSM with conserved R-parity, the LOSP has to decay into the gravitino and SM particles. It decays typically with a long lifetime

$$\tau_{\rm losp} \approx \frac{8\pi}{\alpha_{\rm losp}} \frac{M_{\rm pl}^2 m_{3/2}^2}{m_{\rm losp}^5} \gtrsim 2 \text{ months} \left(\frac{m_{3/2}}{100 \text{ GeV}}\right)^2 \left(\frac{200 \text{ GeV}}{m_{\rm losp}}\right)^5 \gg 1 \text{ s} \sim t_{\rm BBN} \tag{3.21}$$

due to the extremely weak interactions of the gravitino, where $\alpha_{\rm losp}$ is a dimensionless number of at most $\mathcal{O}(1)$. If these decays occur during or after BBN, the emitted SM particles can change the primordial abundances of light elements [39, 88, 89]. Even though the LOSP decays much faster than an unstable gravitino, because $(m_{3/2}/m_{\rm losp})^5 \ll 1$, we obtain a similar problem as afflicted with any late-decaying particle.

Specific setups like the Constrained MSSM have been studied in [90, 91, 92, 93, 94]. Only in exceptional regions of the parameter space [95, 96, 97], a stau LOSP not orders of magnitude heavier than the gravitino could be consistent with all constraints, allowing reheating temperatures $T_{\rm R} \sim 10^9$ GeV. Generically a conservative upper bound on the reheating temperature $T_{\rm R} \lesssim$ few $\times\, 10^8$ GeV is found. Since then the maximally produced baryon asymmetry is too small, various LOSP candidates have been investigated more model-independently [89, 98] to identify best-case scenarios: A sneutrino [99, 100, 101, 102, 103, 104] actually could allow large enough reheating temperatures with reasonable masses due to its invisible decays. A stop [105, 106] or a general neutralino [6] are not reconcilable with thermal leptogenesis for masses below a TeV. They would definitely require $m_{3/2} < 10$ GeV.

Altogether, to reconcile supersymmetry with big bang nucleosynthesis, that requires some initial baryon asymmetry, one might finally search for solutions of the LOSP decay problem, because it is the strongest conflict between thermal leptogenesis and gravitino dark matter. This conflict is embodied in (3.20) taking into consideration the restriction to small gravitino masses, $m_{3/2} \leq 10$ GeV. A finite number of solutions has been proposed: The gravitino could be degenerate in mass with the LOSP, so that its decay products are low-energetic enough to circumvent BBN bounds [95]. Enlarging the hidden sector there may be additional hidden sector decay modes, so that the gravitino decays early enough [107, 108]. A super light gravitino together with an also super light neutralino circumvents the problem [109]. Of course, a combination of various approaches is possible [110, 111, 112, 113]. In Chapter 4 we discuss extensively the possibility of late-time entropy production. The impact of the R-parity violating solution on the axion multiplet is discussed in Chapter 5. As the prime example for a solution with decoupled decay products, we present the axino solution to the gravitino problem in Chapter 6 and show that the scenario is tested with on-going experiments.

3.4 Cosmology of the Axion Multiplet

Quantum chromodynamics (QCD) has a rich vacuum structure, that gives rise to the so-called Θ-term in the Lagrangian

$$\mathcal{L}_\Theta = \Theta \frac{g_s^2}{32\pi^2} G^{\mu\nu} \widetilde{G}_{\mu\nu} \,, \tag{3.22}$$

where $\widetilde{G}_{\mu\nu}$ is the dual of the QCD field strength tensor $G^{\mu\nu}$. This term violates parity (P) and time reversal symmetry (T). Imposing CPT symmetry, the combined symmetry of parity and charge conjugation (CP) must be violated. Measurements of the electric dipole moment of the neutron d_n constrain CP violation in the strong sector. In particular [114],

$$d_n \sim 10^{-15}\, \widetilde{\Theta}\ \mathrm{e} \cdot \mathrm{cm} < 10^{-25}\ \mathrm{e} \cdot \mathrm{cm} \,, \tag{3.23}$$

where $\widetilde{\Theta} = \mathrm{Arg\ Det}\ M_{\mathrm{quarks}} + \Theta$. The complex quark mass matrix M_{quarks} breaks CP symmetry explicitly. Thus CP cannot be invoked to be exactly conserved and $\mathrm{Arg\ Det}\ M_{\mathrm{quarks}} \neq 0$. The *strong CP problem* is then formulated in the question: Why should a sum of two independent parameters result in zero?

This is a fine-tuning problem of Standard Model QCD. Its standard solution is the *Peccei-Quinn (PQ) mechanism* [3, 4].[7] Introducing a global $U(1)_{\mathrm{PQ}}$ symmetry, which is necessarily spontaneously broken, replaces $\widetilde{\Theta}$ by a field that conserves CP dynamically. In other words, the vacuum expectation value of this field drives $\widetilde{\Theta}$ dynamically to zero. Any spontaneously broken symmetry gives rise to a Goldstone particle. The PQ pseudo-Goldstone boson is known as axion a [116, 117].

[7] All other proposed solutions seem to fail. For a review of axions and the strong CP problem see [115].

Name	Spin	P_R	Gauge Eigenstates	Mass Eigenstates
Higgs bosons	0	+1	$H_u^0\ H_d^0\ H_u^+\ H_d^-$	$h^0\ H^0\ A^0\ H^\pm$
squarks $\widetilde{q}$	0	-1	$\widetilde{u}_L\ \widetilde{u}_R\ \widetilde{d}_L\ \widetilde{d}_R$ $\widetilde{s}_L\ \widetilde{s}_R\ \widetilde{c}_L\ \widetilde{c}_R$ $\widetilde{t}_L\ \widetilde{t}_R\ \widetilde{b}_L\ \widetilde{b}_R$	(same)
sleptons $\widetilde{l}$	0	-1	$\widetilde{e}_L\ \widetilde{e}_R\ \widetilde{\nu}_L^e\ \widetilde{\nu}_R^e$ $\widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{\nu}_L^\mu\ \widetilde{\nu}_R^\mu$ $\widetilde{\tau}_L\ \widetilde{\tau}_R\ \widetilde{\nu}_L^\tau\ \widetilde{\nu}_R^\tau$	(same)
quarks q	1/2	+1	$u_L\ u_R\ d_L\ d_R$ $s_L\ s_R\ c_L\ c_R$ $t_L\ t_R\ b_L\ b_R$	(same)
leptons l	1/2	+1	$e_L\ e_R\ \nu_L^e\ \nu_R^e$ $\mu_L\ \mu_R\ \nu_L^\mu\ \nu_R^\mu$ $\tau_L\ \tau_R\ \nu_L^\tau\ \nu_R^\tau$	(same)
neutralinos	1/2	-1	$\widetilde{B}^0\ \widetilde{W}^0\ \widetilde{H}_u^0\ \widetilde{H}_d^0$	$\chi_1^0\ \chi_2^0\ \chi_3^0\ \chi_4^0$
charginos	1/2	-1	$\widetilde{W}^\pm\ \widetilde{H}_u^+\ \widetilde{H}_d^-$	$\chi_1^\pm\ \chi_2^\pm$
gluinos	1/2	-1	$\widetilde{g}^a$	(same)
gauge bosons (γ, Z, $W^\pm$)	1	+1	$B_\mu\ W_\mu^a$	$A_\mu\ Z_\mu^0\ W_\mu^\pm$
gluons	1	+1	g_μ^a	(same)
gravitino (with goldstino)	3/2 (1/2)	-1	$\Psi_{3/2}$	(same)
PQ scalars	0	+1	$a \quad \phi_{\rm sax}$	(same)
axino	1/2	-1	$\widetilde{a}$	(same)

Table 3.4: Physical particles as approximated

Name	Bosons	Fermions	$(\mathrm{SU}(3)_{\mathrm{C}}, \mathrm{SU}(2)_{\mathrm{L}})_Y$
Axion, saxion, axino	a, ϕ_{sax}	$\widetilde{a}$	$(\mathbf{1}, \mathbf{1})_0$

Table 3.5: Axion supermultiplet

Its Lagrangian is obtained by the replacement $\Theta \to a/f_a$ in (3.22), where f_a is refered to as axion decay constant. Since in supersymmetry particles reside in supermultiplets, the axion is accompanied by another real scalar, the saxion ϕ_{sax}, and the axino $\widetilde{a}$ carrying the fermionic degrees of freedom. The members of the axion multiplet (Table 3.5) are neutral under the SM gauge group.

The axion has not been observed yet. However, its physics provides a lower limit on the axion decay constant [13, 118]

$$f_a \gtrsim 6 \times 10^8 \text{ GeV}, \tag{3.24}$$

where the most constraining bound stems from supernova cooling considerations. There are two—so called invisible—axion models accommodating such a large axion decay constant. The simplest case is the Kim-Shifman-Vainstein-Zakharov (KSVZ) axion model [119, 120] incorporating a new heavy quark Q with

$$\mathcal{L}_{\mathrm{ksvz}} = -\bar{\sigma} Q_L Q_R + (\text{h.c.}) - V(|\sigma|^2) - \frac{\widetilde{\Theta}}{32\pi^2} G^{\mu\nu} \widetilde{G}_{\mu\nu}, \tag{3.25}$$

where the axion arises as the phase of the singlet scalar field σ. The vacuum expectation value of σ sets the axion decay constant, $\langle\sigma\rangle = f_{\mathrm{PQ}} = f_a$. Here, the Higgs doublets H_u and H_d are neutral under $U(1)_{\mathrm{PQ}}$. By coupling σ to the Higgs doublets, one can introduce a PQ symmetry without necessarily introducing heavy quarks. This is the Dine-Fischler-Srednicki-Zhitnitskii (DFSZ) axion model [121, 122]. The periodicity of the axion field is changed, which implies $f_a = f_{\mathrm{PQ}}/N$ with $N = 6$.

Both models have been supersymmetrised [123, 124, 125, 126]. The supersymmetrised axion interaction from (3.22) reads

$$\int d^2\vartheta \frac{g_s^2}{16\sqrt{2}\pi^2 f_a} \Phi_a \mathcal{W}^\alpha \mathcal{W}_\alpha + \text{ h.c.}, \tag{3.26}$$

where the axion is found as the pseudoscalar component of the chiral multiplet $\Phi_a = (\phi_{\mathrm{sax}} + ia)/\sqrt{2} + \vartheta\widetilde{a} + (\text{F term})$. ϑ are Grassmann variables. $\mathcal{W}_\alpha$ denotes the vector multiplet containing the gluino and gluon field strength. Then $\mathcal{W}^\alpha \mathcal{W}_\alpha|_{\theta\theta} = -i2\lambda^a\sigma^m\delta_m\bar{\lambda}^a - \frac{1}{2}G^a_{\mu\nu}G^{a\mu\nu} + \frac{i}{2}G^a_{\mu\nu}\widetilde{G}^{a\mu\nu} + D^2$. An analogous interaction is present for the hypercharge gauge multiplet. In superstring models most probably σ couples to both, heavy quarks and the Higgs doublets [127]. Since we discuss the KSVZ and DFSZ models explicitly, our analysis can be applied to such general axion models. We would like to stress that by its nature the axion multiplet interacts with SM particles, but the interaction strength is suppressed by f_a.

The axion receives a tiny mass from instanton effects [12]

$$m_a \simeq 0.62 \text{ meV} \left(\frac{10^{10} \text{ GeV}}{f_a}\right) . \tag{3.27}$$

Saxion and axino masses are split from the almost vanishing axion mass, when SUSY is broken. Most probably the saxion mass $m_{\rm sax}$ is around the soft mass scale, $m_{\rm sax} \sim m_{\rm soft}$. The axino mass should also be near this scale. Thus

$$m_{\widetilde{a}} \sim m_{\rm sax} \sim m_{\rm soft} \sim m_{3/2} . \tag{3.28}$$

However, the precise value of the axino mass depends on the model, specified by the SUSY breaking sector and how SUSY breaking is mediated to the axion multiplet. Since the axino mass can also be much smaller than $m_{\rm soft}$ [126, 128, 129, 130, 131, 132, 133], we consider the case of a light axino in Chapter 6.

3.4.1 Axino

After reheating, reactions like $gg \leftrightarrow \widetilde{g}\widetilde{a}$ or $\widetilde{q}q \leftrightarrow g\widetilde{a}$ drive the axino into thermal equilibrium, if the reheating temperature is larger than its decoupling temperature [134]

$$T_{\widetilde{a}}^{\rm dcp} \simeq 10^{11} \text{ GeV} \left(\frac{f_a}{10^{12} \text{ GeV}}\right)^2 \left(\frac{0.1}{\alpha_{\rm s}}\right)^3 , \tag{3.29}$$

where $\alpha_{\rm s} = g_s^2(\mu)/4\pi$ evaluated at the scale relevant for the processes under consideration. The equilibrium axino yield is equal to (3.14).

Even if axinos do not enter thermal equilibrium after inflation, they are, nevertheless, regenerated by thermal scatterings and decays in the thermal plasma. In the KSVZ model with heavy exotic quarks the resulting density can be estimated as [135, 136, 137]

$$\Omega_{\widetilde{a}}^{\rm ksvz} h^2 \simeq 7.8 \times 10^6 \left(\frac{m_{\widetilde{a}}}{1 \text{ TeV}}\right) \left(\frac{T_{\rm R}}{10^9 \text{ GeV}}\right) \left(\frac{10^{12} \text{ GeV}}{f_a}\right)^2 . \tag{3.30}$$

Recent studies [138, 139] report that this abundance can be suppressed, if the KSVZ quarks are lighter than the scale of PQ symmetry breaking. Furthermore, the axino yield is reported to become independent of the reheating temperature in the DFSZ model[8],

$$\Omega_{\widetilde{a}}^{\rm dfsz} \sim 10^3 \left(\frac{m_{\widetilde{a}}}{1\ {\rm TeV}}\right) \left(\frac{10^{12}\ {\rm GeV}}{f_a}\right)^2 (+\Omega_{\widetilde{a}}^{\text{freeze-in}}) \tag{3.31}$$

for $T_{\rm R} \gtrsim 10^4$ GeV, where we also indicate that the DFSZ axino is in addition produced via "freeze-in" from heavy Higgsino and/or Higgs decays [140], like $\widetilde{H} \rightarrow \widetilde{a} + h$. The yield from freeze-in is again proportional to f_a^{-2} and independent of the reheating temperature, but dependent on the Higgsino and Higgs masses.

The axino might decay dominantly into a gluino-gluon pair with width [135]

$$\Gamma(\widetilde{a} \rightarrow \widetilde{g} g) = \frac{\alpha_s^2}{16\pi^3} \frac{m_{\widetilde{a}}^3}{f_a^2} \left(1 - \frac{m_{\widetilde{g}}^2}{m_{\widetilde{a}}^2}\right)^3 . \tag{3.32}$$

Assuming the absence of phase space suppression, this gives an estimate for its lifetime

$$\tau_{\widetilde{a}}^{\widetilde{g}g} \simeq 6\ {\rm s} \left(\frac{1\ {\rm TeV}}{m_{\widetilde{a}}}\right)^3 \left(\frac{f_a}{10^{14}\ {\rm GeV}}\right)^2 , \tag{3.33}$$

where we evaluated the strong coupling constant at the relevant scale, i.e., $\alpha_s(m_{\widetilde{a}}) \simeq 0.1$. The involved operator exists for any axion model that is able to solve the strong CP problem. The DFSZ superfield is directly coupled to the Higgs superfields. Therefore, the DFSZ axino may dominantly decay into a Higgsino-Higgs pair. Using $\mathcal{L} = \frac{2\mu}{f_a}(H_u^0 \widetilde{H}_d^0 + H_d^0 \widetilde{H}_u^0)\,\widetilde{a} + {\rm h.c.}$, we obtain in the decoupling limit[9]

$$\begin{aligned}\Gamma(\widetilde{a} \rightarrow \widetilde{H} + h) \;\; &\simeq \;\; \frac{m_{\widetilde{a}}}{4\pi} \sum_i |N_{\chi_i^0 \widetilde{H}_d^0} \sin\beta + N_{\chi_i^0 \widetilde{H}_u^0} \cos\beta|^2 \left(\frac{\mu}{f_a}\right)^2 \\ &\times \{(1 + x_\chi - x_h)^3 (1 + x_\chi + x_h)^3 ((1 - x_\chi)^2 - x_h^2)\}^{\frac{1}{2}}\end{aligned} \tag{3.34}$$

with $x_\chi = m_{\chi_i^0}/m_{\widetilde{a}}$ and $x_h = m_h/m_{\widetilde{a}}$. μ is the usual supersymmetric μ-parameter in (3.3), $\tan\beta = v_u/v_d$ is the ratio of the Higgs vacuum expectation values and

[8] The studies agree upon the effect, while different suppressions are reported. In addition, [139] reports a strong dependence on the infra-red regulator used in the axino thermal production computation.

[9] The result seems quite insensitive to the possibility of the Giudice-Masiero mechanism to generate the μ-term, but this might be different for the saxion decay into two light Higgses [141].

$N_{\chi_i^0 \widetilde{H}_d^0}$ and $N_{\chi_i^0 \widetilde{H}_u^0}$ denote the Higgsino fractions of the i-th neutralino. We consider the case of degenerate, maximally mixed Higgsinos, which are then decoupled from the gauginos. For sufficiently large $\tan\beta$ the mixing factor in (3.34) then becomes about 1/2. As an estimate we give

$$\tau_{\widetilde{a}}^{\widetilde{H}h} \approx 10^{-2}\ \mathrm{s} \left(\frac{1\ \mathrm{TeV}}{m_{\widetilde{a}}}\right) \left(\frac{10^2\ \mathrm{GeV}}{\mu}\right)^2 \left(\frac{f_a}{10^{14}\ \mathrm{GeV}}\right)^2 . \tag{3.35}$$

We would like to stress that both estimates, (3.33) and (3.35), assume from the beginning a rather heavy axino.

Altogether, requiring a consistent cosmology the existence of the axino in the particle spectrum implies similar problems as the gravitino. Since the axino is a typical late-decaying particle, the discussion in Sec. 3.3 can be done in analogy. In short, since the axino couples more strongly than the gravitino, it may decay early enough. But for the same reason, if there is a bound on the reheating temperature, this bound can be much stronger. Therefore, the opportunity of axino cold dark matter [135], for example, excludes thermal leptogenesis. If the reheating temperature is too large for axino dark matter, the axino might be required to decay before the LOSP freezes out around $t_{\mathrm{losp}}^{\mathrm{fo}} \sim 10^{-9}\ \mathrm{s} \ll 1\ \mathrm{s} \sim t_{\mathrm{BBN}}$. Otherwise the axino decay may produce too many LOSPs. Corresponding constraints on PQ parameters are tightened. This is a typical consequence of the requirement of a consistent cosmology and will be one issue in the following chapters.

3.4.2 Saxion

Due to supersymmetry the couplings of the saxion have the same strength as the axino couplings. Thus the saxion is produced by thermal scatterings as efficiently as the axino. Its equilibrium yield as scalar is smaller by a factor 2/3 only. Its dominant decay producing MSSM particles is into a pair of gluons with [142]

$$\Gamma(\phi_{\mathrm{sax}} \to gg) = \frac{\alpha_s^2}{32\pi^3} \frac{m_{\mathrm{sax}}^3}{f_a^2} . \tag{3.36}$$

Thus its lifetime is comparable to the axino lifetime (3.33). In the DFSZ model the saxion may decay into a pair of light Higgses with [141, 143]

$$\Gamma(\phi_{\mathrm{sax}} \to hh) = \frac{m_{\mathrm{sax}}}{8\pi} \left(1 - \frac{4m_h^2}{m_{\mathrm{sax}}^2}\right)^{\frac{1}{2}} \left(\frac{\mu}{f_a}\right)^2 \left(\frac{\mu}{m_{\mathrm{sax}}}\right)^2 . \tag{3.37}$$

For $\mu \sim m_{\text{sax}}$ this leads to an estimate of the saxion lifetime as

$$\tau_{\text{sax}}^{hh} \simeq 10^{-2}\ \text{s} \left(\frac{300}{m_{\text{sax}}}\right)^3 \left(\frac{f_a}{10^{14}\ \text{GeV}}\right)^2 . \tag{3.38}$$

In addition, the saxion may decay as well into two axions with [144]

$$\Gamma(\phi_{\text{sax}} \to aa) \simeq \frac{x^2}{64\pi}\frac{m_{\text{sax}}^3}{f_a^2}, \tag{3.39}$$

where the self-coupling x can be of $\mathcal{O}(1)$. Comparing (3.36) and (3.39) we see that for large x this is the dominant decay channel. No MSSM particles are produced. Thus bounds on such decays do not apply. The produced axions represent a form of dark radiation, i.e., decoupled, relativistic particles not present in the Standard Model. As discussed below, the emerging energy density of axions might be negligible, if the saxion decays early enough. On the other hand, if the saxion decays into axions while dominating the energy density of the Universe, the Universe becomes filled with relativistic energy. This were in contradiction to observations. In particular, this occurs if the saxion is copiously produced from coherent oscillations. Often $x \ll 1$ is required. There are concrete models realising this [133].

Since the saxion corresponds to a flat direction of the scalar potential lifted by SUSY breaking effects, it can develop a large field value during inflation. It begins to oscillate around the potential minimum when the Hubble parameter becomes comparable to the saxion mass, $H \sim m_{\text{sax}}$. This corresponds to the production of non-relativistic particles. The temperature at the onset of oscillations is

$$T_{\text{sax}}^{\text{osc}} \simeq 2.2 \times 10^{10}\ \text{GeV} \left(\frac{m_{\text{sax}}}{1\ \text{TeV}}\right)^{\frac{1}{2}} \left(\frac{228.75}{g_*(T_{\text{sax}}^{\text{osc}})}\right)^{\frac{1}{4}} . \tag{3.40}$$

We find in agreement with [144]: If $T_{\text{R}} < T_{\text{sax}}^{\text{osc}}$, the produced saxion abundance is,

$$\begin{aligned} \frac{\rho_{\text{sax}}^{\text{osc}}}{s} &= \frac{1}{8}T_{\text{R}}\left(\frac{\phi_{\text{sax}}^{\text{i}}}{M_{\text{pl}}}\right)^2 \\ &\simeq 2.1 \times 10^{-3}\ \text{GeV}\left(\frac{T_{\text{R}}}{10^7\ \text{GeV}}\right)\left(\frac{f_a}{10^{14}\ \text{GeV}}\right)^2\left(\frac{\phi_{\text{sax}}^{\text{i}}}{f_a}\right)^2 \end{aligned} \tag{3.41}$$

where $\phi_{\text{sax}}^{\text{i}}$ denotes the initial amplitude of the oscillations and where we have assumed the simplest saxion potential, $V = \frac{1}{2}m_{\text{sax}}^2\phi_{\text{sax}}^2$. For $T_{\text{R}} > T_{\text{sax}}^{\text{osc}}$, the produced saxion abundance is independent of T_R and given by

$$\begin{aligned}\frac{\rho_\mathrm{sax}^\mathrm{osc}}{s} &= \frac{1}{8} T_\mathrm{sax}^\mathrm{osc} \left(\frac{\phi_\mathrm{sax}^\mathrm{i}}{M_\mathrm{pl}}\right)^2 \\ &\simeq 4.8\ \mathrm{GeV} \left(\frac{m_\mathrm{sax}}{1\ \mathrm{TeV}}\right)^{\frac{1}{2}} \left(\frac{f_a}{10^{14}\ \mathrm{GeV}}\right)^2 \left(\frac{\phi_\mathrm{sax}^\mathrm{i}}{f_a}\right)^2 \left(\frac{228.75}{g_*(T_\mathrm{sax}^\mathrm{osc})}\right)^{\frac{1}{4}} . \end{aligned} \tag{3.42}$$

Often a low initial amplitude $\phi_\mathrm{sax}^\mathrm{i} \lesssim f_a$ is assumed. The upper bound is indicated in the left column of Tab. 7.1.

