Strong magnetic fields in lattice QCD

P.V.Buividovich, M.N.Chernodub,T.K. Kalaydzhyan, D.E. Kharzeev, E.V.Luschevskaya, M.I. Polikarpov

ITEP Lattice

arXiv:1003.2180, arXiv:0910.4682, arXiv:0909.2350, arXiv:0909.1808, arXiv:0907.0494, arXiv:0906.0488, arXiv:0812.1740

Lattice simulations with magnetic fields

1. Chiral Magnetic Effect

1.1 CME on the lattice

1.2 Vacuum conductivity induced by magnetic field

1.3 Quark mass dependence of CME

1.4 Dilepton emission rate

2. Other effects induced by magnetic field

2.1 Chiral symmetry breaking

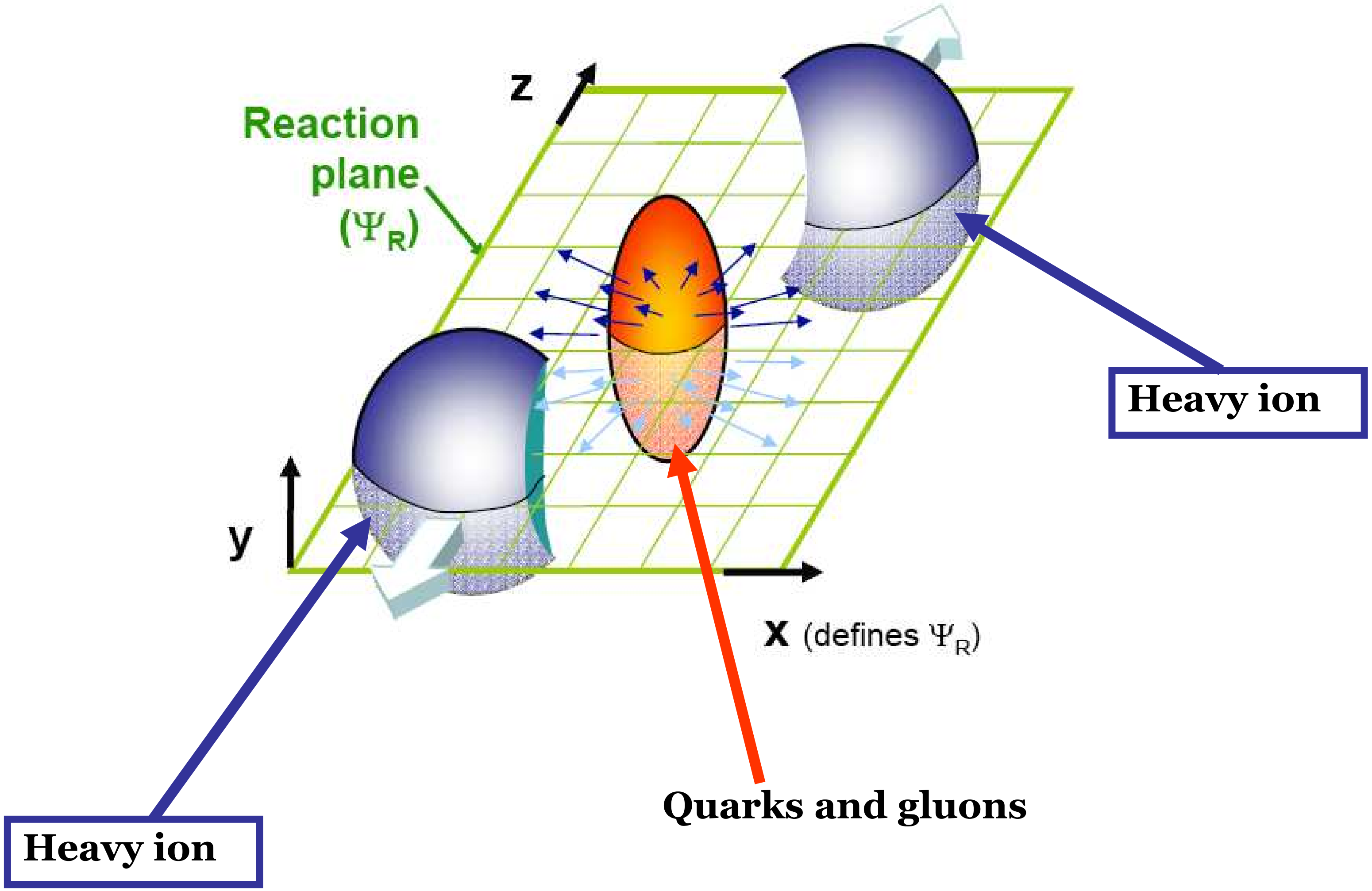
2.2 Magnetization of the vacuum

2.3 Electric dipole moment of quark along the direction of the magnetic field

Main picture by McLerran,Kharzeev,Fukushima

Magnetic fields in non-central collisions

[Fukushima, Kharzeev, Warringa, McLerran '07-'08]



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