3.4.3 Axion

Due to the similar coupling strength the axion is produced thermally as efficiently as the saxion [145]. Its equilibrium yield is the same as the saxion yield. However, due to its tiny mass (3.27) its thermal relic density is in any case negligibly small. The axion decays harmlessly into two photons with a lifetime many orders of magnitude larger than the age of the Universe [12], while sparticles cannot decay into the axion due to R-parity. A large axion density may be produced non-thermally by vacuum misalignment and topological defects.

The density produced by vacuum misalignment [146, 147, 148] is usually given as

$$\Omega_a^\mathrm{mis} h^2 \sim a_0^2 \left(\frac{N f_a}{10^{12}\ \mathrm{GeV}}\right)^{7/6} , \tag{3.43}$$

where a_0 comprises model-dependent factors and might well be of $\mathcal{O}(1)$. This leads to the known cosmological upper bound $f_a \lesssim 10^{12}$ GeV to avoid overproduction. Since axions from vacuum misalignment are non-relativistic at the onset of structure formation, (3.43) also tells us that it is a perfect dark matter candidate on its own.

In the presence of supersymmetric dark matter—or any other dark matter population—we require $\Omega_a/\Omega_\mathrm{DM} = r \ll 1$, which gives an upper bound

$$f_a < 10^{10}\ \mathrm{GeV} \quad \text{for} \quad a_0 = N = 1\ ,\ r = 0.04\,. \tag{3.44}$$

Note that this bound becomes tighter for larger N, but see Sec. 5.2. It is considered in the left column of Table 7.1.

Topological defects occur, if the Peccei-Quinn symmetry is restored after inflation, which is the case if $T_\mathrm{R} > T_\mathrm{PQ} \sim f_\mathrm{PQ} = N f_a$. This leads to the formation of disastrous domain walls [149], if $N > 1$, and cosmic strings [150]. The abundance of axions from cosmic strings $\Omega_a^{\rm str}$ exceeds that from vacuum misalignment, $\Omega_a^{\rm str} \sim 10 \times \Omega_a^{\rm mis}$ [151, 152]. Depending on the axion model and the value of the axion decay constant, the occurrence of topological effects might have to be forbidden, which would imply an upper bound on the reheating temperature

$$T_{\rm R} < T_{\rm PQ} \sim N f_a \, .$$

Altogether, the existence of the axion multiplet in the particle spectrum implies several generic problems.

Chapter 4

Late-time Entropy Production

In Sec. 3.3 it has been argued that one should search for solutions of the LOSP decay problem to reconcile supersymmetry with big bang nucleosynthesis. If entropy is produced after the LOSP is frozen out, its density is diluted. Thus, late-time entropy production can naively resolve this conflict for any LOSP within or without a specific model. The relic density prior to its decay,

$$\Omega_{\mathrm{losp}} = \Delta^{-1}\,\Omega^{\mathrm{fo}}_{\mathrm{losp}}\,, \tag{4.1}$$

is reduced compared to its freeze-out density $\Omega^{\mathrm{fo}}_{\mathrm{losp}}$ by the dilution factor Δ. In this chapter we investigate comprehensively to what extent the LOSP decay problem can be solved by late-time entropy production. In Sec. 4.3 we show how BBN constraints on a general neutralino with a gravitino LSP with $m_{3/2} = 100$ GeV are softened by $\Delta > 1$. The impact of this scenario on the gravitational wave background is examined in Sec. 4.5.

4.1 Thermal Leptogenesis and Gravitino Yield with late Entropy Production

From (2.19) we see that a significant dilution, $\eta_{\mathrm{B}}^{\mathrm{max}} \to \eta_{\mathrm{B}}^{\mathrm{max}}/\Delta$ with $\Delta \gg 1$, can only be compensated by a larger $M_{\nu^1_{\mathrm{R}}}$, since all the other parameters are chosen already to be optimal. Due to the requirement $T_{\mathrm{R}} \gtrsim M_{\nu^1_{\mathrm{R}}}$ this gives a linear shift of the required reheating temperature. Since the gravitino density (3.16) depends also linearly on the reheating temperature and is diluted in the same way as the baryon density, such a compensation seems to give a trivial shift of the problem to higher reheating temperatures. However, there are aspects that do not show up in (2.19) and (3.20), in addition to the impact on the LOSP decay problem.

Most importantly, in the domain of large $M_{\nu_{\mathrm{R}}^1}$ washout processes reduce the efficiency factor κ_0 exponentially. In the case of hierarchical neutrinos, this domain corresponds to $M_{\nu_{\mathrm{R}}^1} > 4 \times 10^{13}$ GeV. From this we would obtain $\Delta < 2 \times 10^4$ [153]. However, while at low $M_{\nu_{\mathrm{R}}^1}$ many numeric examples and an analytic approximation for κ_0 [25] exist in the literature, the situation for larger $M_{\nu_{\mathrm{R}}^1}$ is less well-studied. As an additional complication, for $T \gg 10^9$ GeV more spectator processes are in equilibrium and thus should be taken into account. Hence, it is not clear whether the maximal value κ_0^{peak} can be reached for $M_{\nu_{\mathrm{R}}^1} \sim 4 \times 10^{13}$ GeV. Consequently, we expect an upper bound

$$\Delta < \Delta^{\max} \sim 10^3 \ldots 10^4 \,. \tag{4.2}$$

We would like to stress that this is an intrinsic bound of the problem. It is stronger than bounds from perturbativity of Yukawa couplings ($\Delta < 10^5$) or the requirement of a reheating temperature below the GUT scale ($\Delta < 10^7$).

We remark that according to Fig. 6b of [16] there is a much stronger bound with roughly $\Delta^{\max} < 10^2$ for quasi-degenerate neutrino masses. Thus, thermal leptogenesis with late-time entropy production requires hierarchical neutrinos even more than thermal leptogenesis already does.

Late-time entropy production leads to a strong reduction of the allowed parameter space for successful thermal leptogenesis. Since the required minimal $M_{\nu_{\mathrm{R}}^1}$ is increased, the range of allowed values for κ_0 and the neutrino mass parameters is reduced. However, the same region of parameter space is already favoured by the need to keep the reheating temperature as low as possible in order to avoid the overproduction of gravitinos. Therefore, late-time entropy production does not reduce the parameter space of thermal leptogenesis with gravitino dark matter.

In (3.16) one has to consider the impact of the running couplings and masses due to the shift of the reheating temperature. For example, if we increase T_{R} from 3×10^9 GeV to 3×10^{13} GeV and choose $\Delta = 10^4$ to compensate, $\Omega_{3/2}^{\mathrm{tp}}$ decreases by 25%.[1] Note that this effect is unavoidable and softens the tension between thermal

[1] Besides, the electroweak contributions double their contribution to the total yield to about

leptogenesis and gravitino dark matter already before considering the impact of entropy production on the LOSP decay problem.

Another possibility is a gravitino with such a small mass that it enters thermal equilibrium after reheating. Then its relic abundance becomes independent of the reheating temperature, which allows $T_{\mathrm{R}} \gg M_{\nu_{\mathrm{R}}^1}$. Taking into account the lower limit on the mass of a warm dark matter particle [87], it turns out that in standard cosmology its relic energy density exceeds the observed dark matter density. However, already a Δ of a few dilutes the gravitino sufficiently to make it viable warm dark matter again [111, 112]. For $\Delta \simeq 10^3$ it forms cold dark matter with $m_{3/2} \simeq 1$ MeV [113, 154, 155]. Note that for these small masses the LOSP decays before BBN, so the decay problem is absent.

4.2 Entropy Production by Decaying Matter

In this section we discuss briefly how decaying matter can produce considerable entropy in the early universe [12, 156]. We consider a non-relativistic and long-lived particle species ϕ with chemical potential $\mu = 0$ in a flat Friedmann-Lemaitre-Robertson-Walker Universe (2.2). When ϕ drops out of chemical equilibrium, its yield Y_ϕ freezes out, see Sec. 2.2. Y_ϕ could also be generated from inflaton decay or thermally after reheating, if ϕ never enters chemical equilibrium. The contribution of non-relativistic particles to the energy density decreases as $\rho_{\mathrm{mat}} \propto a^{-3}$. Since the energy density of radiation in the Universe (2.14) decreases $\propto a^{-4}$, $\rho_{\mathrm{mat}}/\rho_{\mathrm{rad}}$ grows $\propto a$. Since a grows with time, at some time $t_\phi^=$ or temperature $T_\phi^=$ the unstable species ϕ comes to dominate the energy density automatically, if its lifetime $\tau_\phi > t_\phi^=$. If the Universe has been dominated by radiation before, it enters a phase of matter domination that lasts roughly till ϕ decays exponentially at τ_ϕ. Here we assume that everything released by the particle decay is rapidly thermalised, i.e., on timescales $\Delta t \ll H^{-1} \simeq$ expansion time. At some intermediate time $t \simeq t_\phi^= \left(\tau_\phi/t_\phi^=\right)^{3/5}$ the radiation produced in decays of ϕ starts to become the dominant component of the radiation energy density. The temperature of the Universe begins to fall more slowly, $T \propto a^{-3/8}$, than the usual $T \propto a^{-1}$. The Universe is never reheated, since the temperature decreases at all times. From $t \simeq t_\phi^= \left(\tau_\phi/t_\phi^=\right)^{3/5}$ till $t \simeq \tau_\phi$, the entropy per comoving volume S is growing $\propto a^{15/8}$.

30%.

At τ_ϕ the Universe becomes purely radiation-dominated again with $T \propto a^{-1}$ and a temperature $T_\phi^{\rm dec} = T(\tau_\phi)|_{\text{rad-dom}}$, where we use the time–temperature relation for a radiation-dominated Universe,

$$t|_{\text{rad-dom}} = \left(\frac{45}{2\pi^2 g_*(T)}\right)^{\frac{1}{2}} M_{\rm pl} T^{-2} \,. \tag{4.3}$$

This is the temperature after significant entropy production $T^{\rm after}$, which would be identified as the reheating temperature in the approximation of simultaneous decay of all ϕ particles. We identify $T^{\rm after} = T_\phi^{\rm dec}$. If ρ_ϕ never dominates over $\rho_{\rm rad}$, ϕ decays never produce a significant amount of entropy relative to the initial entropy. Then the produced entropy is negligible. However, if ρ_ϕ dominates over $\rho_{\rm rad}$ before τ_ϕ, the produced entropy dilutes significantly any relic density by a factor Δ.

The dilution factor Δ is defined as the ratio of entropy per comoving volume after ϕ decay $S_{\rm f}$ over the initial entropy per comoving volume $S_{\rm i}$ and can be expressed as

$$\Delta = \frac{S_{\rm f}}{S_{\rm i}} \simeq 0.82 \left\langle g_*^{1/3}\right\rangle^{3/4} \frac{m_\phi Y_\phi \tau_\phi^{1/2}}{M_{\rm pl}^{1/2}} \,, \tag{4.4}$$

where the angle brackets indicate the appropriately-averaged value of $g_*^{1/3}$ over the decay interval. For simplicity we use g_* only, since the temperatures occurring in this chapter are above 1 MeV, where $g_{*s} = g_*$. We see how Δ is determined by the properties of the unstable particle, i.e., its mass m_ϕ and lifetime τ_ϕ. Meanwhile it is assumed that $\tau_\phi > t_\phi^{=}$. The pre-decay yield Y_ϕ of the unstable particle depends on both its interactions and the earlier cosmology.

For convenience we would like to rephrase (4.4) in terms of temperatures. Without entropy production after the generation of the pre-decay yield, it is constant till the particle decays. With $\rho = mn$ we find

$$m_\phi Y_\phi = \frac{\rho_\phi}{s} = \left.\frac{\rho_{\rm rad}}{s}\right|_{T=T_\phi^{=}} = \frac{3}{4} T_\phi^{=} \,, \tag{4.5}$$

where we have used (2.16), (2.14) and $\rho_\phi = \rho_{\rm rad}$ at $T_\phi^{=}$.

Using (4.3) we can replace the particle lifetime in (4.4) as

$$\tau_\phi^{\frac{1}{2}} = \left(\frac{45}{2\pi^2}\right)^{\frac{1}{4}} g_*^{-\frac{1}{4}}(T_\phi^{\rm dec})\, M_{\rm pl}^{\frac{1}{2}} \left(T_\phi^{\rm dec}\right)^{-1} . \tag{4.6}$$

Inserting (4.5) and (4.6) into (4.4) we obtain

$$\Delta = 0.75 \, \frac{\left\langle g_*^{1/3} \right\rangle^{3/4}}{g_*^{1/4}(T_\phi^{\rm dec})} \, \frac{T_\phi^{=}}{T_\phi^{\rm dec}} \, . \tag{4.7}$$

This linear growth in temperature can also be expressed in terms of energy densities, since

$$\rho_\phi = n_\phi m_\phi = s Y_\phi m_\phi = \frac{2\pi^2}{45} \, \frac{3}{4} \, g_*(T) \, T^3 \, T_\phi^{=} \, , \tag{4.8}$$

where we have used (2.16) and (4.5). Taking this together with (2.14) we find

$$\frac{\rho_\phi}{\rho_{\rm rad}} = \frac{T_\phi^{=}}{T} \, . \tag{4.9}$$

Thus, for $T = T_\phi^{\rm dec}$ we see that

$$\frac{T_\phi^{=}}{T_\phi^{\rm dec}} = \frac{\rho_\phi}{\rho_{\rm rad}} (T_\phi^{\rm dec}) \, , \tag{4.10}$$

where $\rho_{\rm rad}$ is the density of the "old" radiation, i.e., it does not include the radiation from ϕ decays.

Going back to times earlier than BBN, thus towards temperatures higher than $T_{\rm BBN}$, the first cosmological event important for our scenario is the freeze-out of the LOSP. Standard computations of relic abundances rely on the assumption of radiation domination during freeze-out. If the Universe is dominated by matter during LOSP freeze-out, the LOSP relic abundance is increased. Taking the later dilution by entropy production into account, the overall effect remains a reduction [157]. The effects of different cosmological scenarios on relic densities have been studied [158, 159, 160] and there are computer codes [161]. In particular, the neutralino has been investigated, also considering the production of neutralinos in the decay of a dominating matter particle [162, 163, 164, 165]. Since it is the easiest case to study, we take $T_\phi^{=} < T_{\rm losp}^{\rm fo}$. Thereby the Universe is radiation-dominated during LOSP freeze-out happening at $T_{\rm losp}^{\rm fo}$ and the standard computations hold.[2] Later we will find that the window between BBN and LOSP freeze-out is favoured intrinsically by the scenario.

[2] Using the simple estimate $H(T_{\rm losp}^{\rm fo}) \sim \Gamma(T_{\rm losp}^{\rm fo})$, where Γ is the rate of LOSP annihilations, one finds that for $T_\phi^{=} = T_{\rm losp}^{\rm fo}$ the LOSP abundance is increased by a factor of only $\sqrt{2}$ compared to the standard case of radiation domination, while the freeze-out temperature stays nearly constant.

Sticking to this particular window, we can evaluate (4.7),

$$\Delta \simeq 0.75 \times 10^3 \left(\frac{m_{\rm losp}}{100\ {\rm GeV}}\right)\left(\frac{4\ {\rm MeV}}{T_\phi^{\rm dec}}\right), \tag{4.11}$$

where we have inserted $T_\phi^{=} = T_{\rm losp}^{\rm fo} \simeq m_{\rm losp}/25$ and $\left\langle g_*^{1/3}\right\rangle \simeq 2.2 \simeq g_*^{1/3}(T_\phi^{\rm dec})$ with $g_*(T_\phi^{\rm dec}) = 10.75$, exploiting the fact that for $4\ {\rm MeV} \leq T \leq 4\ {\rm GeV}$ the effective relativistic degrees of freedom are known [13]. If we compare (4.11) and (4.2), we see that the cosmological window between BBN and LOSP freeze-out is not only the first and easiest but also sufficiently large to produce enough entropy to come close to the upper limit on Δ set by thermal leptogenesis itself.

This discussion assumes that there is no further entropy production after the generation of Y_ϕ. Otherwise, Y_ϕ would be diluted like any other relic abundance, i.e., $Y_\phi \to Y_\phi' = \Delta_1^{-1} Y_\phi$. There are two possibilities for the impact of such an earlier entropy increase $\Delta_1 > 1$. i) Despite the dilution, ϕ dominates the Universe for some time. Then the later entropy production by the decay of ϕ is simply reduced by a factor Δ_1, as we see from (4.4) since lifetime and mass of the unstable particle are unchanged. ii) The relic abundance of ϕ becomes so small that the particle never dominates the energy density of the Universe. Then (4.4) does not hold and $S \simeq$ const., i.e., $\Delta = 1$.

After an arbitrary number of late events of entropy production Δ_i labelled by $i = 1, 2, \ldots$, where the index implies a time-ordering with larger i corresponding to later decays, the total dilution factor is

$$\Delta_{\rm tot} = \prod_i \Delta_i \,. \tag{4.12}$$

Here, "late" indicates that all decays happen after the freeze-out of all unstable particles supposed to produce significant entropy, so that their relic abundances are diluted by each earlier decay. This implies

$$\Delta_i = \max\left\{\frac{\Delta_i(\Delta_{j<i} = 1)}{\prod_{j<i}\Delta_j}, 1\right\}, \tag{4.13}$$

where $\Delta_i(\Delta_{j<i} = 1)$ refers to the dilution factor obtained from (4.4) without considering the other dilutions in the calculation of Y_ϕ. As mentioned, we set $\Delta_i = 1$, if a decaying particle does not come to dominate the energy density of the

Universe. One can convince oneself that the total dilution is simply given by the largest individual dilution factor,

$$\Delta_{\rm tot} = \max\left\{\Delta_i(\Delta_{j<i} = 1)\right\}. \qquad (4.14)$$

The upper bound (4.2) limits $\Delta_{\rm tot}$. The dilution of the LOSP abundance can be smaller than $\Delta_{\rm tot}$ if some decays happen before LOSP freeze-out. Thus, we see from (4.13) with (4.2) how our requirement of sufficient entropy production after LOSP freeze-out restricts the possibility of earlier entropy production.

4.3 BBN Constraints on Neutralino LOSP

In this section we present constraints from big bang nucleosynthesis on a neutralino next-to-LSP (NLSP) in the case of a gravitino with a mass of $m_{3/2} = 100$ GeV being the LSP. Thus in this section the LOSP is the lightest neutralino. We investigated those bounds excluding the possibility of entropy production in [6] and found for masses below a TeV a maximal gravitino mass of a few GeV. In the following we assume that the neutralino is diluted after its freeze-out by a factor $\Delta = 10^3$. It is trivial to infer the impact of arbitrary Δs. BBN constraints on a stau NLSP with Δ up to 2×10^4 have been studied in [166, 167], where it has been found that interesting parameter regions are allowed for dilution factors $\Delta \sim 10^3$.

As explicated in Sec. 2.3 the key quantities to determine constraints are the thermal neutralino density after freeze-out and its branching ratios. Both depend on the composition of the neutralino. For a discussion of the neutralino decay channels, branching ratios and more details we refer to [6, 11]. A numerical package, micrOMEGAs 2.2 [36, 168], has been used to compute the neutralino number density before dilution and decay, cp. Sec. 2.2. SOFTSUSY 2.0 [169] has been used to compute the physical mass spectrum from the soft SUSY breaking parameters. To determine model-independent constraints within the MSSM we take all points that are not ruled out by LEP up to a mass of 2 TeV, while we fix the masses of the sfermions to be above 2 TeV. To keep our analysis as general as possible we do not fix all supersymmetric parameters according to a specific scenario, but instead we set the soft SUSY breaking parameters at the low energy scale. We keep the majority of the parameters fixed and vary the gaugino and Higgsino mass parameters to study how the lifetime and number density vary with the mass and composition of the lightest neutralino. We plot these points against the hadronic and electromagnetic BBN bounds in Fig. 4.1–4.3. The bounds are taken from [39] and the different curves are explained in the figure caption. The vertical axis corresponds to the fraction of the energy density that decays to electromagnetic or hadronic products. A $\Delta > 1$ shifts all points downwards on this axis by a factor of Δ. Therefore, it is easy to infer constraints for arbitrary Δ once a plot with fixed Δ is given.

Hadronic bounds are generally more constraining. However, it has been found that large gravitino masses, for which light neutralinos have a low hadronic branching ratio, are excluded by the electromagnetic bounds.

In Fig. 4.1 we consider a mixed bino-wino NLSP. The large dip corresponds to resonant annihilation into the pseudo-scalar Higgs, which happens for our choice of parameters at a neutralino mass $m_\chi \sim 1150$ GeV. To increase $\eta_{\rm B}^{\rm max}$ we are more interested in the region of small NLSP masses, since small NLSP masses allow more easily for small gluino masses in (3.20).

Thanks to the dilution by entropy production the wino overcomes the electromagnetic bounds for any mass even for masses close to the gravitino mass. If the neutralino is mainly bino the electromagnetic bounds are more involved. For a bino-like neutralino, masses below about 450 GeV are excluded. Smaller and smaller masses become allowed when the wino component increases, so that there is allowed space for binos with a non-negligible wino component and $m_\chi \sim 200$ GeV or even smaller masses.

The hadronic bounds exclude most of the parameter space for a bino-wino with dominant bino component even with $\Delta = 10^3$. The mixed bino-wino states with $m_\chi \sim 200$ GeV mentioned above are found on the less conservative ^{6}Li/^{7}Li exclusion line for a decaying particle of 100 GeV mass. Thus we find many points that should not be considered as strictly excluded with masses around 200 GeV and also mixed bino-wino states that are allowed with masses smaller than 200 GeV. Winos with $m_\chi \lesssim 400$ GeV overcome even any less conservative bound. For 400 GeV $\lesssim m_\chi \lesssim 1100$ GeV the wino could violate the less conservative bound, while even larger masses become allowed again.

Altogether, the situation is qualitatively different for bino and wino. While the wino safely overcomes all bounds, especially at low masses, a bino-like neutralino

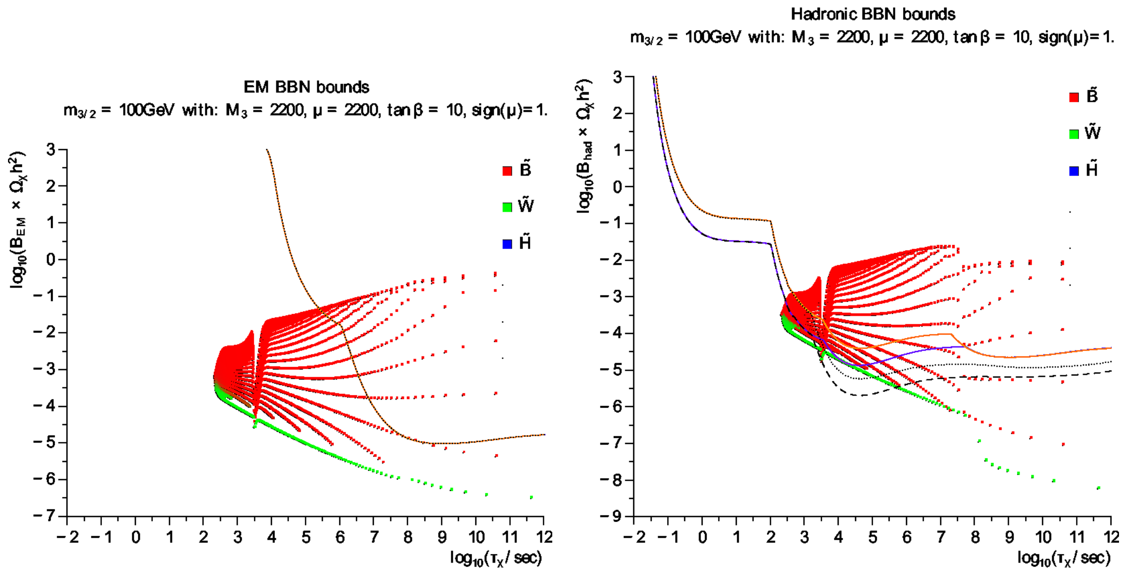


Figure 4.1: Energy density of the bino-wino neutralino decaying into electromagnetic/hadronic products compared with the BBN electromagnetic (left) and hadronic constraints (right) for the case of a 100 GeV gravitino mass and a dilution factor $\Delta = 10^3$. The bounds are taken from [39]: the continuous (dashed) lines correspond to more (less) conservative bounds for the ^{6}Li to ^{7}Li ratio, and the region between the curves should not be considered as strictly excluded. The red/upper and violet/lower curves in the hadronic plots are the constraints for 1 TeV and 100 GeV decaying particle mass, respectively. The mass increases from right to left as heavier particles decay faster. The composition goes from bino at the top to wino at the bottom while the colours give the dominant component. The deformation between the left and right panel is due to the mass dependence of the hadronic branching ratio with lighter NLSPs having lower branching ratios to hadrons. In contrast the electromagnetic branching ratio is always nearly one.

with reasonable mass stays excluded even for much larger dilution factors that would be in contradiction with successful thermal leptogenesis (4.2). However, there is also some space for bino-wino mixed states that are mainly bino with masses below 200 GeV.

In Fig. 4.2 we consider a mixed bino-Higgsino NLSP. The dip is broader in this case, since the Higgsino component that couples to the pseudo-scalar Higgs is larger. Thanks to the dilution the Higgsino overcomes the electromagnetic bounds like the wino for all masses, even though Δ should not be much smaller than

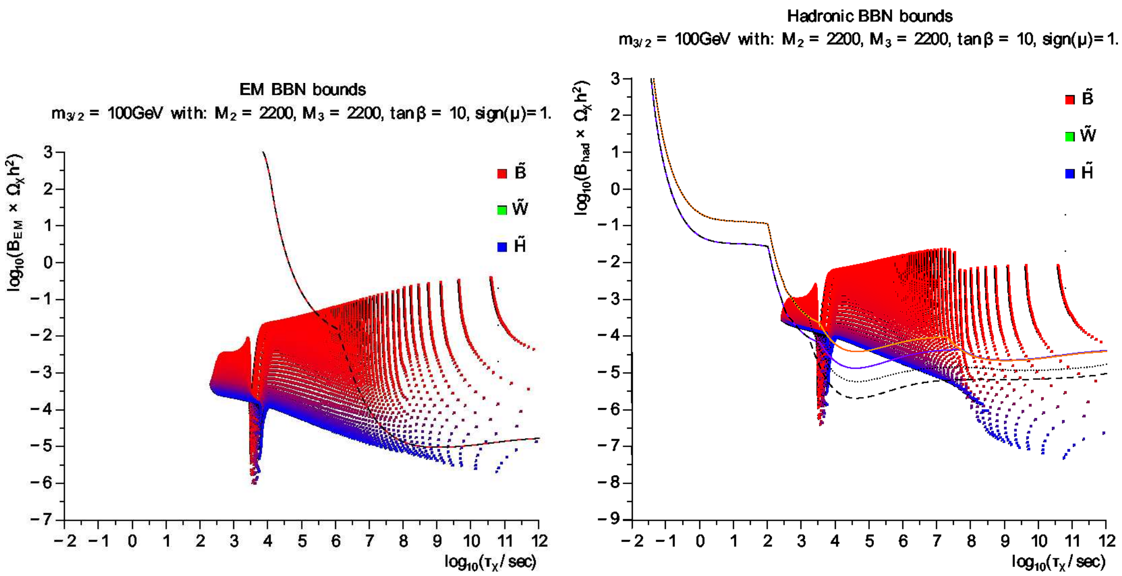


Figure 4.2: Energy density of the bino-Higgsino neutralino decaying into electromagnetic/hadronic products compared with the BBN electromagnetic (left) and hadronic constraints (right) for the case of a 100 GeV gravitino mass and a dilution factor $\Delta = 10^3$. The bounds are taken from [39]: the continuous (dashed) lines correspond to more (less) conservative bounds for the ^{6}Li to ^{7}Li ratio, and the region between the curves should not be considered as strictly excluded. The red/upper and violet/lower curves in the hadronic plots are the constraints for 1 TeV and 100 GeV decaying particle mass, respectively. The mass increases from right to left as heavier particles decay faster. The composition goes from bino at the top to Higgsino at the bottom while the colours give the dominant component. The deformation between the left and right panel is due to the mass dependence of the hadronic branching ratio with lighter NLSPs having lower branching ratios to hadrons. In contrast the electromagnetic branching ratio is always nearly one.

roughly 10^2 to allow for light Higgsino neutralinos. For the bino the situation is comparable to the case of mixed bino-wino. No mixed bino-Higgsino state with a dominant bino component is allowed with masses as low as 200 GeV, though.

Again, the hadronic bounds exclude most of the bino parameter space. Exceptions are found in the dip and at very large masses. There are states with comparable bino and Higgsino components and $m_\chi \gtrsim 200$ GeV—thus not excluded by the electromagnetic bounds—violating the less conservative hadronic bound. Higgsino neutralinos lighter than 250 GeV escape even these constraints, while they are excluded for 670 GeV $\lesssim m_\chi \lesssim$ 1100 GeV.

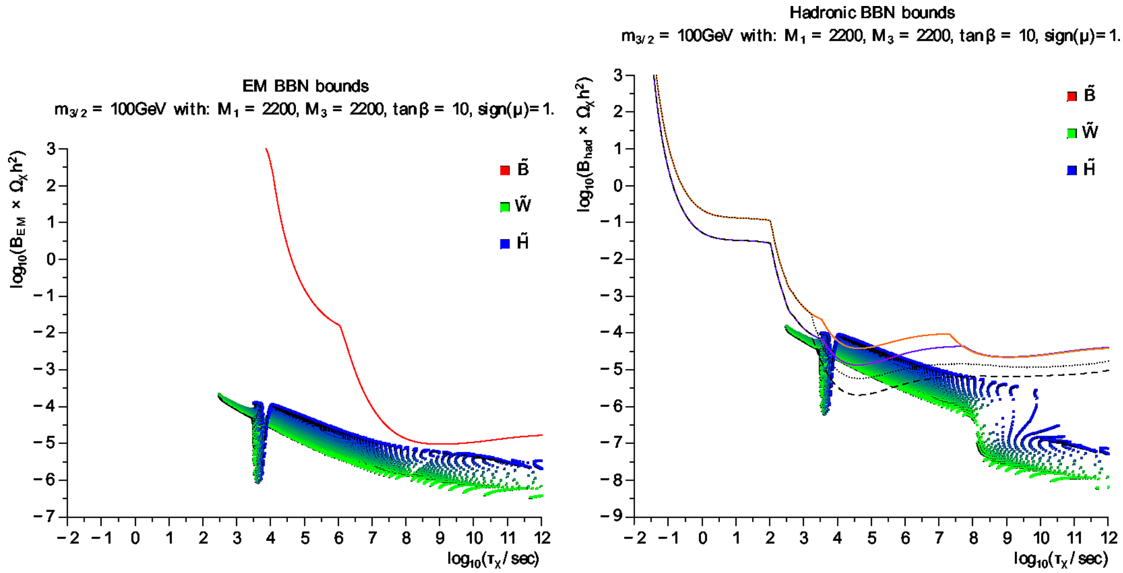


Figure 4.3: Energy density of the wino-Higgsino neutralino decaying into electromagnetic/hadronic products compared with the BBN electromagnetic (left) and hadronic constraints (right) for the case of a 100 GeV gravitino mass and a dilution factor $\Delta = 10^3$. The bounds are taken from [39]: the continuous (dashed) lines correspond to more (less) conservative bounds for the ^{6}Li to ^{7}Li ratio, and the region between the curves should not be considered as strictly excluded. The red/upper and violet/lower curves in the hadronic plots are the constraints for 1 TeV and 100 GeV decaying particle mass, respectively. The mass increases from right to left as heavier particles decay faster. The composition goes from Higgsino at the top to wino at the bottom while the colours give the dominant component. The deformation between the left and right panel is due to the mass dependence of the hadronic branching ratio with lighter NLSPs having lower branching ratios to hadrons. In contrast the electromagnetic branching ratio is always nearly one.

Altogether, we find that for mixed bino-Higgsino only states that are mainly Higgsino allow for preferable small masses but then even down to the gravitino mass. Considering only the conservative hadronic bound from the ^{6}Li to ^{7}Li ratio, in addition maximally mixed states with masses in the region around 230 GeV become allowed. $\Delta > 10^3$ would allow for larger bino components in the mixed bino-Higgsino.

In Fig. 4.3 we consider a mixed wino-Higgsino NLSP. Thanks to the dilution it overcomes the electromagnetic bounds for all masses and mixings. We are especially interested in the small mass region. By vertical shifts of all points we can search for the minimal dilution factor $\Delta^{\rm min}$ to overcome the electromagnetic bounds at small masses. A wino close to the gravitino mass becomes allowed for

$$\Delta^{\rm min}_{\widetilde{W}} \simeq 25\,. \tag{4.15}$$

Larger Δ allows for more wino masses and eventually for light Higgsinos at

$$\Delta^{\rm min}_{\widetilde{H}} \simeq 90\,. \tag{4.16}$$

Considering the hadronic constraints, winos with $m_\chi < 400$ GeV and Higgsinos with $m_\chi < 200$ GeV satisfy all bounds. Wino-Higgsinos with larger mass can be in conflict with the less conservative bound, and the wino overcomes the more conservative one completely. Disregarding the dip, Higgsinos are excluded for a window 700 GeV $\lesssim m_\chi \lesssim$ 1300 GeV.

In summary, entropy production after LOSP freeze-out can allow for a gravitino LSP of 100 GeV mass with a light neutralino LOSP that is the NLSP. This reconciles thermal leptogenesis and gravitino dark matter within the scenario of a light neutralino NLSP. However, this depends on the composition of the lightest neutralino. The wino is the best case due to its small freeze-out abundance. Also a light Higgsino becomes allowed for reasonable dilution factors. A light bino-like neutralino, which is typical for the Constrained MSSM, stays excluded for $m_{3/2} = 100$ GeV even if the possibility of entropy production after freeze-out is exploited.

Coming back to thermal leptogenesis, strictly speaking none of the points in the figures allows for a sufficiently high reheating temperature, since the gluino mass has been fixed at 2.2 TeV. However, in the parameter space regions with smaller neutralino masses, the gluino mass can be lowered without affecting our considerations at all. Consequently, all allowed points with a neutralino mass below a TeV can be compatible with thermal leptogenesis.

4.4 Search for a Viable Candidate

In this section we discuss candidates for the entropy-producing particle ϕ of the previous sections. After enumerating the required properties in general, we exemplify in detail an implementation of the scenario with the axion multiplet.

4.4.1 General Requirements on ϕ

To dilute the LOSP relic density, ϕ must *i) decay after LOSP freeze-out.* But, for sure, it *ii) decays before BBN.* Thus the lifetime τ_ϕ or equivalently the decay temperature T_ϕ^{dec} is constrained to a window. The particle has to be *iii) produced in the early Universe such that it dominates the energy density before BBN.* Meanwhile we stick to the case where its relic density *iv) does not come to dominate before LOSP freeze-out.* Thus the relic density prior to its decay $\rho_\phi = Y_\phi m_\phi s$ is also constrained to a window. Requirements iii)+iv) imply that the dominance of ϕ has to grow with the expansion, which is true for non-relativistic matter. So ϕ is implicitly assumed to become non-relativistic before BBN. The requirements i)+ii) and iii)+iv) constrain two different quantities τ_ϕ and ρ_ϕ, which are determined in different ways but by the same properties of ϕ, namely its couplings and mass.

Requirements i)+ii) constrain only the total decay rate $\Gamma_\phi^{\mathrm{tot}} = \tau_\phi^{-1}$. In fact, the branching ratios of ϕ into the LSP and LOSP are also constrained. The branching ratio into the LOSP $B_{\phi\to\mathrm{losp}+\dots}$ must be so small that the *v) LOSP decay problem is not reintroduced by the decay.* Branching ratios into the LSP are always restricted by overproduction. Especially when the LSP is already produced thermally as in our scenario of thermal leptogenesis with gravitino dark matter, ϕ should *vi) not produce too many LSPs in its decays.* Since ϕ even dominates the energy density of the Universe at its decay, the requirements v)+vi) force the corresponding branching ratios to be—at least—close to zero.

In addition, ϕ must be *vii) compatible with gravitino dark matter.* For example, the gravitino would become unstable due to the existence of ϕ, if it could decay into ϕ. This would take away the explanation for the observed dark matter abundance, if the gravitino lifetime were too short, or by itself be in conflict with other observations.

Finally, *viii) unavoidable by-products of ϕ have to be harmless.* For example, such by-products are the supermultiplet partners in SUSY. They are harmless, if they do not violate ii) or vii), are free of the problems solved by v)+vi) and do not introduce new problems on their own.

No.	Requirement	Comment
i	$T_\phi^{\text{dec}} < T_{\text{losp}}^{\text{fo}}$	to have effect on Ω_{losp}
ii	$T_\phi^{\text{dec}} > T_{\text{BBN}}$	not to spoil BBN
iii	$\frac{\rho_\phi}{\rho_{\text{rad}}}(T_\phi^{\text{dec}}) > 1$	$\mathcal{O}(10) < \Delta < 10^4$
iv	$\frac{\rho_\phi}{\rho_{\text{rad}}}(T_{\text{losp}}^{\text{fo}}) < 1$	for standard LOSP freeze-out
v	$B_{\phi\to\text{losp}+\ldots} \simeq 0$	from LOSP decay problem
vi	$B_{\phi\to\Psi_{3/2}+\ldots} \simeq 0$	from overproduction ($\Omega_{3/2}^{\text{tp}} \simeq \Omega_{\text{DM}}$)
vii	e.g. $\tau_{3/2} \gg t_0$	compatibility with gravitino dark matter
viii	ii) and v)–vii)	for by-products; no new problems

Table 4.1: List of requirements for our scenario of entropy produced by ϕ to dilute the LOSP.

Altogether, the properties of ϕ seem to be highly constrained. We summarise the requirements in Table 4.1. The number of free parameters—mass and couplings—is finite. Since they enter in different ways for different constrained quantities, it is not a matter of course that the scenario of late-time entropy production is viable at all. Especially if Y_ϕ is produced thermally via scatterings, the same coupling might be responsible for the production and the late decay.

On the other hand, many extensions of the SM contain or predict super-weakly interacting and hence long-lived particles, cf. Chapter 3. Such particles generically satisfy i), if not by definition. In order to ensure that they are harmless, one usually demands that they decay before BBN conform to ii). Thermal leptogenesis places the upper limit (4.2) on the maximally allowed dilution, which implies that for decay right before BBN iv) has to hold at least approximately. Considering high reheating temperatures and the growth of $\rho_{\text{mat}}/\rho_{\text{rad}} \propto a$, it is probable that the energy density of late-decaying particles dominates over the radiation energy density at their decay. Thus, iii) can be considered as fulfilled generically, which in fact normally poses a problem. Besides, the decay into superparticles usually has to be suppressed in order to avoid producing too much dark matter and further late-decaying particles like the LOSP or the gravitino, in case it is not the LSP.

Consequently, v) and vi) are generic, too, possibly amended by $B_{\phi\to\mathrm{lsp}+\dots} \simeq 0$, if the gravitino is not the LSP. In any case the scenario has to be compatible with whatever is supposed to form the dark matter, so that vii) is generic. Also viii) arises as a generic requirement on any late-decaying particle and is particularly constraining in supersymmetric models.

In summary, ϕ is severely constrained such that the scenario of entropy production to dilute the LOSP density might appear unappealing. However, in extensions of the SM containing long-lived particles, in principle i)+ii) and v)–viii) are no new requirements and are present—in appropriate form—without considering entropy production at all. If $T_\phi^{\mathrm{dec}} \sim T_{\mathrm{BBN}}$, successful thermal leptogenesis favours the situation of iv). Finally, for the corresponding high reheating temperatures iii) is generic. Thus, all the requirements of Tab. 4.1 either have to be fulfilled or are generically fulfilled. In other words, the solution of the generic problems of long-lived particles may well cause the entropy production desired to solve the LOSP decay problem and thereby reconcile thermal leptogenesis and gravitino dark matter.

With a specific candidate at hand the details have to be worked out. One has to determine whether a candidate is excluded, not useful or can be the solution and how generically this is true. As an example, we investigate the axion multiplet of Sec. 3.4.

4.4.2 Example: Axion Multiplet

We investigate the saxion ϕ_{sax} as candidate for the entropy-producing particle ϕ of the previous sections. Thus the axion a and the axino $\widetilde{a}$ are unavoidable by-products, while it will become clear why the axino is no candidate itself. However, the existence of the saxion is motivated from the strong CP problem and not from our scenario. Since the PQ symmetry would have been restored at the considered reheating temperatures ($\gtrsim 10^{12}$ GeV), axion models with $N > 1$ like the DFSZ model are excluded due to the formation of domain walls.

Thermally Produced Multiplet

As described in Sec. 3.4 the saxion is thermally produced after inflation. It becomes non-relativistic at a temperature $T_{\rm sax}^{\rm nr} \simeq 0.37\, m_{\rm sax}$ around its mass [134]. From $0.37\, m_{\rm sax} \simeq T_{\rm sax}^{\rm nr} > T_{\rm sax}^{=} = T_{\rm losp}^{\rm fo} \simeq m_{\rm losp}/25$ would arise a lower bound

$$m_{\rm sax} > \frac{m_{\rm losp}}{9.25}\,. \tag{4.17}$$

We will find that this is weaker than the lower bound on the saxion mass from early enough decay (4.21) and thus in nearly all cases and at least in the interesting ones does not yield any constraint. With (4.5) we see that if the saxion lives long enough, it dominates the energy density of the Universe below the temperature

$$T_{\rm sax}^{=} = \frac{4}{3} Y_{\rm sax}^{\rm eq} m_{\rm sax} \simeq 1.6\ {\rm GeV} \left(\frac{m_{\rm sax}}{1\ {\rm TeV}}\right)\,. \tag{4.18}$$

We avoid matter domination during LOSP freeze-out by requiring $T_{\rm sax}^{=} < T_{\rm losp}^{\rm fo} \simeq m_{\rm losp}/25$, which gives an upper bound

$$m_{\rm sax} < 2.5\ {\rm TeV} \left(\frac{m_{\rm losp}}{10^2\ {\rm GeV}}\right)\,. \tag{4.19}$$

As we know (4.7), considerable entropy is produced only, if $T_{\rm sax}^{\rm dec} \ll T_{\rm sax}^{=}$. Assuming the saxion to decay dominantly into a gluon pair (3.36), the saxion decay temperature can be derived from (4.3) with $T = T_{\rm sax}^{\rm dec}$ and $t = 1/\Gamma_{\rm sax}^{gg}$. This yields [170]

$$T_{\rm sax}^{\rm dec} \simeq 27\ {\rm MeV} \left(\frac{10^{12}\ {\rm GeV}}{f_a}\right) \left(\frac{m_{\rm sax}}{1\ {\rm TeV}}\right)^{\frac{3}{2}} \left(\frac{\alpha_{\rm s}}{0.1}\right) \left(\frac{10.75}{g_*(T_{\rm sax}^{\rm dec})}\right)^{\frac{1}{4}}\,. \tag{4.20}$$

Here, $\alpha_{\rm s}$ has to be evaluated at $m_{\rm sax}$. As we do not consider an extremely large range of saxion masses, $\alpha_{\rm s}(m_{\rm sax})$ does not vary significantly. Besides, in the range of parameters considered, $g_*(T_{\rm sax}^{\rm dec})$ remains approximately constant. Therefore, we drop the explicit dependence on $\alpha_{\rm s}$ and $g_*(T_{\rm sax}^{\rm dec})$ in the following equations. Together with the bound (2.28) from early enough decay, we obtain the lower limit

$$m_{\rm sax} > 180\ {\rm GeV} \left(\frac{T_{\rm min}^{\rm after}}{4\ {\rm MeV}}\right)^{\frac{2}{3}} \left(\frac{f_a}{10^{12}\ {\rm GeV}}\right)^{\frac{2}{3}}\,. \tag{4.21}$$

If we compare this lower bound with (4.17), we see that (4.21) is stronger as long as

$$\left(\frac{f_a}{1.5\times 10^{10}\ {\rm GeV}}\right)^{\frac{2}{3}} \gtrsim \left(\frac{m_{\rm losp}}{10^2\ {\rm GeV}}\right)\,. \tag{4.22}$$

In any case, the saxion mass is constrained to a window. Since a saxion mass in the TeV range, 10^2 GeV $\lesssim m_{\mathrm{sax}} \lesssim 1$ TeV, is expected, one might conclude from this discussion that the requirements i)–iv) of Tab. 4.1 are naturally fulfilled.

Inserting (4.18) and (4.20) into (4.7) we obtain

$$\Delta \simeq 13 \left\langle g_*^{1/3} \right\rangle^{3/4} \left(\frac{f_a}{10^{12}\ \mathrm{GeV}} \right) \left(\frac{1\ \mathrm{TeV}}{m_{\mathrm{sax}}} \right)^{\frac{1}{2}} . \tag{4.23}$$

For simplicity, we replace $\left\langle g_*^{1/3} \right\rangle$ for the moment by 2.2, the value estimated for (4.11). We insert the bounds on the saxion mass (4.19) and (4.21) to find

$$14 \left(\frac{f_a}{10^{12}\ \mathrm{GeV}} \right) \left(\frac{10^2\ \mathrm{GeV}}{m_{\mathrm{losp}}} \right)^{\frac{1}{2}} < \Delta < 55 \left(\frac{f_a}{10^{12}\ \mathrm{GeV}} \right)^{\frac{2}{3}} \left(\frac{4\ \mathrm{MeV}}{T_{\mathrm{min}}^{\mathrm{after}}} \right)^{\frac{1}{3}} . \tag{4.24}$$

The lower bound on Δ shows that (4.21) is always stronger than (4.17), since the inequality (4.22) is always true as long as significant entropy is produced and if this were not the case, (4.17) would not be considered at all.

If saxions are part of the particle spectrum, (4.24) shows two things. i) It is likely that saxions produce significant entropy in their decays. To avoid it, one would have to restrict the reheating temperature such that they never enter equilibrium, or to choose safe values for f_a and m_{sax}, e.g. $m_{\mathrm{sax}} = 1$ TeV and $f_a = 10^{10}$ GeV. ii) The corresponding dilution factor is much smaller than the maximal value allowed by cosmology and preferred as a solution of the LOSP decay problem.

The dilution factor can be increased by a larger axion decay constant, which makes the saxion more weakly interacting. From (4.24) we see that we need $f_a \simeq 5.2 \times 10^{13}$ GeV to reach the maximum $\Delta \simeq 0.75 \times 10^3$ of (4.11). This increases the decoupling temperature (3.29) and thereby the reheating temperature required to have the saxions in thermal equilibrium. If they did not enter equilibrium, the yield would be $Y_{\mathrm{sax}} \ll Y_{\mathrm{sax}}^{\mathrm{eq}}$, and the saxion would be useless for our purpose. From the requirement $T_{\mathrm{R}} > T_{\mathrm{sax}}^{\mathrm{dcp}}$ and (3.29) we derive the upper bound

$$f_a \lesssim 1.0 \times 10^{12}\ \mathrm{GeV} \left(\frac{T_{\mathrm{R}}}{4 \times 10^{12}\ \mathrm{GeV}} \right)^{\frac{1}{2}} \left(\frac{\alpha_{\mathrm{s}}}{0.03} \right)^{\frac{3}{2}} , \tag{4.25}$$

where $\alpha_{\mathrm{s}}(4 \times 10^{12}\ \mathrm{GeV}) \simeq 0.03$. Already such a T_{R} corresponds—at least in the case of heavy gravitinos—to an allowed but relatively large dilution factor $\Delta \sim 10^3$, cf. (3.19) and (4.2). For the small Δs of (4.24) the situation becomes worse and is in fact inconsistent with itself.

To summarise, if thermally produced saxions are to deliver the desired entropy, we need a large axion decay constant. Then we also need a large reheating temperature to make the saxion enter thermal equilibrium. This results in an overproduction of gravitinos, if they are produced without entering equilibrium, so the scenario is not viable.

On the other hand, if the gravitino is so light that it enters equilibrium after reheating, the relic gravitino density becomes independent of the reheating temperature. Moreover, there could be another saxion production mechanism, which is the alternative we will concentrate on in the next section.

Let us therefore continue discussing the requirements of Tab. 4.1, turning to the decay products of the saxion. Due to R-parity conservation, it must produce sparticles in pairs and thus cannot decay into single gravitinos. Besides, the decay into gravitino pairs is negligible, since it is suppressed by an additional factor of $M_{\rm pl}^2$. Consequently, requirement vi) is satisfied without any effort.

To fulfil requirement v) the decay into any other sparticle pair must be kinematically forbidden, i.e., $m_{\rm sax} < 2\,m_{\rm losp}$. This is the case if the saxion is lighter or not much heavier than the gravitino. Given that one expects both the gravitino and the saxion mass to be of order $m_{\rm soft}$, such a spectrum does not seem unlikely. It is understood that requirement v) does not apply for a light gravitino produced in thermal equilibrium, since the LOSP decays early enough before BBN and does not overproduce gravitinos. To avoid axion overproduction from saxion decay (3.39), we require $x \ll 1$, here.

Up to now, we went through the requirements i)–vi) of Tab. 4.1. We do not see any incompatibilities between the saxion producing entropy and gravitino dark matter. Hence, vii) is fulfilled automatically as well. Facing viii) we have to take care of the unavoidable by-products.

Axion One might expect to obtain severe constraints from cold axion production (3.43). However, for considerable entropy production by the saxion this bound no longer holds, since the Universe is dominated by the saxion—thus matter, not radiation—at the onset of axion oscillations. Then the axion density is given by [171]

$$\Omega_a h^2 \simeq 0.21 \left(\frac{T_{\rm sax}^{\rm dec}}{4\ \rm MeV}\right)\left(\frac{f_a}{10^{15}\ \rm GeV}\right)^2 . \tag{4.26}$$

If we require $\Omega_a/\Omega_{\rm DM} = r \ll 1$, we find

$$\left(\frac{f_a}{10^{14}\ \mathrm{GeV}}\right)^2 \lesssim \left(\frac{r}{0.02}\right)\left(\frac{4\ \mathrm{MeV}}{T_{\rm sax}^{\rm dec}}\right), \tag{4.27}$$

so values of $f_a > 10^{12}$ GeV are indeed allowed. The bound from $\Omega_a \ll \Omega_{3/2} \simeq \Omega_{\rm DM}$ is self-consistently cured by the decaying saxion.

Altogether, there is no problem at all with the axion in our scenario.

Axino This is different for the axino $\widetilde{a}$. Here, we demand $m_{\widetilde{a}} > m_{3/2}$ to keep the gravitino as LSP. Then the mass range for the axino becomes similar to that of the saxion. With a light gravitino and axino NLSP, one would obtain another NLSP decay problem. The situation would be worse than our starting point. If the axino should not produce gravitinos, which would lead to $Y_{3/2} \sim Y_{\widetilde{a}}^{\rm eq} = Y_{3/2}^{\rm eq}$, there must be another decay channel kinematically open, i.e., $m_{\widetilde{a}} > m_{\rm losp}$. In the most interesting case $m_{\rm losp}$ is close to the gravitino mass and the axino fulfils requirement vi).

All decays of the axino—$\widetilde{a} \to \widetilde{g} + g$, $\widetilde{a} \to \widetilde{H} + h$, or others—finally produce LOSPs. The case of a heavy axino that decays after LOSP freeze-out has been studied in [172] for a neutralino dark matter scenario including the weaker decay into a neutralino and the re-annihilation of neutralinos. Since we require a rather large $T_{\rm R}$, the thermally produced axino density (3.30) is large. The resulting neutralino density is many magnitudes larger than the thermal relic abundance (cf. e.g. Fig. 4.1), which in our scenario reintroduces the LOSP decay problem. In fact, the problem becomes much worse. Thus requirement v) is badly violated by the axino.

If we require the axino to decay before LOSP freeze-out, so that we do not have to care about the produced number of LOSPs since they thermalise normally, we find by a derivation analogous to that of (4.21) the lower bound on the axino mass

$$m_{\widetilde{a}} \gtrsim 60\ \mathrm{TeV}\left(\frac{m_{\rm losp}}{10^2\ \mathrm{GeV}}\right)^{\frac{2}{3}}\left(\frac{f_a}{10^{13}\ \mathrm{GeV}}\right)^{\frac{2}{3}}\left(\frac{0.1}{\alpha_{\rm s}}\right)^{\frac{2}{3}}\left(\frac{g_*(T_{\widetilde{a}}^{\rm dec})}{100}\right)^{\frac{1}{6}}. \tag{4.28}$$

Since the gravitino problem could also be solved by making the gravitino comparably unnaturally heavy, such a large axino mass is not considered as a solution here. Furthermore, such an axino would produce considerable entropy with $\Delta_{\widetilde{a}} \simeq 29$.

From the discussion at the end of Sec. 4.2 we know that this would spoil our scenario, since $\Delta_{\widetilde{a}}$ would dilute the saxion but not the LOSP. Thus, the situation is also inconsistent. The required axino mass to achieve $\Delta_{\widetilde{a}} = 1$ would be larger than about 10^4 TeV. For the required early decay and relatively large value of f_a, we find that the axino mass is required to be large also in the DFSZ model. Actually, the decay into Higgsino and Higgs, $\widetilde{a} \to \widetilde{H}h$, becomes subdominant then. Thus the same bound applies for both models in this case.

Altogether, requirement viii) of Tab. 4.1 is badly violated by the axino. Consequently, the thermally produced saxion—and obviously also the axino itself—is ruled out as viable particle to produce significant entropy after LOSP freeze-out. The exception to this conclusion is a light gravitino in thermal equilibrium after reheating, since it allows for high reheating temperatures and the LOSP decay problem is absent.

One may worry then if the strong CP problem can be solved by the Peccei-Quinn mechanism in scenarios of standard thermal leptogenesis with very light gravitino dark matter only. Going through the equations (3.29)–(3.33), especially, from (4.28), we see that the axino becomes harmless for smaller axion decay constants $f_a \lesssim 10^{10}$ GeV with an acceptable axino mass $m_{\widetilde{a}} \gtrsim 1.2$ TeV. Investigating other axino decay channels in the KSVZ model, we find that they require $m_{\widetilde{a}}$ to be larger than the expected gluino mass, which keeps (3.32) to be the dominant decay channel. Independently of f_a the axino must be sufficiently heavier than the gluino, which is expected to be among the heavier superparticles due to the running of its mass. This lower bound is considered in the left column of Tab. 7.1. For instance, with a gluino mass $m_{\widetilde{g}} = 1$ TeV and the parameter values appearing in (4.28) the axino mass is required to be larger than about 1.35 TeV. The bounds in the DFSZ model are derived in Chapter 5.

Since its decay into the gravitino is suppressed like $(f_a/M_{\mathrm{pl}})^2$, the contribution to the gravitino density from axino decay is negligible. However, by inspection of (4.23) we see that in this case the saxion is unable to produce a significant amount of entropy. Then also the axion abundance (3.43) restricts f_a to values smaller than about 10^{10} GeV.

In summary, by making the axino harmless we find that the thermally produced multiplet may also exist in scenarios of thermal leptogenesis with gravitino dark matter that does not enter equilibrium after reheating. However, the axion decay constant is restricted to a small window, 6×10^8 GeV $\lesssim f_a \lesssim 10^{10}$ GeV. Moreover, the thermally produced multiplet is in fact useless for our purpose. This is due to two generic features of the considered scenario: i) Superpartners have similar couplings and masses. ii) The same coupling—or at least couplings of the same strength—are responsible for production and late decay of the entropy-producing particle.

4.4.3 Generic Thermally Produced Particle

The negative result for the saxion can be generalised to other late-decaying particles that are produced in thermal equilibrium by processes controlled by the same coupling as the decay. As the simplest estimate, let us assume that the particle ϕ under consideration couples to SM particles via non-renormalisable interactions suppressed by an energy scale Λ and that the rate of reactions keeping ϕ in thermal equilibrium at high temperatures can be written as

$$\Gamma_\phi^{\mathrm{prod}} = x\,\frac{T^3}{\Lambda^2}\,, \tag{4.29}$$

where x is a model-dependent, dimensionless quantity containing couplings and kinematical factors, for example. The freeze-out from thermal equilibrium occurs for $H \simeq \Gamma_\phi^{\mathrm{prod}}$, which yields the decoupling temperature

$$T_\phi^{\mathrm{dcp}} \simeq \left(\frac{\pi^2 g_*(T_\phi^{\mathrm{dcp}})}{90}\right)^{\frac{1}{2}} \frac{\Lambda^2}{x M_{\mathrm{pl}}} \simeq \frac{2.1\,\Lambda^2}{x \times 10^{18}\ \mathrm{GeV}}\,. \tag{4.30}$$

For the decay we estimate

$$\Gamma_\phi^{\mathrm{dec}} = y\,\frac{m_\phi^3}{\Lambda^2}\,, \tag{4.31}$$

where y contains model-dependent factors. Generically, we expect $x \lesssim y$, where kinematic factors and the relation between number density and temperature tend to lead to a somewhat smaller x. For instance, for the saxion we find $x \simeq 6 \times 10^{-7}$ and $y \simeq 3 \times 10^{-6}$. We obtain the temperature after the decay as discussed in Sec. 4.2,

$$T_\phi^{\mathrm{dec}} \simeq \left(\frac{45}{2\pi^2 g_*(T_\phi^{\mathrm{dec}})}\right)^{\frac{1}{4}} \frac{(y m_\phi^3 M_{\mathrm{pl}})^{\frac{1}{2}}}{\Lambda} \simeq 1.1 \times 10^9\,\frac{y^{\frac{1}{2}} m_\phi^{\frac{3}{2}}\ \mathrm{GeV}^{\frac{1}{2}}}{\Lambda}\,, \tag{4.32}$$

assuming a sufficiently late decay to yield $g_*(T_\phi^{\rm dec}) = 10.75$. Together with the analogy of (4.18), which holds for any thermally produced scalar, and (4.7), we find the dilution factor

$$\Delta \simeq 1.1 \times 10^{-12} \frac{\Lambda}{(y m_\phi \, \mathrm{GeV})^{\frac{1}{2}}} \,, \tag{4.33}$$

estimating as before $\langle g_*^{1/3} \rangle \simeq g_*^{1/3}(T_\phi^{\rm dec})$.

Now we can use (3.19), (4.30), $\Omega_{3/2}^{\rm tp} \leq \Omega_{\rm DM}$ and $T_{\rm R} > T_\phi^{\rm dcp}$ to obtain a lower limit on Δ and thus a constraint on the model parameters,

$$\Delta \gtrsim \frac{6.8}{x} \left(\frac{\Lambda}{10^{14}\ \mathrm{GeV}}\right)^2 \left(\frac{M_{\widetilde{g}}(m_{\rm Z})}{10^3\ \mathrm{GeV}}\right)^2 \left(\frac{100\ \mathrm{GeV}}{m_{3/2}}\right) . \tag{4.34}$$

Furthermore, (4.7), (2.28) and (4.18) yield an upper limit on Δ, which can be combined with (4.34), resulting in

$$\frac{\Lambda}{(x m_\phi)^{\frac{1}{2}}} \lesssim 2.1 \times 10^{13}\ \mathrm{GeV}^{\frac{1}{2}} \left(\frac{10^3\ \mathrm{GeV}}{M_{\widetilde{g}}(m_{\rm Z})}\right) \left(\frac{m_{3/2}}{100\ \mathrm{GeV}}\right)^{\frac{1}{2}} . \tag{4.35}$$

Plugging this bound into (4.33) yields the maximal dilution factor that can be realised with a thermally produced generic scalar,

$$\Delta \lesssim 24 \left(\frac{x}{y}\right)^{\frac{1}{2}} \left(\frac{10^3\ \mathrm{GeV}}{M_{\widetilde{g}}(m_{\rm Z})}\right) \left(\frac{m_{3/2}}{100\ \mathrm{GeV}}\right)^{\frac{1}{2}} . \tag{4.36}$$

Using further combinations of (2.28) and (4.32)–(4.34), we find that this maximal dilution is reached for

$$\Lambda \simeq 1.9 \times 10^{14}\ \mathrm{GeV}\, \frac{x^{\frac{3}{4}}}{y^{\frac{1}{4}}} \left(\frac{10^3\ \mathrm{GeV}}{M_{\widetilde{g}}(m_{\rm Z})}\right)^{\frac{3}{2}} \left(\frac{m_{3/2}}{100\ \mathrm{GeV}}\right)^{\frac{3}{4}} , \tag{4.37}$$

$$m_\phi \simeq 79\ \mathrm{GeV} \left(\frac{x}{y}\right)^{\frac{1}{2}} \left(\frac{10^3\ \mathrm{GeV}}{M_{\widetilde{g}}(m_{\rm Z})}\right) \left(\frac{m_{3/2}}{100\ \mathrm{GeV}}\right)^{\frac{1}{2}} . \tag{4.38}$$

Thus, we conclude that the generalised scenario allows for the production of some entropy, but we do not expect a dilution factor large enough to solve the LOSP decay problem. In order to avoid this conclusion, we have to consider a situation where the mechanisms for production and decay are different, so that the decoupling temperature and the decay temperature are no longer connected.

4.4.4 $\phi_{\rm sax}$ as Oscillating Scalar

As mentioned in Sec. 3.4, the saxion is also produced from coherent oscillations. Since we consider reheating temperatures higher than $T_{\rm sax}^{\rm osc}$ to enable thermal leptogenesis, the produced saxion abundance is independent of $T_{\rm R}$ and given by (3.42). In this way production and decay are disconnected as demanded at the end of the previous section. There is an additional free parameter, $\phi_{\rm sax}^{\rm i}$.

The saxion density is constrained by requirement iv), i.e., that it should not dominate before LOSP freeze-out. For the limiting case of domination onset at $T_{\rm losp}^{\rm fo}$ we obtain from (4.5)

$$\frac{\rho_{\rm sax}^{\rm osc}}{s} = \frac{3}{4} T_{\rm sax}^{=} = \frac{3}{4} T_{\rm losp}^{\rm fo} \,. \tag{4.39}$$

Equalising (3.42) and (4.39) we find for the initial amplitude

$$\left(\frac{\phi_{\rm sax}^{\rm i}}{M_{\rm pl}}\right)^2 = 6\,\frac{T_{\rm sax}^{=}}{T_{\rm sax}^{\rm osc}} \tag{4.40}$$

or equivalently with $T_{\rm sax}^{=} = T_{\rm losp}^{\rm fo} \simeq m_{\rm losp}/25$

$$\left(\frac{\phi_{\rm sax}^{\rm i}}{f_a}\right) \simeq 2.6 \times 10^4 \left(\frac{10^{10}\ {\rm GeV}}{f_a}\right) \left(\frac{8.4\ {\rm GeV}}{m_{\rm sax}}\right)^{\frac{1}{4}} \left(\frac{m_{\rm losp}}{10^2\ {\rm GeV}}\right)^{\frac{1}{2}} \left(\frac{g_*(T_{\rm sax}^{\rm osc})}{228.75}\right)^{\frac{1}{8}} . \tag{4.41}$$

The simplest expectation for the initial amplitude is $\phi_{\rm sax}^{\rm i} \sim M_{\rm pl}$ or $\phi_{\rm sax}^{\rm i} \sim f_a$. Interestingly, the estimate (4.41) yields an initial amplitude $f_a < \phi_{\rm sax}^{\rm i} \sim \sqrt{f_a M_{\rm pl}} < M_{\rm pl}$, if we choose the harmless value $f_a = 10^{10}$ GeV found above. According to (4.7) and (4.20), the maximal dilution (4.11) is achieved for a saxion mass $m_{\rm sax} = 8.4$ GeV on the lower boundary from early enough decay (4.21). The axion multiplet enters thermal equilibrium after reheating, which gives the known limit $m_{\tilde a} \gtrsim 1.2$ TeV (4.28), avoiding problems due to the axino. Thus, we have identified a working scenario where $\rho_{\rm sax}^{\rm osc} \gg \rho_{\rm sax}^{\rm eq}$, which enables significant entropy production while satisfying all requirements.

Smaller f_a are possible, too, provided that they respect the lower bound (3.24). Larger f_a and $\phi_{\rm sax}^{\rm i}$ were not only in conflict with the scenario presented but also with standard cosmology, cf. Tab. 7.1. Furthermore, larger $m_{\rm sax}$ are allowed, while they lead following (4.20) to smaller Δs. From the naturalness point of view, the required small saxion mass—compared to $m_{\tilde a}$ and $m_{\rm soft}$—for maximal Δ might be the biggest concern. Nevertheless, we can conclude that the saxion as oscillating scalar can produce the desired entropy to soften the LOSP decay problem without violating any constraint from cosmology or observations.

We would like to stress that our scenario does not contain more requirements than the scenario with axion multiplet but no entropy production. Instead, we only have to change the allowed windows for some parameters, most importantly $\phi^{\rm i}_{\rm sax}$ and $m_{\rm sax}$. Avoiding axion overproduction by vacuum misalignment becomes even easier. Other restrictions, in particular those on f_a and $m_{\tilde{a}}$, are the same as in the standard scenario, where also often $x \ll 1$ is required in (3.39). Note also that the initial amplitude of the saxion oscillations is allowed to take larger values than in the standard scenario.

4.5 Gravitational Wave Background Signature

In this section we make use of cosmological perturbation theory[3] to derive a simple transfer function that determines the effect of an early matter-dominated era—generalised to any power-law expansion of the Universe—on the gravitational wave background from inflation. We compute the signature of late-time entropy production and argue that the scenario investigated in the previous sections can be falsified by future observations of the gravitational wave background.

Gravitational waves are the tensor perturbations h_{ij} of the space-time metric,

$$ds^2 = a^2(\eta)(-d\eta^2 + (\delta_{ij} + 2h_{ij})dx^i dx^j)\,, \tag{4.42}$$

where $a(\eta)$ denotes the scale factor, cp. (2.2). For computational convenience we use the conformal time η that is defined by $d\eta = dt/a(t)$, where t denotes the physical time used in (2.2). The perturbation h_{ij} is traceless, $h^i_i = 0$, and divergence free, $\partial^i h_{ij} = 0$. The energy density of gravitational waves is then given by [174, 175]

$$\rho_{\rm gw}(\mathbf{x}, t) = \frac{\langle \dot{h}_{ij}(\mathbf{x}, t)\dot{h}^{ij}(\mathbf{x}, t)\rangle}{8\pi G_{\rm N} a^2}\,. \tag{4.43}$$

In this section an overdot indicates the derivative with respect to conformal time η. In Fourier space, the evolution of a gravitational wave mode h in a Friedmann universe (neglecting anisotropic stresses) is determined by [173]

$$\ddot{h} + 2\frac{\dot{a}}{a}\dot{h} + k^2 h = 0\,. \tag{4.44}$$

[3] For a pedagogical introduction see, for instance, [173].

Introducing $x = k\eta$, and assuming power law expansion, $a \propto \eta^q$, this equation has the simple general solution

$$h = \frac{x}{a(\eta)} \left(c_1 j_{q-1}(x) + c_2 y_{q-1}(x) \right) , \tag{4.45}$$

where j_n and y_n denote the spherical Bessel functions of order n as defined, e.g., in [176]. One might replace x/a by x^{1-q} and adjust the pre-factors correspondingly. With this it becomes evident that on super-Hubble scales, $x < 1$, the j-mode is constant while the y-mode behaves as x^{-2q+1}. From this general solution together with (4.43) one infers that $\rho_{\rm gw} \propto a^{-4}$ as soon as the wavelength is sub-Hubble, $x > 1$. (On super-Hubble scales the "energy density" of a mode is not a meaningful concept.)

It is reasonable to assume that both modes have similar amplitudes after inflation, where $x \ll 1$ for all modes of interest. If $q > 1/2$, the y-mode is decaying and soon after inflation we may approximate the solution by the j-mode. Note that a constant value of q corresponds to a constant background equation of state (2.6) and

$$q = 2/(3w + 1) \,. \tag{4.46}$$

For a non-inflating $(3w + 1 > 0)$ universe, $q \geq 1/2$ corresponds to $w \leq 1$ and comprises all cases of interest. During inflation $-1/3 > w \gtrsim -1$ and $q \lesssim -1$.

In standard cosmology, the Universe is radiation-dominated after reheating until the time of equality and matter-dominated afterwards. Therefore $q = 1$ until equality, where $\rho_{\rm rad} = \rho_{\rm mat}$, and $q = 2$ thereafter. Since the energy density in gravitational waves scales like radiation, its fraction is constant on scales which enter the horizon during the radiation-dominated era and scales like $a(\eta_k) \propto \eta_k^2 \propto 1/k^2$ for scales which enter during the matter dominated era. A good approximation to the transfer function $T^2_{\rm eq}(k)$, which relates the energy density per logarithmic k-interval to the amplitude of the gravitational wave spectrum after inflation in standard cosmology, is given in [177]. With this we obtain (for simplicity we neglect changes in the number of effective degrees of freedom and the minor effect of today's vacuum domination)

$$\Omega_{\rm gw}(k) \equiv \frac{1}{\rho_{\rm c}} \frac{d\rho_{\rm gw}(k)}{d\log(k)} = \Omega_{\rm rad} \frac{r\Delta^2_{\mathcal{R}}}{12\pi^2} T^2_{\rm eq}(k) \,, \tag{4.47}$$

where

$$T^2_{\rm eq}(k) = (1 + 1.57\eta_{\rm eq} k + 3.42(\eta_{\rm eq} k)^2)(\eta_{\rm cmb} k)^{-2} \,. \tag{4.48}$$

Here $\Delta_{\mathcal{R}}$ is the amplitude of density fluctuations from inflation as measured in the CMB by WMAP [18], $\Delta^2_{\mathcal{R}} \simeq 2 \times 10^{-9}$, and $\Omega_{\rm rad} \simeq 5 \times 10^{-5}$. The ratio r is the tensor to scalar ratio which depends on the inflationary model. $\eta_{\rm eq}$ and $\eta_{\rm cmb}$ are the conformal time at matter-radiation equality and at CMB decoupling, respectively. This standard spectrum is indicated by the dotted line in Fig. 4.4. For (4.47) a Harrison-Zeldovich spectrum is assumed. For different primordial spectra with spectral index $n_s \neq 1$ and $n_T \neq 0$ of the primordial scalar and tensor fluctuations from inflation, the result (4.47) has to be multiplied by $(k/k_c)^{n_T}$ if the amplitude $\Delta_{\mathcal{R}}$ and r (which then are scale dependent) are determined at the pivot scale k_c. Changes in the background by non-standard evolution of the Universe have previously been studied in [178, 179, 180].

We consider now the scenario of late-time entropy production. We denote the (conformal) time when the entropy-producing particle ϕ begins to dominate the energy density by $\eta_{\rm b}$. We assume that ϕ decays briefly before nucleosynthesis at time $\eta_{\rm e}$. We then compute the final gravitational wave spectrum by matching the radiation solution ($q = 1$) before $\eta_{\rm b}$ to the matter solution ($q = 2$) at $\eta_{\rm b}$ and back to the radiation solution at $\eta_{\rm e}$. After $\eta_{\rm e}$ the Universe follows the standard evolution, so the resulting spectrum simply has to be multiplied by the standard transfer function $T^2_{\rm eq}(k)$. The generic shape of the resulting transfer function T is clear from the general solution: On super-Hubble scales the solution remains constant and $T = 1$. Scales that enter the horizon during the matter-dominated phase at $\eta_b < \eta_k = 1/k < \eta_e$ are suppressed by a factor $a(\eta_k)/a(\eta_e) = (k\eta_e)^{-2}$ since $\rho_{\rm gw} \propto a^{-4}$ while $\rho_{\rm mat} \propto a^{-3}$. Scales which have already entered before matter domination are maximally suppressed by a factor $a(\eta_{\rm b})/a(\eta_{\rm e}) = (\eta_{\rm b}/\eta_{\rm e})^2$.

For sufficiently long matter domination, $\eta_{\rm e}/\eta_{\rm b} \geq 4$, we find the following simple and accurate analytic approximation to the exact result for the transfer function of an intermediate matter-dominated phase (see Appendix A):

$$T^2(k; \eta_e, \eta_b) \simeq \frac{1}{\frac{\eta_{\rm e}^2}{\eta_{\rm b}^2}\left(\frac{2\pi c}{k\eta_b} - \frac{2\pi}{k\eta_e} + 1\right)^{-2} + 1}\,, \tag{4.49}$$

where the best-fit gives $c = 0.5$. The presently observable gravitational wave spectrum is then simply

$$\Omega_{\rm gw}(k) = \Omega_{\rm rad}\frac{r\Delta^2_{\mathcal{R}}}{12\pi^2}T^2_{\rm eq}(k)T^2(k; \eta_e, \eta_b)\,, \tag{4.50}$$

with the fitting formula for $T(k; \eta_e, \eta_b)$ from (4.49). The time when matter dom-

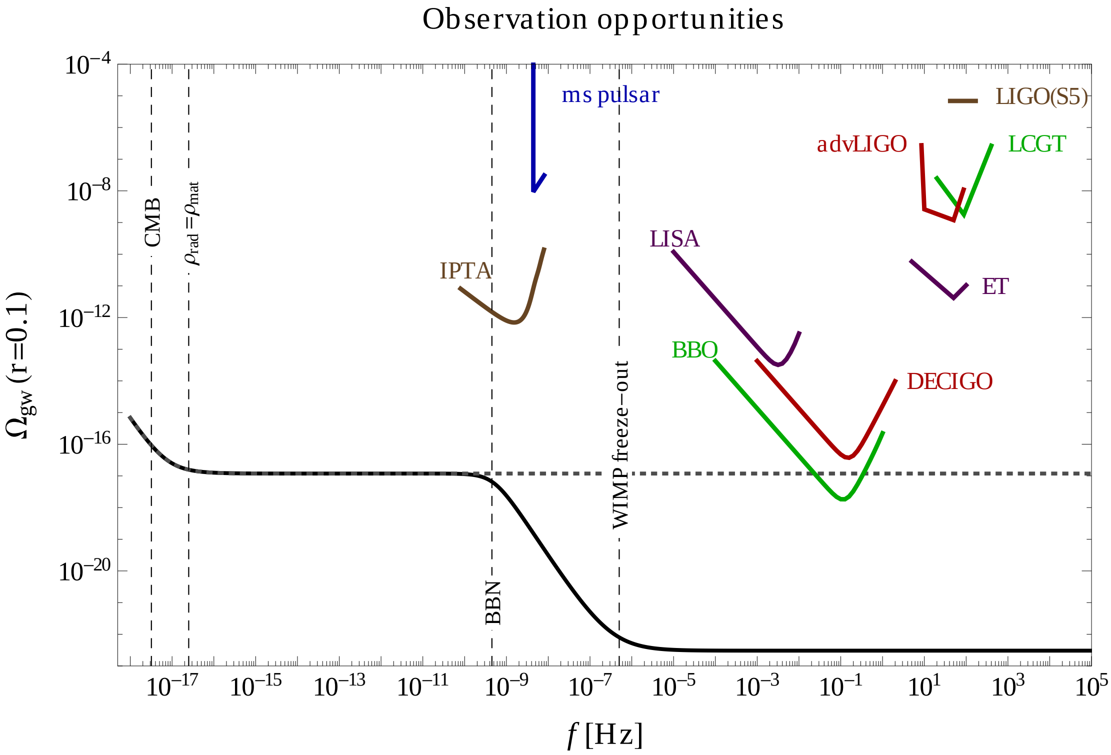


Figure 4.4: Observation opportunities of late-time entropy production in the gravitational wave background. Depicted is the intensity of inflationary gravitational waves vs. their frequency $f = k/(2\pi)$ observed today. The thick solid line shows the expectation for the scenario of late-time entropy production as presented. For comparison the dotted line shows a perfectly flat spectrum. In addition to the spectra, sensitivity curves of existing[4] and future[5] gravitational wave observatories are plotted. Various important and suggestive frequencies/scales are highlighted: CMB indicates the scale of best sensitivity of CMB experiments, and the other frequencies relate to the horizon scale at the indicated event.

ination begins η_b is, for illustration, chosen to be the time of LOSP freeze-out. Using the general formula [181] for conformal time,

$$\eta = 1.5 \times 10^5 \text{ s} \left(\frac{100 \text{ GeV}}{T}\right) g_{\text{eff}}^{-1/6}(T)\,, \tag{4.51}$$

we find $\eta_b \sim 3 \times 10^6$ s, if $T^{\text{fo}}_{\text{losp}} \sim 4$ GeV. The resulting gravitational wave background for $\eta_e \simeq \eta_{\text{BBN}}$ is indicated as the thick solid line in Fig. 4.4.

We compare the spectra with present[4] and future[5] gravitational wave experiments; see footnotes and references therein for details on the experiments.

Interestingly, pulsar timing arrays probe the scale of BBN, where the Universe is surely radiation-dominated. It would be particularly interesting, if they became sensitive to inflationary gravitational waves. The present CMB limit on r from [53, 192] is $r < 0.2$ on CMB scales.

If significant entropy has been produced after reheating, this will be observable in the gravitational wave background from inflation: On CMB scales one will detect the unmodified background from inflation, for example, with the Planck satellite or with a future CMB polarimeter[6]. However, on higher frequencies like mHz or Hz probed by the gravitational wave detectors indicated in Fig. 4.4 no signal will be detected. In other words, if these experiments will detect the signal from the inflationary gravitational wave background as expected from the CMB, this will rule out significant entropy production at cosmic temperatures smaller than $\sim 10^9$ GeV, which corresponds to the sensitivity of BBO.

Even though our derivation presented here is for an intermediate matter-dominated phase, the qualitative result remains true also for a phase of (ultra-short) thermal inflation [198]. Such a phase would dilute gravitational waves on sub-Hubble scales even more strongly, and would render them undetectable for the experiments indicated in Fig. 4.4, while not affecting CMB scales. A (possibly) observable gravitational wave spectrum created after thermal inflation were easily distinguishable from the primordial one by its shape [198].

[4]Existing measurements are LIGO(S5) (Laser Interferometer Gravitational wave Observatory) [182] and the millisecond (ms) pulsar bound [183]. The Parkes- and European Pulsar Timing Array have recently reported sensitivities [184].

[5]Future ground-based observatories are advLIGO [185] (we have scaled up the sensitivity curve found in this reference by roughly a factor of 10, which corresponds to the ratio between the actual LIGO(S5) sensitivity and the one predicted in [185]), LCGT (Large scale Cryogenic Gravitational wave Telescope) [186] and ET (Einstein Telescope) [187]. The limits shown in Fig. 4.4 assume two-detector correlations with an observation time $t_{\rm obs} = 4$ months.
Proposed future satellite missions are LISA (Laser Interferometer Space Antenna) [188], BBO (Big Bang Observatory) [189] and DECIGO (DECI-hertz interferometer Gravitational wave Observatory) [190]. For the satellite missions we assume $t_{\rm obs} = 10$ y.
IPTA [191] is the International Pulsar Timing Array project.

[6]Examples are: ground-based (Keck array [193], QUIET [194]), ballon-bourne (Ebex [195]) or space-bourne (CMBPol [196], Pixie [197]).

Furthermore, from our derivation it is clear that for some other, non-inflationary, intermediate epoch starting at time η_{b} and ending at η_{e}, with equation of state $p = w\rho$ with $-1/3 < w \leq 1$, the inflationary gravitational wave spectrum will be suppressed (or enhanced for $w > 1/3$) by a factor $\alpha(k)$ with

$$\begin{aligned} \Omega_{\mathrm{gw}}^{\mathrm{final}} &= \alpha(k) \times \Omega_{\mathrm{gw}}^{\mathrm{std}} \quad \text{with} \\ \alpha(k) &= \begin{cases} 1 & \text{if } k < \eta_{\mathrm{e}}^{-1} \\ (k\eta_{\mathrm{e}})^{2(3w-1)/(3w+1)} & \text{if } \eta_{\mathrm{e}}^{-1} < k < \eta_{\mathrm{b}}^{-1} \\ \left(\frac{\eta_{\mathrm{e}}}{\eta_{\mathrm{b}}}\right)^{2(3w-1)/(3w+1)} & \text{if } \eta_{\mathrm{e}}^{-1} < k. \end{cases} \end{aligned} \tag{4.52}$$

Our fitting formula for $T(k)$ reproduces this behaviour for an intermediate matter-dominated era, $w = 0$. For $w \neq 0$ the exponents ± 2 in the denominator would have to be replaced by $\pm 2(1 - 3w)/(1 + 3w)$.

Of course there is the caveat that many inflationary models predict only a very low gravitational wave background that cannot be measured by proposed experiments, neither in the CMB nor directly on smaller scales. In this case, an experiment that would be able to detect the background generated in the intermediate matter dominated era as predicted in [199] would be desirable. Such a background is, however, suppressed with respect to the amplitude from inflation by the ratio of the corresponding Hubble rates. Thus there is no hope to detect the gravitational wave background generated in such a short and late period of matter domination.

We summarise the results of this chapter at the end of the thesis, see Chapter 7.

Chapter 5

Broken R-parity

It has been shown that in the case of small R-parity breaking thermal leptogenesis, gravitino dark matter and primordial nucleosynthesis are naturally consistent [200]. Obviously, the violation of R-parity must be small enough to keep the gravitino long-lived enough. This is a decaying dark matter scenario with various possibly observable consequences in the sky, while the violation of R-parity might also be tested at the LHC. Since a consistent cosmology should also enable a solution to the strong CP problem, we investigate in this chapter to what extent the restrictions on the axion multiplet are softened, if R-parity is broken.

The small R-parity breaking couplings allow the LOSP to decay into pairs of Standard Model particles,

$$\text{LOSP} \overset{\not{R}}{\rightarrow} \text{SM} + \text{SM} \ . \tag{5.1}$$

These decays happen instantaneously compared to the Hubble time after LOSP freeze-out or the R-parity-conserving decay into the gravitino. Thus they do not endanger the success of BBN. In this way the LOSP decay problem is circumvented, which at the same time relaxes the constraint on the axino lifetime and thus the allowed range of the axino mass and possibly of other parameters. It is allowed to decay right before BBN, instead of the requirement with conserved R-parity to decay before LOSP freeze-out. Thus its decay temperature can be lowered by three orders of magnitude. In addition, saxion decays are allowed to produce LOSPs at any time before BBN.

The effect on the LOSP decay is the crucial impact of broken R-parity on the constraints on the axion multiplet. All members of the axion multiplet obtain additional R-parity breaking couplings, but R-parity breaking interactions and decays are suppressed by the Peccei-Quinn scale and additionally by the small R-parity breaking couplings. Thus they are produced and decay in the same way as if R-parity were conserved. One exception to this statement occurs, if the axino is the NLSP.

5.1 Axino and Saxion with Conserved R-parity

Axino As discussed in Sec. 4.4.2, the axino has to decay before LOSP freeze-out, if R-parity is conserved. Otherwise, the LOSP decay problem were worsened. The requirements arising from and on the KSVZ axino have been outlined already in Sec. 4.4.2.

In the DFSZ model the decay into Higgsino and Higgs (3.34) can become dominant at low masses, that become allowed for smaller f_a. This leads to a lower bound

$$m_{\widetilde{a}}^{\mathrm{dfsz}} \gtrsim 5.4 \times 10^2\ \mathrm{GeV} \left(\frac{f_a}{10^{11}\ \mathrm{GeV}}\right)^2 \left(\frac{100\ \mathrm{GeV}}{\mu}\right)^2 \left(\frac{m_{\mathrm{losp}}}{10^2\ \mathrm{GeV}}\right)^2 \left(\frac{g_*(T_{\widetilde{a}}^{\mathrm{dec}})}{100}\right)^{\frac{1}{2}} . \tag{5.2}$$

The upper bound on the Peccei-Quinn scale from late axino decay, $f_a \lesssim 10^{10}$ GeV, may thus be softened to $f_a \lesssim 10^{11}$ GeV and for $f_a \lesssim 10^{10}$ GeV we observe that the open decay channel into Higgsino-Higgs suffices in the DFSZ model. Other decay channels are subdominant.

Altogether, only spectra with $m_{\widetilde{a}} > m_{\widetilde{g}}$ are allowed in all axion models. This can be viewed as a problem, because it requires the axino to be heavier than naturally expected. The bound may become softened to $m_{\widetilde{a}} > m_{\widetilde{H}} + m_h$ in the DFSZ model for a small μ-parameter. This is considered in the left column of Tab. 7.1.

Saxion If the saxion is heavy enough to produce superparticle pairs, its decay could lead to the same worsening of the LOSP decay problem as the axino decay. In this situation the lower bound on the saxion mass from the decay into a pair of gluons (3.36) becomes in the end

$$m_{\mathrm{sax}} \gtrsim 7.6 \times 10^2\ \mathrm{GeV} \left(\frac{m_{\mathrm{losp}}}{10^2\ \mathrm{GeV}}\right)^{\frac{2}{3}} \left(\frac{f_a}{10^{10}\ \mathrm{GeV}}\right)^{\frac{2}{3}} \left(\frac{g_*(T_{\mathrm{sax}}^{\mathrm{dec}})}{100}\right)^{\frac{1}{6}} . \tag{5.3}$$

If the DFSZ saxion decays dominantly into Higgs pairs (3.37), we can derive an upper bound on the saxion mass

$$\begin{aligned} m_{\rm sax} &\lesssim 740\ \text{GeV} \left(1 - \frac{4m_h^2}{m_{\rm sax}^2}\right)^{\frac{1}{2}} \left(\frac{10^{11}\ \text{GeV}}{f_a}\right)^2 \\ &\times \left(\frac{\mu}{300\ \text{GeV}}\right)^4 \left(\frac{10^2\ \text{GeV}}{m_{\rm losp}}\right)^2 \left(\frac{100}{g_*(T_{\rm sax}^{\rm dec})}\right)^{\frac{1}{2}} \end{aligned} \tag{5.4}$$

We see that for $f_a \lesssim 10^{10}$ GeV the DFSZ saxion mass is only required to allow for the decay into a pair of light Higgses.

In the following, we re-consider the bounds on the saxion-axion-axion self-coupling x in (3.39) for smaller $f_a \lesssim 10^{10}$ GeV. As mentioned in Sec. 3.4, the produced axions represent a form of dark radiation, i.e., decoupled, relativistic particles not present in the Standard Model. During BBN the energy density of dark radiation $\rho_{\rm dr}$ is constrained to be less than the energy density of one additional neutrino species (2.31), which translates into

$$\left.\frac{\rho_{\rm dr}}{\rho_{\rm SM}}\right|_{\rm BBN} \lesssim 0.14\,. \tag{5.5}$$

Here $\rho_{\rm SM}$ denotes the energy density as expected from the Standard Model and

$$\rho_{\rm dr}|_{\rm BBN} = B_{aa}\,\rho_{\rm sax}|_{T_{\rm sax}^{\rm dec}} \left(\frac{g_{*s}(T_{\rm BBN})}{g_*(T_{\rm sax}^{\rm dec})}\right)^{\frac{4}{3}} \left(\frac{T_{\rm BBN}}{T_{\rm sax}^{\rm dec}}\right)^4, \tag{5.6}$$

where $g_{*s}(T_{\rm sax}^{\rm dec}) = g_*(T_{\rm sax}^{\rm dec})$. Written as bound on the branching ratio B_{aa} of the saxion into two axions (5.5) reads

$$B_{aa} \lesssim 0.4\left(1 + 50\pi^2 x^2\right)^{\frac{1}{2}} \left(\frac{10^{10}\ \text{GeV}}{f_a}\right) \left(\frac{m_{\rm sax}}{10^2\ \text{GeV}}\right)^{\frac{1}{2}} \left(\frac{Y_{\rm sax}^{\rm eq}}{Y_{\rm sax}}\right) \left(\frac{g_*(T_{\rm sax}^{\rm dec})}{10.75}\right)^{\frac{1}{12}}. \tag{5.7}$$

In this inequality we have approximated the decay width of the saxion as $\Gamma_{\rm sax} \simeq \Gamma_{\rm sax}^{gg} + \Gamma_{\rm sax}^{aa}$. Thus our conclusion should hold qualitatively for any axion model. Then the branching ratio reduces to

$$B_{aa} \simeq \frac{\Gamma_{\rm sax}^{aa}}{\Gamma_{\rm sax}^{aa} + \Gamma_{\rm sax}^{gg}} = \frac{x^2}{x^2 + 2\alpha_s^2/\pi^2}\,. \tag{5.8}$$

The value for f_a appearing in (5.7) corresponds to the upper bound on f_a from axino decay and the axion energy density Ω_a (3.44). Since in our scenario the reheating temperature is fixed at rather large values, all members of the axion multiplet enter thermal equilibrium after inflation for such small values of f_a. Therefore the saxion yield $Y_{\rm sax}$ cannot be smaller than the equilibrium value $Y_{\rm sax}^{\rm eq} \simeq 1.21 \times 10^{-3}$. The appearing values for f_a and $m_{\rm sax}$ are chosen to show the worst situation in the considered scenario. Like a smaller f_a or a larger $m_{\rm sax}$, also a larger x leads to an earlier decay, which corresponds to a smaller $\Omega_{\rm sax}$ at its decay. Thus there is a self-curing effect for large x. The bound (5.7) represents indeed an implicit equation for the self-coupling x. Evaluating it for x it turns out that there is no constraint on x at all in the scenario under consideration.

We point out that the absence of any bound on x is due to the expectation $m_{\rm sax} \sim m_{\rm susy}$ and the restriction of f_a to small values appropriate for the considered scenario. Furthermore, $Y_{\rm sax}$ could be much larger than $Y_{\rm sax}^{\rm eq}$, if the saxion is produced from coherent oscillations after inflation.

The saxion might decay before LOSP freeze-out, if the self-coupling is strong enough. Neglecting conservatively the saxion decay into gluons and Higgses we find

$$x \gtrsim 0.9 \left(\frac{10^2~\text{GeV}}{m_{\rm sax}}\right)^{\frac{3}{2}} \left(\frac{f_a}{10^{10}~\text{GeV}}\right) \left(\frac{m_{\rm losp}}{10^2~\text{GeV}}\right) \left(\frac{g_*(T_{\rm sax}^{\rm dec})}{61.75}\right)^{\frac{1}{4}} . \tag{5.9}$$

In this situation there is no additional constraint on the saxion mass. Note that the decay into axions does not produce a significant amount of entropy.

If a new best-fit value demands additional radiation energy in the Universe [201], $\Delta N_{\rm eff} > 0$ in (2.30), like (2.32) or as discussed in Sec. 6.1, we can determine parameter values from (5.7), such that the additional energy is formed by axions from saxion decay. For $f_a \leq 10^{10}$ GeV and $m_{\rm sax} \geq 10^2$ GeV the maximal $\Delta N_{\rm eff}$ is 0.6. However, when these requirements are relaxed also larger $\Delta N_{\rm eff}$ are possible. For example, we obtain $\Delta N_{\rm eff} \simeq 1$ with a rather small saxion mass $m_{\rm sax} = 10$ GeV, $f_a = 10^{10}$ GeV and a self-coupling $x = 0.1$.

5.2 R-parity Violating Case

Axino Since in the case of broken R-parity the LOSP decay problem is absent, the axino may decay right before BBN. Produced superparticles decay promptly into particles of the Standard Model, which thermalise normally. Then the lower bound (4.28) becomes

$$m_{\widetilde{a}} \gtrsim 410 \text{ GeV} \left(1 - \frac{m_{\widetilde{g}}^2}{m_{\widetilde{a}}^2}\right)^{-1} \left(\frac{f_a}{10^{13} \text{ GeV}}\right)^{\frac{2}{3}} \left(\frac{T_{\text{min}}^{\text{dec}}}{4 \text{ MeV}}\right)^{\frac{2}{3}}, \tag{5.10}$$

where we still assume the decay channel $\widetilde{a} \to \widetilde{g}g$ to dominate and take $T_{\text{min}}^{\text{dec}}$ as lower bound on the temperature after the particle decay. From (4.23) we know that the axino and/or the saxion might dominate the energy density, when they decay late. We know as well that the occurring dilution is so small that there arises no constraint from entropy production by the thermally produced axion multiplet. As in the following we omit the dependence on $g_*(T_{\widetilde{a}}^{\text{dec}} = T_{\text{min}}^{\text{dec}}) = 10.75$. We see that due to $T_{\text{min}}^{\text{dec}} \ll T_{\text{nlsp}}^{\text{fo}}$ the axino mass bound becomes so weak that masses much smaller than the expected gluino mass would become allowed.

In this situation the decays of the axino always depend on which channels are kinematically open and thus, in principle, on the full spectrum. This is a qualitative difference to the R-parity conserving case. Particularly interesting is the possibility of an axino next-to-NLSP, which we will assume in the following. Then its dominant decay is fixed into the NLSP, in other words, into the LOSP. From (5.10) we see that the LOSP were allowed to be a gluino. The LHC raised previous lower bounds for such a long-lived gluino already to $m_{\widetilde{g}} \gtrsim 550$ GeV [202, 203, 204].

From (5.2) we see that the DFSZ model would in addition allow for a Higgsino LOSP, if the corresponding decay channel were kinematically open.

Since the lightest neutralino is likely one of the lightest superparticles in the spectrum, one interesting decay is into neutralino and photon with [135]

$$\Gamma(\widetilde{a} \to \chi_i^0 + \gamma) = \frac{\alpha_{\text{em}}^2 C_{a\chi_i^0\gamma}^2}{128\pi^3} \frac{m_{\widetilde{a}}^3}{f_a^2} \left(1 - \frac{m_{\chi_i^0}^2}{m_{\widetilde{a}}^2}\right)^3, \tag{5.11}$$

where $C_{a\chi_i^0\gamma} = (C_{aBB}/\cos\Theta_W) N_{\chi_i^0 \widetilde{B}^0}$, while $N_{\chi_i^0 \widetilde{B}^0}$ is the bino fraction of the i-th neutralino and Θ_W denotes the weak mixing angle. We take the electromagnetic coupling constant $\alpha_{\text{em}}(\mu) = \alpha_{\text{em}}(m_{\widetilde{a}}) \simeq 1/128$. The axion to two B bosons coupling C_{aBB} varies for different implementations of different axion models.[1] For

[1]For example, in the DFSZ model with (d^c, e) unification, $C_{aBB} = 8/3$. In the KSVZ model, for different electromagnetic charges of the heavy quark $e_Q = 0, -1/3, 2/3$, $C_{aBB} = 0, 2/3, 8/3$, respectively. Below the QCD scale, C_{aBB} is reduced by 1.92 [205].

simplicity we set $C_{aBB} = N_{\chi_1^0 \widetilde{B}^0} = 1$. We find

$$m_{\widetilde{a}} \gtrsim 45 \text{ GeV} \left(1 - \frac{m_{\chi_1^0}^2}{m_{\widetilde{a}}^2}\right)^{-1} \left(\frac{f_a}{10^{10}\text{ GeV}}\right)^{\frac{2}{3}} \left(\frac{T_{\text{min}}^{\text{dec}}}{4\text{ MeV}}\right)^{\frac{2}{3}} \tag{5.12}$$

and conclude that a neutralino with a substantial bino component can be the LOSP, if the axino is the next-to-NLSP. Thus a superparticle spectrum with $m_{\widetilde{g}} > m_{\widetilde{a}} > m_{\chi_1^0}$ is now easily possible without any BBN conflict.

If the axino decay is fixed into a sneutrino, it decays via an intermediate neutralino into a photon and a sneutrino-neutrino pair, i.e., $\widetilde{a} \to \gamma\chi^{0*} \to \gamma\widetilde{\nu}\nu$. This process is suppressed compared to (5.11) by an additional power of α_{em} and further factors depending on the neutralino composition and the exact spectrum. Thus in case of a sneutrino LOSP an axino next-to-NLSP might be possible only for parameter values at the boundaries of the allowed region and a tuned spectrum. Consequently, this situation is disfavoured.

For other sfermion LOSPs the situation depends much more strongly on the axion model. In the DFSZ model we estimate the kinematically unsuppressed decay width of the axino into a stop-top pair using the Lagrangian of [206] as

$$\Gamma(\widetilde{a} \to \widetilde{t} + t) \simeq \frac{m_{\widetilde{a}}}{16\pi} \left(\frac{m_t X_u}{f_a}\right)^2 , \tag{5.13}$$

where m_t denotes the fermion mass and $X_u = 1/(\tan^2\beta + 1)$. This decay arises from a dimension-four operator that becomes important at low decay temperatures and accordingly small masses. If only this channel were open, the resulting lower mass bound on the axino would become

$$m_{\widetilde{a}} \gtrsim 2.4 \times 10^2 \text{ GeV} \left(\frac{f_a}{10^{12}\text{ GeV}}\right)^2 \left(\frac{T_{\text{min}}^{\text{dec}}}{4\text{ MeV}}\right)^2 \left(\frac{\tan\beta}{10}\right)^4 \left(\frac{173\text{ GeV}}{m_t}\right)^2 . \tag{5.14}$$

The lower bound would practically be given by f_a and the requirement to have the channel kinematically unsuppressed, since a long-lived stop with mass below 249 GeV is excluded according to the CDF experiment [207]. It might well happen that the decay of the next-to-NLSP ($\widetilde{a}$) into the NLSP ($\widetilde{t}$) = LOSP and its superpartner (t) is kinematically forbidden. The axino would decay violating R-parity. This is the same if the $\widetilde{a}$ is the NLSP. We comment on this excluded case below.

The decay width into other sfermions is given by (5.13), if m_t is replaced by the corresponding fermion mass and $X_u \to X_d = \tan^2\beta/(1 + \tan^2\beta)$ if appropriate.

For example, the decay width into stau and tau is (5.13) with the replacements $m_t \to m_\tau \simeq 1.78$ GeV and $X_u \to X_d$. The corresponding lower bound on the axino mass becomes

$$m_{\tilde{a}} \gtrsim 2.3 \times 10^2\ \text{GeV} \left(\frac{f_a}{10^{12}\ \text{GeV}}\right)^2 \left(\frac{T_{\text{min}}^{\text{dec}}}{4\ \text{MeV}}\right)^2 . \tag{5.15}$$

Thus, for f_a below 10^{12} GeV also this channel is restricted by the requirement of kinematic accessibility only. Since $m_\tau/m_{\tilde{\tau}} \ll 1$, this is not a strong constraint, if the stau is not much heavier than the gravitino. Consequently, if the stau is the LOSP, the DFSZ axino may be the next-to-NLSP.

Since the other leptons are lighter, in these cases the bound becomes tighter. While the smuon stays possible, the selectron would require $m_{\tilde{a}} \gtrsim 1$ TeV even for parameter values at the boundaries. These considerations expand to the quarks lighter than the top. The superpartners of the bottom, charm, and strange quarks are possibilities. For the down squark the Peccei-Quinn scale needs to be close to the lower limit. The situation for the up squark is even worse than the one for the selectron.

In the KSVZ model axino-sfermion-fermion interactions are one-loop-suppressed in the low-energy effective theory [208, 209], which weakens axino to sfermion-fermion decays substantially relative to the DFSZ case. The tree-level decay via intermediate neutralino into other sfermion-fermion pairs is comparable to the decay into sneutrino-neutrino. Consequently, the KSVZ axino is disfavoured, if the axino decay is fixed into a sfermion-fermion pair.

If the axino were the NLSP, i.e., for a spectrum with $m_{\text{losp}} > m_{\tilde{a}} > m_{3/2}$, it would decay either into the gravitino or via R-parity violation. Both decay modes are strongly suppressed by the Planck scale or by the Peccei-Quinn scale and the R-parity violating coupling, respectively. Therefore, the lifetime of the axino would always become much larger than the time of BBN. This is independent of whether R-parity is broken by bilinear or trilinear couplings [210, 211, 212]. Since $\Omega_{\tilde{a}} \gg 1$, such a late decay would spoil the predictions of BBN. We conclude that an axino NLSP stays excluded in the R-parity violating scenario. By the way, along the same reasoning an axino LSP with the required broken R-parity is excluded.

Saxion If R-parity is broken also superparticle pairs from saxion decays are harmless. If its dominant decay channel is into a pair of (massless) gluons, the most severe bound on the saxion mass is found to be similar to (5.10). So for $f_a \sim 10^{10}$ GeV the saxion mass is practically not constrained in the R-parity violating case. However, considerations concerning the self-coupling x are not affected by R-parity violation.

Since also bounds on the initial amplitude of saxion oscillations $\phi^{\rm i}_{\rm sax}$ are not affected by R-parity violation, we quantify them here to see how far they can be relaxed, if we allow for considerable entropy production as described in Chapter 4. The dilution factor due to the decays of saxions produced in coherent oscillations is given by

$$\begin{aligned}\Delta \;\simeq\; & 4\times 10^{-10}\left(\frac{T_{\rm R}}{2\times 10^{9}\ {\rm GeV}}\right)\left(\frac{f_a}{10^{10}\ {\rm GeV}}\right)^{3} \\ & \times\left(\frac{\phi^{\rm i}_{\rm sax}}{f_a}\right)^{2}\left(\frac{1\ {\rm TeV}}{m_{\rm sax}}\right)^{\frac{3}{2}}\left(\frac{g_*(T^{\rm dec}_{\rm sax})}{10.75}\right)^{\frac{1}{4}}. \end{aligned} \tag{5.16}$$

Values for Δ smaller than one lead to the standard scenario with $\Delta = 1$. In this discussion we fix the reheating temperature at its lower boundary from thermal leptogenesis, $T_{\rm L}^{\rm min} = 2\times 10^9$ GeV or $2\Delta\times 10^9$ GeV with $\Delta > 1$. Requiring standard cosmology, i.e. $\Delta \leq 1$, the tightest bound on the initial amplitude is found for the maximal axion decay constant $f_a = 10^{10}$ GeV and a small saxion mass, for concreteness $m_{\rm sax} = 10^2$ GeV. It is $\phi^{\rm i}_{\rm sax} \lesssim 7\times 10^{13}$ GeV. In comparison, the loosest bound is found for the minimal axion decay constant $f_a = 6\times 10^8$ GeV and a rather large saxion mass, for concreteness $m_{\rm sax} = 1$ TeV. It is $\phi^{\rm i}_{\rm sax} \lesssim 1.4\times 10^{15}$ GeV. Thus both bounds are far below the Planck scale. Conservatively allowing for smaller saxion masses as well, we summarise these bounds in Table 7.1 as $\phi^{\rm i}_{\rm sax} \lesssim (10^{13}$–$10^{15})$ GeV.

In the case of considerable entropy production, for a certain Δ the reheating temperature has to become larger than the temperature at the onset of saxion oscillations, i.e., $T_{\rm R} > T^{\rm osc}_{\rm sax}$. In this case larger initial amplitudes may be allowed, while at the same time larger axion decay constants become allowed. The tightest bound on the initial amplitude in the scenario with $\Delta = \Delta^{\rm max} = 10^4$ is found for the maximal axion decay constant $f_a = 4\times 10^{12}$ GeV and the minimal saxion mass $m_{\rm sax} \sim 300$ GeV from the decay into a gluon pair. It is $\phi^{\rm i}_{\rm sax} \lesssim 4\times 10^{14}$ GeV. In comparison, the loosest bound is found for the minimal axion decay constant $f_a = 6\times 10^8$ GeV and a rather large saxion mass, for concreteness $m_{\rm sax} = 1$ TeV. It is $\phi^{\rm i}_{\rm sax} \lesssim 5\times 10^{16}$ GeV. We summarise these bounds, that are still far below the

Planck scale, in Table 7.1 as a range $\phi^{\rm i}_{\rm sax} \lesssim 5 \times (10^{14}\text{–}10^{16})$ GeV.

Axion First, the lower limit on the axion decay constant from axion physics (3.24) is not changed by the additional R-parity violating interactions. Since here the reheating temperature is fixed at about $2\Delta \times 10^9$ GeV from the requirement of successful standard thermal leptogenesis, the situation is particularly interesting. There are two possible cases: i) topological defects are not created after inflation, so we do not have to care about them or ii) they occur and we have to take them into consideration. To avoid topological defects completely,

$$f_a > 2 \times 10^9 \text{ GeV} \frac{\Delta}{N}\,, \tag{5.17}$$

which could lead, depending on N, to a stronger lower bound on f_a than (3.24). In the case without entropy production ($\Delta = 1$), this favours models with $N \geq 4$ such as the DFSZ model. If (5.17) is violated, $N = 1$—fulfilled by the KSVZ model—still avoids domain walls and the axion density from strings, $\Omega_a^{\rm str} \sim 10 \times \Omega_a^{\rm mis}$, gives a tighter upper bound than (3.44),

$$f_a < 1.3 \times 10^9 \text{ GeV} \quad \text{for} \quad a_0' = N = \Delta = 1\;,\; r = 0.04\,, \tag{5.18}$$

where a_0' comprises model-dependent factors at the production of axions from cosmic strings. Combining the above considerations, only the small interval $(1.3\text{–}2) \times 10^9$ GeV for f_a might be excluded. In this sense, the allowed band for f_a is not changed.

With entropy production ($\Delta > 1$) the reheating temperature is raised to compensate the dilution of the baryon asymmetry, so the Peccei-Quinn symmetry becomes restored for a larger range of values of f_a. We consider entropy production after the QCD phase transition by late particle decay and estimate the axion abundance from cosmic strings as $\Omega_a^{\rm str} \sim 10 \times \Omega_a^{\rm mis}/\Delta$ with $\Omega_a^{\rm mis}$ as in (3.43). Then the upper bound (5.18) on f_a is softened by a factor of $\Delta^{6/7}$, because the axions are diluted, while the amount produced remains the same. The maximal $\Delta^{\rm max} \sim 10^4$ corresponds to $f_a \lesssim 4 \times 10^{12}$ GeV, so in this situation the Peccei-Quinn symmetry is probably restored, because $T_{\rm R} \sim 2 \times 10^{13}$ GeV. We therefore list this upper bound on f_a in Table 7.1. Larger values are possible, if the symmetry is not restored and if one accepts the axino to be heavier than the gluino. We know already that if the Universe is dominated by matter at the onset of axion oscillations at

$T_a^{\mathrm{osc}} \sim 1$ GeV the upper bound on f_a disappears, see (4.27). In this case the axion decay constant is constrained more strongly by Ω_a^{str} and too late axino and saxion decays. Altogether, a large dilution factor opens up—at least for $N = 1$—more parameter space, but the situation depends on the time of entropy production.

The given bounds from the different production mechanisms of axions refer in each case to "standard values" in parameter space. They can be relaxed or circumvented in "non-standard" scenarios. Constructing models that realise a small Ω_a is beyond the scope of this thesis.

We summarise the results of this chapter at the end of the thesis, see Chapter 7 and especially Table 7.1.

Chapter 6

Dark Radiation

It has been recognised that an axino LSP of mass $m_{\widetilde{a}} \lesssim \mathcal{O}\,(\text{keV})$ with a gravitino next-to-LSP of mass $m_{3/2} \sim \mathcal{O}\,(100 \text{ GeV})$ might provide a natural solution to the cosmological gravitino problem [110]. Since any gravitino abundance decays invisibly into axino-axion pairs,

$$\Psi_{3/2} \to \widetilde{a} + a\,, \tag{6.1}$$

reheating temperatures as high as $T_{\mathrm{R}} \sim 10^{15}$ GeV were claimed to be possible. Due to their suppressed couplings axion and axino are indeed decoupled from the thermal bath at such late times when the gravitino decays. The axino is light to avoid overproduction. At the same time, the LOSP can decay into axino and some Standard Model partner like the photon,

$$\text{LOSP} \to \widetilde{a} + \gamma\,. \tag{6.2}$$

Altogether, the decay problems and overproduction constraints pointed out in Sec. 3.3 are circumvented. It seems that successful thermal leptogenesis is possible in supergravity, while the PQ mechanism is implemented. An extensive investigation of the phenomenological viability of the considered mass hierarchy, $m_{\text{soft}} \sim m_{\text{losp}} > m_{3/2} > m_{\widetilde{a}}$, reported on numerous restrictions of the PQ and MSSM parameter space [142]. In particular, much tighter constraints on the reheating temperature dependent on the PQ scale and/or arising from the assumed bino LOSP were found.

In this chapter we point out that the reheating temperature is also constrained from the allowed amount of relativistic particles decoupled from the thermal bath. We have dubbed such particles “dark radiation“, if they are not contained in the Standard Model, so they are neither photons nor neutrinos. Such dark radiation arises from the gravitino decays, because the axions and axinos with m_a, $m_{\tilde{a}} \ll m_{3/2}$ are emitted relativistically and thus contribute to the radiation energy density. Actually, we will show that by requiring successful leptogenesis the gravitino problem can turn out as a fortune within this scenario, because the gravitino decay might explain a possible increase in the radiation energy density of the Universe after the time of BBN.

6.1 Observational Constraints

We discussed constraints on the radiation energy density given in terms of the effective number of neutrino species N_{eff} during BBN in Sec. 2.3. Here, we assume that $N_{\text{eff}}^{\text{BBN}} = N_{\text{eff}}^{\text{SM}} = 3.046$ and elaborate on the possibility that N_{eff} increased after the time of primordial nucleosynthesis, i.e., $\Delta N_{\text{eff}}^{\text{BBN}} = 0 < \Delta N_{\text{eff}}^{\text{CMB}}$. It is a new opportunity to determine the amount of radiation in the Universe from observations of the CMB alone with precision comparable to that of BBN. The current constraints from observations of the CMB are much stronger than previous ones [213] but still weaker than those from BBN. Measurements from the WMAP satellite [18] using the first and third acoustic peaks and the ground-based Atacama Cosmology Telescope (ACT) [192] using observations of the third through the seventh peaks are complementary, since they span a broad range of scales. This allows an estimate from the CMB alone [192]

$$\text{(CMB alone - ACT)} \quad N_{\text{eff}}^{\text{CMB}} = 5.3 \pm 1.3 \qquad (68\%\ \text{CL}). \tag{6.3}$$

Recently the ground-based South Pole Telescope (SPT) published [53]

$$\text{(CMB alone - SPT)} \quad N_{\text{eff}}^{\text{CMB}} = 3.85 \pm 0.62 \qquad (68\%\ \text{CL}). \tag{6.4}$$

Combined with measurements of today’s Hubble expansion rate H_0 and baryon acoustic oscillations (BAO) the current constraints are [192]

$$\text{(ACT + WMAP + BAO} + H_0) \quad N_{\text{eff}}^{\text{CMB}} = 4.56 \pm 0.75 \qquad (68\%\ \text{CL}) \tag{6.5}$$

and [53]

$$\text{(SPT + WMAP + BAO} + H_0) \quad N_{\text{eff}}^{\text{CMB}} = 3.86 \pm 0.42 \qquad (68\%\ \text{CL}), \tag{6.6}$$

respectively. These limits are consistent with $N_{\rm eff}^{\rm SM}$ at the 2-σ level and any deviation may well be due to systematic errors. However, there is a hint of tension and the central values are off the standard value by $\Delta N_{\rm eff} \sim 0.81$-$2.3$.[1] More important, the Planck satellite will significantly increase the precision of CMB observations to $\Delta N_{\rm eff} \simeq 0.26$ [215] or even better [216]. Thus Planck could reveal a difference between $N_{\rm eff}^{\rm SM}(= N_{\rm eff}^{\rm BBN})$ and $N_{\rm eff}^{\rm CMB}$ at the level of 3- to 5-σ, if the central values from the current measurements are accurate; see also [217]. This could be taken as another hint for physics beyond the two standard models. Improvements in the determination of $N_{\rm eff}^{\rm BBN}$ would become crucial.

To have an effect on the effective number of neutrino species measured from the CMB, it is necessary that the additional radiation was generated before the observable modes of the CMB have reentered the horizon. This requirement constrains the maximum lifetime $\tau^{\rm max} < 1650$ y $\simeq 5.2 \times 10^{10}$ s [218]. This time is before the time of matter-radiation equality, $\tau_{\rm CMB}^{\rm max} < t_{\rm eq} \sim 10^{12}$ s.

We should search for explanations from particle physics for such an increase in radiation [219, 220, 221], especially, because other explanations are missing, if the current mean values are accurate. The late emergence of visible radiation like photons is practically excluded. This would not only spoil the success of BBN, but also induce a chemical potential for the photons that is bounded by CMB observations [54, 55, 56].

6.2 Emergence of Dark Radiation

The discussed observations constrain the emergence of dark radiation. Not to affect BBN and to have an effect on measurements of $N_{\rm eff}$ from the CMB, the lifetime of a decaying matter particle τ is constrained to lie in the range

$$t_{\rm BBN}^{\rm end} \sim 20 \text{ min} \simeq 1.2 \times 10^3 \text{ s} < \tau < 5.2 \times 10^{10} \text{ s} \simeq \tau_{\rm CMB}^{\rm max} \tag{6.7}$$

or, equivalently, the cosmic temperature at the particle decay $T_{\rm BBN}^{\rm end} \sim 33$ keV $> T^{\rm dec} > 5$ eV $\simeq T_{\rm CMB}^{\rm min}$. Since $\tau_{\rm CMB}^{\rm max} < t_{\rm eq}$ in standard cosmology the Universe is radiation dominated at these times, so that $T \propto t^{-1/2}$.

[1] We do not take into account Lyman-α forest data, which may probe $N_{\rm eff}$ at even later times and seems to favour even larger central values [214] disfavouring the standard value at 2-σ, but also seem to be afflicted with large systematic uncertainties.

We estimate the gravitino decay width with the following considerations. In the gravitino centre-of-mass frame its two-body decay width is in general given by $\Gamma_{3/2} = |\mathcal{M}|^2|\vec{p}_1|/(8\pi m_{3/2}^2)$. If both decay products are much lighter than the gravitino, their equal with opposite sign 3-momenta, $|\vec{p}_1|=|\vec{p}_2|$, are well approximated by the leading term in $|\vec{p}_1| = \frac{1}{2}(m_{3/2} - \mathcal{O}(m_1, m_2))$. The squared amplitude $|\mathcal{M}|^2$ is on dimensional grounds $\propto m_{3/2}^4/M_{\rm pl}^2$. We have to sum over the outgoing axino spin states and average over the incoming gravitino spin states, which yields a factor $2/4 = 1/2$. The rest of the squared amplitude will be a factor $1/6 \times (1 \pm c'(m_{\tilde{a}}/m_{3/2})^2 \pm c''(m_{\tilde{a}}/m_{3/2})^4 \pm \ldots)$, where at the leading order the gravitino polarisation sum gives a factor $2/3$. We assume that a detailed analysis renders the typical additional factor $1/4$. The coefficients $c', c'', \ldots$ are usually $1 \leq c', c'' \leq 12$. Altogether, we estimate the gravitino decay width

$$\Gamma_{3/2} \simeq \frac{m_{3/2}^3}{192\pi M_{\rm pl}^2}$$

with an expected error of the $\mathcal{O}\left(m_{\tilde{a}}/m_{3/2}\right)$. As $\tau = \Gamma^{-1}$, we obtain the gravitino lifetime

$$\tau_{3/2} \simeq 2.3 \times 10^9 \text{ s} \left(\frac{10^2 \text{ GeV}}{m_{3/2}}\right)^3 \tag{6.8}$$

or, equivalently, $T_{3/2}^{\rm dec} \simeq 24 \text{ eV}(m_{3/2}/(10^2 \text{ GeV}))^{3/2}$. We see that the gravitino decay is expected to happen within the considered range (6.7). The mass range for the gravitino to decay within (6.7) is $10^4 \text{ GeV} \gtrsim m_{3/2} \gtrsim 35 \text{ GeV}$. Thus most sparticle spectra expected at the LHC are allowed from this perspective and as soon as SUSY is discovered the gravitino next-to-LSP mass is constrained to a small window.

Since the axino is the LSP, the gravitino next-to-LSP decays into axino and axion. Other gravitino channels emitting gluons or photons are never restrictive. Due to an additional PQ vertex suppressed by the PQ scale, the branching ratio of such processes is extremely small. We find the emitted energy at least twelve orders of magnitude below current limits.

Qualitatively, this holds true even if R-parity is broken as long as the breaking is not too large. R-parity violating decays of the gravitino into two standard model particles were still gravitational decays and additionally suppressed by the R-parity violating coupling. Therefore, the viability of the late emergence of dark radiation in this scenario does not depend on conserved R-parity. We expect the upper bound on R-parity violating couplings from leptogenesis to be more constraining.

Since the axion and also the axino are much lighter than the decaying gravitino, both are emitted with relativistic momenta, $p_a \simeq p_{\tilde{a}} \simeq m_{3/2}/2$. Therefore, the energy density in dark radiation $\rho_{\rm dr}$ consisting of the axion and the axino is given by the energy density of the gravitino at its decay. It is

$$\rho_{\rm dr}|_{T_{3/2}^{\rm dec}} = \rho_a|_{T_{3/2}^{\rm dec}} + \rho_{\tilde{a}}|_{T_{3/2}^{\rm dec}} = \rho_{3/2}|_{T_{3/2}^{\rm dec}} = m_{3/2} Y_{3/2}^{\rm tp} s(T_{3/2}^{\rm dec})\,. \tag{6.9}$$

Note that for the times considered in this chapter $g_{*s} = 3.91$ is constant. If we did not make use of the sudden-decay approximation, the resulting energy density would differ by a factor $\sqrt{\pi}/2 \simeq 0.89$ only [135]. We estimate the thermally produced gravitino yield (3.16) as

$$Y_{3/2}^{\rm tp} \simeq 1.2 \times 10^{-11} \left(\frac{m_{\tilde{g}}(m_Z)}{1\ \rm TeV}\right)^2 \left(\frac{10^2\ \rm GeV}{m_{3/2}}\right)^2 \left(\frac{T_{\rm R}}{10^{10}\ \rm GeV}\right)\,. \tag{6.10}$$

Since at times after BBN g_{*s} is constant, the energy density of dark radiation scales like the energy density of radiation in the thermal bath, even though dark radiation is not coupled to the bath. Thus it is most easy to calculate $\Delta N_{\rm eff}$ at gravitino decay.

Using (6.9) and (2.30) the change of the effective number of neutrino species by the gravitino decay at any time after BBN is given by

$$\Delta N_{\rm eff} \simeq 0.6 \left(\frac{10^2\ \rm GeV}{m_{3/2}}\right)^{\frac{5}{2}} \left(\frac{m_{\tilde{g}}}{1\ \rm TeV}\right)^2 \left(\frac{T_R}{10^{10}\ \rm GeV}\right)\,. \tag{6.11}$$

This is a number of $\mathcal{O}(1)$ and could have been a priori anything. This coincidence for parameter values motivated by completely disconnected reasons is the key observation in this chapter. We remind that the gluino is expected to be among the heaviest sparticles and the gravitino next-to-LSP with $m_{\rm soft} \sim m_{3/2} \sim 10^2$ GeV is not only motivated from SUSY breaking, but also part of the solution of the gravitino problem as described above. This opens up the opportunity for thermal leptogenesis that requires a reheating temperature sufficiently larger than $T_{\rm L}^{\rm min} = 2 \times 10^9$ GeV.

For the approximations made and fixed sparticle masses as in (6.11) we could state that $T_{\rm R} \gtrsim 10^{11}$ GeV is excluded at the 5-σ level referring to the bound (6.5) on the effective number of neutrino species. For a fixed reheating temperature the

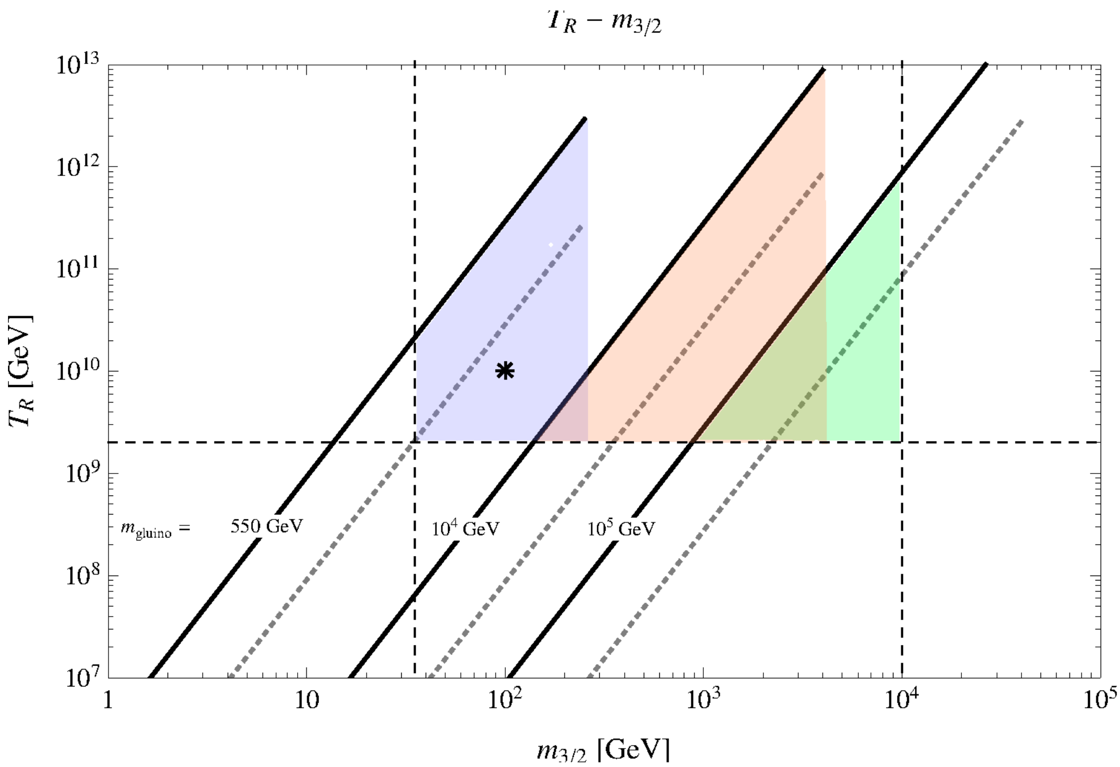


Figure 6.1: The solid lines represent upper bounds on the reheating temperature T_R using (6.11) and (6.5) as function of the gravitino mass $m_{3/2}$ for three different values of the gluino mass $m_{\tilde{g}} = 550$ GeV, 10^4 GeV and 10^5 GeV. We restrict $m_{3/2} < 2\,m_{\tilde{g}}$. The shaded regions show the corresponding parameter space consistent with our scenario. Indicated are bounds from late and early enough gravitino decay (6.7). As discussed successful thermal leptogenesis requires T_R sufficiently larger than 2×10^9 GeV. The dotted lines correspond to values of $\Delta N_\mathrm{eff} = 0.52$. This is the possible Planck 2-σ exclusion limit, if the observed central value coincides with the Standard Model expectation. The star corresponds to the parameter values appearing in (6.11).

gravitino cannot be much lighter, since its mass has the largest exponent in (6.11) and the gluino mass is bounded from below by experiments. The other way around, for a fixed mass ratio a heavier gravitino does not allow for much higher reheating temperatures. Thus we find an upper bound, $T_\mathrm{R} \lesssim 10^{11}$ GeV, four orders of magnitude tighter than in the original work [110]. Different upper bounds on the reheating temperature depending on the gravitino mass are depicted in Fig. 6.1 for different values of the gluino mass. The upper bound does not depend at all on the parameters describing the axion multiplet. Even if the axino were allowed to be heavy, such that it would become nonrelativistic shortly after the decay, the emitted radiation energy would roughly reduce by a factor of 2 only. This is due to the axion that still carries half of the kinetic energy. In this sense, the upper bound does not rely on the very light axino. A dependence on the MSSM parameters enters through the gravitino yield (3.16) only. Since a certain, finite mass gap between gluino and gravitino next-to-LSP is well-motivated and the electroweak contributions are subleading, we find this dependence quite limited.

That ΔN_{eff} in (6.11) is of order one opens up another possibility. An increase in N_{eff} as discussed in the previous section might be predicted by the scenario of successful thermal leptogenesis where a light axino solves the gravitino problem, because the, in general unknown, reheating temperature becomes practically fixed.[2] This is especially appealing in a time where current (on-going) experiments might measure two of the unknowns in (6.11). As mentioned above the Planck satellite mission might reveal a $\Delta N_{\text{eff}} > 0$ and the LHC might measure the gaugino masses, in particular the gluino mass. Furthermore, as soon as SUSY is discovered this will shed light on the mass of a gravitino assumed lighter than the lightest superpartner in the MSSM. The collider phenomenology is not distinguishable from the axino LSP case without gravitino, which could itself be mistaken as the neutralino LSP case. However, in contrast heavy charged LOSPs may leave the detector. In this case dark matter is probably not formed by neutralinos. Interestingly, the reheating temperature in an axino LSP scenario might be probed at the LHC [223, 224].

It is important to know at what time the emitted particles become nonrelativistic. In general, particles become nonrelativistic at the temperature when their momenta become equal to their mass, $p(T^{\text{nr}}) \simeq m$. In the rest frame of a particle decaying into two much lighter particles the decay products carry a momentum $p(T^{\text{dec}}) \simeq m/2$, where m denotes the mass of the decaying particle. After the decay these momenta decrease due to the expansion of the Universe. In our scenario this yields for the axino

$$p_{\tilde{a}}(T) = \frac{m_{3/2}}{2} \left(\frac{g_{*s}(T)}{g_{*s}(T^{\text{dec}}_{3/2})} \right)^{\frac{1}{3}} \frac{T}{T^{\text{dec}}_{3/2}} \,. \tag{6.12}$$

[2] In higher-dimensional theories the gravitino density may be independent of T_{R} for temperatures required by thermal leptogenesis [222]. Larger T_{R} were allowed while the predicted increase in ΔN_{eff} were unchanged.

Assuming $g_{*s}(T_{3/2}^{\text{dec}}) = g_{*s}(T^{\text{nr}})$ we obtain

$$T_{\tilde{a}}^{\text{nr}} \simeq 5 \times 10^{-4}\ \text{meV} \left(\frac{m_{\tilde{a}}}{1\ \text{keV}}\right) \left(\frac{m_{3/2}}{10^2\ \text{GeV}}\right)^{\frac{1}{2}} . \tag{6.13}$$

Since this temperature is much smaller than the temperature of the Universe today $T_0 \simeq 0.237$ meV, axinos with $m_{\tilde{a}} \sim \mathcal{O}\,(\text{keV})$ were surely still relativistic today and thus contribute as radiation to today's energy budget of the Universe. For larger axino masses $m_{\tilde{a}} \sim 200$ keV these axinos became nonrelativistic today. For even larger masses $m_{\tilde{a}} > 1$ MeV these axinos were relativistic at photon decoupling and nonrelativistic today. However, the contribution of these axinos to the dark matter density is suppressed by the small mass ratio $m_{\tilde{a}}/m_{3/2}$. Since the axion is much lighter than the axino, as $m_a \lesssim 10$ meV, the emitted axions are still relativistic today for all possible parameter values.

If we require the gravitino to never dominate the energy density of the Universe, we find a constraint on T_{R} similar to the one found above. This is expected, because the Universe is dominated by radiation around the gravitino decay and the decay emits a substantial amount of radiation energy. However, there is no significant amount of entropy produced in the gravitino decay even if it dominates at its decay, because the decay products do not thermalise. Thus cosmological abundances are not diluted and a period of matter domination after nucleosynthesis and before matter-radiation equality is not excluded by these considerations. Instead, a long period of matter domination is excluded by measurements of the radiation energy density as (6.3), (6.4), (6.5) and (6.6).

Recently, a study investigated the impact of the emission of dark radiation during BBN [225]. This is for particle lifetimes, $0.1\ \text{s} < \tau < 10^3$ s. The situation seems more involved for the time window between $\tau_{\text{CMB}}^{\text{max}} \simeq 5.2\times10^{10}$ s and $t_{\text{eq}} \sim 4\times10^{12}$ s. It seems reasonable that the release of a huge amount of dark radiation from a dominating matter particle at these times affects the CMB in an observable way even though maybe only at lower multipoles. An analysis of this situation is beyond the scope of this thesis and proposed as future research.

6.3 LOSP Decay and Dark Matter

In this scenario neither the gravitino nor the LOSP can account for the observed dark matter, but the axion and the axino might be natural candidates. As their production depends on f_a, we need to consider constraints from LOSP decay. Again, the LOSP has to decay early enough not to spoil the success of BBN. We remind that the constraints on the LOSP are greatly relaxed compared to the gravitino LSP case. It decays much faster, because $f_a m_{\rm losp}/M_{\rm pl} m_{3/2} \ll 1$.

Since the lightest neutralino is likely one of the lightest superparticles, we consider the case of the neutralino being the LOSP and thus its decay into axino and photon. By inverting (5.11) and with the same simplifying assumptions we obtain

$$\Gamma(\widetilde{B}^0 \to \widetilde{a} + \gamma) = \alpha_{\rm em}^2 m_{\widetilde{B}^0}^3/(128\pi^3 f_a^2)$$

with $\alpha_{\rm em} = 1/128$. Using $\Gamma = \tau^{-1}$ we obtain the upper bound

$$f_a \lesssim 2 \times 10^{10}\ \text{GeV} \left(\frac{m_{\widetilde{B}^0}}{100\ \text{GeV}}\right)^{\frac{3}{2}} \left(\frac{\tau_{\widetilde{B}^0}^{\max}}{10^{-2}\ \text{s}}\right)^{\frac{1}{2}}, \tag{6.14}$$

where we demand the bino lifetime $\tau_{\widetilde{B}^0}$ to be at most 0.01 s. By comparison with the bounds in Fig. 2.3 we suppose this lifetime bound to be conservative. If also the decay into $\widetilde{a}$ and Z boson is kinematically unsuppressed, the bound is additionally relaxed by a factor of two. Note that a substantial wino or Higgsino component lowers the relic density of the neutralino, such that a much later decay becomes allowed with $\tau \sim (10^2$–$10^3)$ s, cf. Fig. 4.1 and 4.2. Thus a mixed bino-wino or bino-Higgsino state may allow for f_a even larger than 10^{12} GeV already for neutralino masses close to $m_{3/2} \sim 100$ GeV.

The DFSZ axino couples to Higgsino-Higgs and sfermion-fermion via dimension-4 operators. Thus Higgsino and stau[3] can be the LOSP. These channels easily allow for f_a even larger than 10^{12} GeV as soon as they are kinematically open. Estimates for any LOSP candidate can be inferred from Sec. 5.2 by interchanging LOSP and axino. As long as the LOSP is a fermion this is achieved by $m_{\widetilde{a}} \leftrightarrow m_{\rm losp}$. For LOSP masses as large as the upper bound on the gravitino mass, $m_{3/2} \lesssim 10^4$ GeV as following from (6.8), Peccei-Quinn scales as large as 10^{13} GeV become allowed already for a bino LOSP and other LOSP candidates might even allow for $f_a \sim 10^{16}$ GeV.

In the following three paragraphs we sketch different example scenarios to account for the observed dark matter in the considered setting. Of course this list does not contain all possibilities to achieve a consistent cosmology.

[3] A recent comprehensive study of stau LOSP constraints is found in [224].

I Natural cold axion dark matter: The density of cold axions from vacuum misalignment (3.43) is of the order of the observed dark matter density $\Omega_{\mathrm{DM}} \simeq 0.21$ for an initial misalignment angle Θ of order one and f_a not too far below 10^{12} GeV [151]. As we have seen, constraints from BBN on the decays of different LOSP candidates often allow for large enough Peccei-Quinn scales with the important exception of a bino-like LOSP with $m_{\widetilde{B}^0} \lesssim 500$ GeV. At the same time topological defects do not occur, because the scale of PQ symmetry restoration is larger than the reheating temperature. From (3.30) we see that the axino is required to be light, $\lesssim \mathcal{O}\,(\mathrm{keV})$, to make up only a small fraction of Ω_{DM}. We mention that an admixture of such light axinos could be favoured from problems of small scale structure formation [226]. The DFSZ axino might be allowed to be heavier, $m_{\widetilde{a}}^{\mathrm{dfsz}} \sim 200$ keV $-$ 1 MeV, but this depends on the relative contributions from the different production mechanisms. Altogether, for $f_a \sim 10^{12}$ GeV the dark matter is naturally formed by cold axions and the axino is required to be light.

II Warm axino dark matter: For smaller $f_a \sim 10^{10}$ GeV the generic axion density becomes negligible and the KSVZ axino is overproduced.[4] Its mass would be required to be at most of $\mathcal{O}\,(\mathrm{eV})$ to satisfy hot dark matter constraints. This situation is disfavoured, because we would lack a natural dark matter candidate and models with such small masses seem hard to achieve. The DFSZ axino, however, could constitute warm dark matter with $m_{\widetilde{a}} > \mathcal{O}\,(\mathrm{keV})$ in this case. Thereby it could even be dominantly produced by freeze-in. Altogether, for $f_a \sim 10^{10}$ GeV the dark matter could be formed by warm DFSZ axinos. Much smaller Peccei-Quinn scales are disfavoured for both axion models, because the reheating temperature would need to be lowered, such that standard thermal leptogenesis were excluded.

III Beyond LHC: From the above discussion we have seen that PQ scales as large as 10^{16} GeV with correspondingly heavier sparticles are not forbidden for many LOSP candidates. Surely, Ω_a^{mis} needs to be suppressed by a vanishing misalignment angle in this case and such heavy sparticles are disfavoured if they are supposed to stabilise the Higgs mass. The KSVZ as well as the DFSZ axino could form cold dark matter in this scenario with a much smaller mass gap to the

[4] For the KSVZ model symmetry restoration would not be problematic here. For the DFSZ model we need to ensure that the PQ symmetry is not restored for $T_{\mathrm{R}} \lesssim 10^{10}$ GeV.

gravitino. In this case, a considerable amount of axinos is produced from LOSP decays (6.16), such that a LOSP with a small freeze-out abundance is required, and also the gravitino produces a substantial amount of cold axinos. Anyway, it is possible to have the desired dark radiation in form of axions, a consistent cosmology and no signal at colliders, because all detectable particles are beyond the discovery range of the LHC.

Besides the production from the discussed mechanisms, the axino LSP is also produced in decays of the LOSP. The temperature at which axinos from LOSP decay become nonrelativistic depends on the LOSP lifetime $\tau_{\rm losp}$ as

$$T^{\rm nr}_{\tilde{a}\text{-losp-dec}} \simeq 24\ {\rm meV} \left(\frac{1\ {\rm s}}{\tau_{\rm losp}}\right)^{\frac{1}{2}} \left(\frac{m_{\tilde{a}}}{1\ {\rm keV}}\right) \left(\frac{10^2\ {\rm GeV}}{m_{\rm losp}}\right) \left(\frac{g_*(\tau_{\rm losp})}{10.75}\right)^{\frac{1}{12}} , \qquad (6.15)$$

where $\tau_{\rm losp}$ is to a good approximation independent of the axino mass. Constraining the LOSP lifetime from BBN to be shorter than a second, $\tau_{\rm losp} < 1$ s, we see that it is likely that axinos from the LOSP decay become nonrelativistic after CMB and before today. The contribution of these axinos to today's critical energy density $\Omega^{\rm losp\ decay}_{\tilde{a}}$ is simply given by

$$\begin{aligned} \Omega^{\rm losp\ decay}_{\tilde{a}} &= \frac{m_{\tilde{a}}}{m_{\rm losp}} \Omega^{\rm fo}_{\rm losp} \\ &= 10^{-8} \left(\frac{m_{\tilde{a}}}{1\ {\rm keV}}\right) \left(\frac{10^2\ {\rm GeV}}{m_{\rm losp}}\right) \Omega^{\rm fo}_{\rm losp} , \end{aligned} \qquad (6.16)$$

where the mass hierarchy between LOSP and axino can usually not be reduced much. The LOSP energy density from freeze-out prior to its decay $\Omega^{\rm fo}_{\rm losp}$ is highly model and parameter dependent. For weakly interacting LOSPs simple estimates tend to $\Omega^{\rm fo}_{\rm losp} \sim \Omega_{\rm DM} \sim 0.2$. However, we see that even for large values $\Omega^{\rm fo}_{\rm losp} \sim 10^4 \times \Omega_{\rm DM}$, as they might occur for a bino LOSP as in Fig. 4.2, axinos from LOSP decay with $m_{\tilde{a}} \ll m_{\rm losp}$ give a negligible contribution to Ω after they become nonrelativistic.

On the other hand, axinos from LOSP decay contribute to the radiation energy density, like those from gravitino decay, before they become nonrelativistic. We take into account the scaling of the energy density of dark radiation $\rho_{\rm dr} \propto g_{*s}^{4/3}(T) T^4$, while we simplify exploiting $g_*(\tau_{\rm losp}) = g_{*s}(\tau_{\rm losp})$ and assuming $g_{*s}(\tau_{\rm losp}) > g_{*s}(T) = 3.91$. The resulting change of the effective number of neutrino species is

$$\Delta N_{\rm eff}^{\rm losp\ decay} \simeq 9\times 10^{-6}\,\Omega_{\rm losp}^{\rm fo}\left(\frac{\tau_{\rm losp}}{1\ \rm s}\right)^{\frac{1}{2}}\left(\frac{r_{\tilde a}}{0.5}\right)\left(\frac{10.75}{g_*(\tau_{\rm losp})}\right)^{\frac{1}{12}}, \tag{6.17}$$

where $r_{\tilde a}$ denotes the fraction of the emitted energy that is carried by the axino. If the Standard Model particle emitted together with the axino is also very light or massless like the photon, $r_{\tilde a} = 0.5$ is a good approximation. With the same reasoning as after (6.16) we see that $\Delta N_{\rm eff}$ from LOSP decay is negligible. This is to some extent self-consistent as larger $\tau_{\rm losp}$ becomes allowed only for smaller $\Omega_{\rm losp}^{\rm fo}$.

Saxion As in the case of broken R-parity, the saxion is allowed to produce LOSPs at any time before BBN. Thus constraints from and on the saxion are the same as derived in Sec. 5.2 and summarised in the second column of Table 7.1. Note that in the considered case of $f_a \sim 10^{16}$ GeV we also consider large values of $m_{\rm soft} \gtrsim 10^5$ GeV. Thus the bound is fulfilled in this case as well.

In contrast to the late decaying gravitino as considered in this chapter, the saxion has to decay before BBN even if the production of axion pairs is the dominant decay channel. This is due to its nonnegligible branching ratio into a gluon pair[5]. Thus it leads to an increase in $N_{\rm eff}$ before BBN, so $0 < \Delta N_{\rm eff}^{\rm BBN} = \Delta N_{\rm eff}^{\rm CMB}$. In Sec. 5.1 we considered the constraint on the saxion-axion-axion self-coupling x by requiring that such an increase before BBN should be smaller than one. Even though $\Delta N_{\rm eff} = 1$ from saxion decay is consistent with all existing measurements, for deviations of the effective number of neutrino species $\Delta N_{\rm eff}^{\rm CMB} < 1.78$ it would be an unfortunate case considering the discovery potential of Planck. A 3-σ detection of an increase in $N_{\rm eff}$ after BBN would become impossible. From the discussion in Sec. 5.1 we see that $\Delta N_{\rm eff}$ from saxion decay can well be small for natural parameter values. For $m_{\rm sax} > 900$ GeV it is expected small in any case and especially for $x \ll 1$ and $x > 1$. One can consider cases with a significant deviation from the Standard Model expectation during BBN and an even larger deviation at photon decoupling, i.e., $0 < \Delta N_{\rm eff}^{\rm BBN} < \Delta N_{\rm eff}^{\rm CMB}$. We do not comment further on this possibility. Note that if the DFSZ saxion can decay into a Higgs pair, this decay

[5] Considering a saxion with $m_{\rm sax} \lesssim 40$ MeV produced from coherent oscillations and $T_{\rm R} \lesssim 10^6$ GeV the saxion could actually lead to a $\Delta N_{\rm eff}$ of $\mathcal{O}(1)$ after BBN [220].

might become stronger than the gluon decay channel. This reduces the amount of emitted axions.

Altogether, we see that we have to consider the saxion decay into two axions, especially for the scenario under consideration in this chapter. However, depending on the actually measured value from Planck this leads to acceptable or mild constraints on the saxion mass. In general, the emergence of dark radiation before BBN would be constrained by a better determination of the effective number of neutrino species during BBN.

The results of this chapter are summarised in the following chapter.

Chapter 7

Results and Outlook

In this chapter we summarise our results, conclude and provide an outlook.

7.1 Results

In Chapter 4 we investigated to what extent the LOSP decay problem can be solved by late-time entropy production. If entropy is produced after the LOSP is frozen out, its density is diluted. Possibly it is diluted to such an extent that the LOSP decay becomes harmless for big bang nucleosynthesis (BBN). We have found that

- if the neutrinos have hierarchical masses, thermal leptogenesis is compatible with entropy production diluting the baryon asymmetry as well as the LSP and LOSP relic densities by up to three to four orders of magnitude. (Quasi-degenerate neutrinos allow for less than two orders of magnitude only.)
- this amount of dilution roughly coincides with the maximum amount obtainable, if radiation domination at LOSP freeze-out is required.
- for a gravitino LSP with a mass of 100 GeV, which allows for a reheating temperature suitable for thermal leptogenesis, a neutralino LOSP which is not much heavier can be diluted sufficiently to be compatible with BBN. However, this is only possible if the lightest neutralino contains a large wino or Higgsino component, whereas a bino-like neutralino remains excluded.

- the general requirements on the particle, which produces the desired entropy, are severely constraining, see Table 4.1. On the other hand, in some sense all these requirements either have to be fulfilled by long-lived particles anyway or are generically fulfilled.

- the strong CP problem can be solved by the Peccei-Quinn mechanism, while thermal leptogenesis is successful and the gravitino forms the dark matter. However, the Peccei-Quinn parameters become much more strongly constrained, see left column of Table 7.1.

- generic thermally produced particles cannot produce sufficient entropy. This is due to two conflicting requirements: On the one hand sufficient production requires sufficiently strong couplings, while on the other hand sufficiently late decay requires weak couplings, where later decay corresponds to more entropy production. This holds true for the saxion, so the allowed parameter ranges fail to overlap.

- if the right amount of saxions is produced in coherent oscillations, which is independent of the saxion couplings, a relatively light saxion with a mass around 10 GeV is indeed able to satisfy all requirements.

- combining future CMB polarisation measurements with very sensitive gravitational wave probes such as BBO could rule out significant entropy production at cosmic temperatures smaller than $\sim 10^9$ GeV and thus at times later than $\sim 10^{-25}$ s, see Figure 4.4.

We conclude that, if the Peccei-Quinn mechanism solves the strong CP problem, the potentially dangerous saxion decays can in fact turn out as a fortune, solving the a priori unrelated gravitino problem. Furthermore, the scenario might be independently falsifiable by observations of the gravitational wave background from inflation. This is an additional connection between physical processes, that are often regarded as unconnected.

In Chapter 5 we investigated the impact of R-parity violating solutions to the LOSP decay problem on the cosmological constraints from and on the axion multiplet. Here, the LOSP decays before BBN via R-parity violating couplings. We summarise the results in Table 7.1. Obviously, the most important findings also

[GeV]	standard	$\not{R}$	$\not{R} \wedge \Delta\vert_{T<\Lambda_{\rm QCD}} = \Delta^{\rm max}$
f_a	$\lesssim \mathbf{10^{10}}$	$\lesssim 10^{10}$	$\lesssim 4 \times 10^{12}$
$m_{\tilde{a}}$	$> m_{\tilde{g}}$ (or $> m_{\widetilde{H}} + m_h$ in DFSZ)	$> m_{\text{“losp”}}$	$\gtrsim \max[m_{\text{“losp”}},$ $300\left(\frac{f_a}{10^{11}\,\text{GeV}}\right)^{\frac{2}{3}}]$
$m_{\rm sax}$	$> 760\left(\frac{f_a}{10^{10}\,\text{GeV}}\right)^{\frac{2}{3}}$ (or $> 2m_h$ in DFSZ) or $\in [5\left(\frac{f_a}{10^{10}\,\text{GeV}}\right)^{\frac{2}{3}}, 2m_{\rm losp}]$	$\gtrsim 5\left(\frac{f_a}{10^{10}\,\text{GeV}}\right)^{\frac{2}{3}}$	$\gtrsim 5\left(\frac{f_a}{10^{10}\,\text{GeV}}\right)^{\frac{2}{3}}$
$\phi^{\rm i}_{\rm sax}$	$\lesssim 10^{13}$–10^{15}	$\lesssim 10^{13}$–10^{15}	$\lesssim 5 \times (10^{14}$–$10^{16})$

Table 7.1: Constraints on Peccei-Quinn parameter space for the different scenarios (standard: R-parity conserved ($\Delta = 1$), $\not{R}$: R-parity violated ($\Delta = 1$) and violated R-parity with the maximal entropy dilution at the right time). Units are GeV where not written explicitly. Here, we assume the self-coupling $x \ll 1$. Only if a mass bound is sensitive to the actual value of f_a, its dependence is given. By “losp” we indicate that the bound actually does not hold for any possible LOSP and depends on the axion model. The upper bound on f_a in the standard scenario is boldface to indicate that it arises from Ω_a and the (KSVZ) axino decay. In the $\not{R}$ case it stems from $\Omega_a^{\rm mis}$ only. If, furthermore, matter dominates at $T_a^{\rm osc}$ it stems from $\Omega_a^{\rm str}$.

hold in less restricted scenarios than gravitino dark matter with thermal leptogenesis. We have found that if R-parity is broken,

- the saxion mass becomes practically unconstrained.
- the axino may be anywhere in the superparticle mass spectrum as long as a decay channel into an ordinary superparticle of the MSSM is kinematically open and allowed at tree-level in the low-energy effective theory. This does not hold if the only possible decay is into a light fermion (up-quark, electron, neutrino) and its superpartner. (An axino next-to-LSP stays excluded.)

$\rightarrow$ for the DFSZ axion model, usually considered superparticle spectra become allowed including spectra with stau next-to-LSP.

$\rightarrow$ in any model an open axino decay channel into a neutralino suffices. The sufficient condition is that the axino decay into a neutralino is not too strongly kinematically suppressed.

- constraints on the axion decay constant from axino and saxion decay are softened. The DFSZ model is—depending on the μ-parameter—less restricted from particle decay already in the standard scenario. For example, the saxion mass could be unconstrained, if the Higgs boson is lighter than the LOSP.

Furthermore, we have found that

- if the reheating temperature is fixed at large values like that required by thermal leptogenesis, the constraints from axion overproduction on the decay constant are particularly interesting and depend on the axion model. They might be softened by late-time entropy production, see right column of Table 7.1.
- there is no constraint on the saxion-axion-axion self-coupling as long as $f_a < 10^{10}$ GeV and $m_{\text{sax}} > 100$ GeV. A large self-coupling of $\mathcal{O}(1)$ can remove constraints on the saxion mass, especially in the standard scenario, see (5.9).
- for suitable values of the self-coupling and other model parameters one can obtain any desired amount of additional radiation energy in the Universe formed by axions from saxion decay, cf. (5.7).

We conclude that broken R-parity does not only solve the LOSP decay problem but also makes it easier to solve the strong CP problem. As a consequence, it will be interesting to construct concrete models with i) the naturally expected axino and saxion mass of $\mathcal{O}(m_{\text{soft}})$, possibly along the lines of existing models like [141, 227, 228], ii) a particular self-coupling, and/or iii) a small axion density $\Omega_a \ll \Omega_{\text{DM}}$. Our results are an additional motivation for broken R-parity.

In Chapter 6 we presented the axino solution to the gravitino problem. Any gravitino abundance decays invisibly into axino-axion pairs and the LOSP can decay into axino and its Standard Model partner. Current observations of the cosmic microwave background provide hints for an increase in the effective number of neutrino species after BBN but before photon decoupling. If a mass hierarchy $m_{\text{soft}} > m_{3/2} > m_{\tilde{a}}$ is presupposed to solve the cosmological gravitino problem by— in some sense—the cosmological axino problem, we have found that

- the increase in the effective number of neutrino species in (6.11) by the invisible gravitino decay after BBN but before photon decoupling is of $\mathcal{O}(1)$

for expected (natural) masses and a reheating temperature motivated by successful standard thermal leptogenesis.

- there is a new upper bound on the reheating temperature $T_{\mathrm{R}} \lesssim 10^{11}$ GeV in this scenario. This bound does not at all depend on the Peccei-Quinn parameter space. The only requirement on the MSSM parameter space stems from thermal gravitino production. It is a large enough but finite mass gap between gluino and gravitino next-to-LSP.

- the axion may naturally form the observed amount of cold dark matter for $f_a \sim 10^{12}$ GeV without any conflict with BBN from late LOSP decay, except for a light bino LOSP.

- the DFSZ axino might be able to form warm dark matter for smaller $f_a \sim 10^{10}$ GeV.

- the scenario is safe against other gravitino decay channels and radiation from LOSP decay.

- constraints from and on the saxion are absent or moderate (with the exception of the bound on its initial oscillation amplitude), because it is allowed to produce LOSPs at any time before BBN as in the case of broken R-parity discussed in Chapter 5, cp. Table 7.1.

- the worst-case lower bound on the saxion mass from its decay into an axion pair is still in the TeV range.

- the collider phenomenology is not distinguishable from the axino LSP case without gravitino.

Altogether, we proposed a natural origin of the late emergence of "dark radiation". This origin is motivated from other problems of cosmology and the Standard Model. A consistent cosmology is possible without further ingredients and in the same parameter range.

We point out that the identification of consistent cosmologies and their corresponding parameter space with the desired increase in radiation calls for more comprehensive studies, cp. Figure 6.1. It is interesting that an admixture of thermally produced, light axinos may be favoured from small scale problems of structure formation.

The emission of a huge amount of dark radiation after $\tau_{\max}^{\mathrm{CMB}} \simeq 5.2 \times 10^{10}$ s and before $t_{\mathrm{eq}} \sim 4 \times 10^{12}$ s or photon decoupling should affect the CMB in an observable way. We propose an analysis of this situation as future work.

Even though we have also shown that the LHC cannot rule out the scenario, we want to point out its surprising testability considering that gravitino and axino are generally elusive particles and the reheating temperature is not an experimentally accessible quantity. Combining the new opportunity to measure the effective number of neutrino species from the CMB alone and the potential discovery of supersymmetry at the LHC, two of the unknowns in (6.11) can be determined. Within this scenario the gravitino mass would be constrained to a small window and, therefore, the reheating temperature as well. The viability of thermal leptogenesis in this scenario would be tested and the cosmological gravitino problem would turn out as a fortune when the Planck mission indeed discovers an increased radiation energy density. The other way around, that discovery would provide indirect evidence for this particular scenario.

7.2 Outlook

We have presented scenarios that are featuring dark and visible matter as observed, while solutions to various problems of the Standard Model are enabled. These phenomenological studies often serve as motivation for new model building, for instance, supersymmetric axion model building. The other way around, such models, hopefully, imply new phenomenological consequences.

We would like to point out that there might be better ideas how to solve the LOSP decay problem. More generally, it might be interesting to investigate if other scenarios of dark and visible matter enable a solution to the strong CP problem. However, other crucial cosmological notions like dark energy were missed out and thus assumed to simply coexist without cosmological interplay. No concrete realisation of inflation has been considered, while an inflationary phase had to be assumed. Towards a consistent cosmology these notions should be included in a cosmological overall picture. In particular, the transition from inflation to the hot thermal universe should be considered with more care. Moreover, other extensions of the Standard Model such as an origin of flavour or explanations for mass and mixing textures should be confronted with cosmology. How does their cosmological overall picture look like? Do they enable a solution to the strong CP problem? Our work could serve as a blueprint to address this question and for comparison.

Even though we combined different observations and observables, many others have not been exploited. The Lyman-α forest probes cosmological times much later than the time of photon decoupling. Thanks to galaxy surveys and simulations our current understanding of the formation of structures in the Universe is much better than in the past. Actually, standard structure formation struggles with problems on small scales. Some cosmic ray observatories reported on anomalies in their spectra. Possible signals at the LHC are only used on a basic level in this thesis.

When these observations are taken into account in the future, the research shall benefit from some general insights we have encountered. Constraints from cosmological problems do not simply add up. We have shown that problems can: i) have common viable parameter space, ii) have a common solution, iii) favour or disfavour solutions to other problems, or iv) even provide the solution to another problem. Requiring a consistent cosmology we have found observable consequences of such solutions. Thus, most importantly, they are testable. Combining expected discoveries at the LHC with current observations of the cosmic microwave background and future observations of the gravitational wave background from inflation, respectively, we have been able to i) constrain elusive particles like the gravitino, the axion and its superpartner, ii) give insight to yet unprobed early cosmological times, and iii) get a handle on experimentally inaccessible quantities like the initial temperature of the hot early universe.

Appendix A

Derivation of Transfer Function T

In this appendix we derive the transfer function T that determines the effect of an early matter-dominated era on the gravitational wave background. We define

$$T^2(k;\eta_b,\eta_e) \equiv N^{-1}\frac{\text{output}}{\text{input}} \tag{A.1}$$

as the factor with which we have to multiply a spectrum (4.47) to consider a period of matter domination that occurs—in any case deviating from standard cosmology—in between two phases of radiation domination.

In the most general form (4.45) we can write the "in- and output" ($q = 1$) as

$$h^{\text{in/out}} = \frac{x}{a(\eta)}\left(c_1^{\text{in/out}} j_0(x) + c_2^{\text{in/out}} y_0(x)\right) \tag{A.2}$$

while the solution during matter domination ($q = 2$) is given by

$$h^{\text{md}} = \frac{x}{a(\eta)}\left(c_1^{\text{md}} j_1(x) + c_2^{\text{md}} y_1(x)\right) . \tag{A.3}$$

From matching the solutions and their derivatives, $h' = -\frac{x}{a(\eta)}\left(c_1 j_q(x) + c_2 y_q(x)\right)$, at the beginning η_b and end η_e of the matter dominated phase we obtain four equations with four unknowns:

$$c_1^{\text{in}} j_0(x_b) + c_2^{\text{in}} y_0(x_b) = c_1^{\text{md}} j_1(x_b) + c_2^{\text{md}} y_1(x_b) \tag{A.4}$$

$$c_1^{\text{in}} j_1(x_b) + c_2^{\text{in}} y_1(x_b) = c_1^{\text{md}} j_2(x_b) + c_2^{\text{md}} y_2(x_b) \tag{A.5}$$

$$c_1^{\text{md}} j_1(x_e) + c_2^{\text{md}} y_1(x_e) = c_1^{\text{out}} j_0(x_e) + c_2^{\text{out}} y_0(x_e) \tag{A.6}$$

$$c_1^{\text{md}} j_2(x_e) + c_2^{\text{md}} y_2(x_e) = c_1^{\text{out}} j_1(x_e) + c_2^{\text{out}} y_1(x_e) , \tag{A.7}$$

where $x_b = k\eta_b$ and $x_e = k\eta_e$. Eliminating c_1^{md}, c_2^{md} we can determine c_1^{out}, c_2^{out} as functions of c_1^{in}, c_2^{in}, x_b and x_e, where we exploit that we need to consider in- and output indeed at the beginning and the end of the matter dominated phase only. The ratio output/input in (A.1) becomes

$$\begin{aligned}&\big\{j_1(x_b)y_1(x_b)\left(c_1^{\mathrm{in}}j_1(x_e)-c_2^{\mathrm{in}}y_1(x_e)\right)+j_2(x_b)y_1(x_e)\\&\;\left(c_1^{\mathrm{in}}j_0(x_b)+c_2^{\mathrm{in}}y_0(x_b)\right)+j_1(x_e)\big(c_2^{\mathrm{in}}y_1(x_b)^2-y_2(x_b)\\&\;\left(c_1^{\mathrm{in}}j_0(x_b)+c_2^{\mathrm{in}}y_0(x_b)\right)\big)-c_1^{\mathrm{in}}j_1(x_b)^2y_1(x_e)\big\}\\&\;/\big\{x_b^{-2}\left(c_1^{\mathrm{in}}j_0(x_b)+c_2^{\mathrm{in}}y_0(x_b)\right)\big\}\,.\end{aligned} \tag{A.8}$$

In case there is enough time between inflation and η_b we can set $c_2^{\mathrm{in}} = 0$ and (A.8) becomes independent of the initial conditions and equal to

$$\begin{aligned}&\big\{-j_1^2(x_b)y_1(x_e)+j_1(x_b)j_1(x_e)y_1(x_b)\\&\quad+j_0(x_b)\left(j_2(x_b)y_1(x_e)-j_1(x_e)y_2(x_b)\right)\big\}/\\&\big\{j_0(x_b)\left(j_2(x_b)y_1(x_b)-j_1(x_b)y_2(x_b)\right)\big\}\,.\end{aligned} \tag{A.9}$$

The factor N^{-1} in the definition (A.1) mods out that the Universe is radiation dominated if not dominated by matter and normalises the transfer function. The normalisation factor is found by investigation of the limit $k \to \infty$. Altogether, we determine

$$N = \frac{\eta_e}{\eta_b}\frac{j_0(x_e)}{j_0(x_b)}\,. \tag{A.10}$$

In summary, $T^2(k;\eta_b,\eta_e)$ is now given by (A.10) and (A.9). The function appears highly oscillatory. However, since only the average square of a solution h enters (4.43) and approximately $\langle\dot{h}\dot{h}\rangle \simeq \langle k^2h^2\rangle$, we give the analytic approximation (4.49) and call it transfer function as well. It is a good approximation down to the percent level for $\eta_e/\eta_b \gtrsim 4$.

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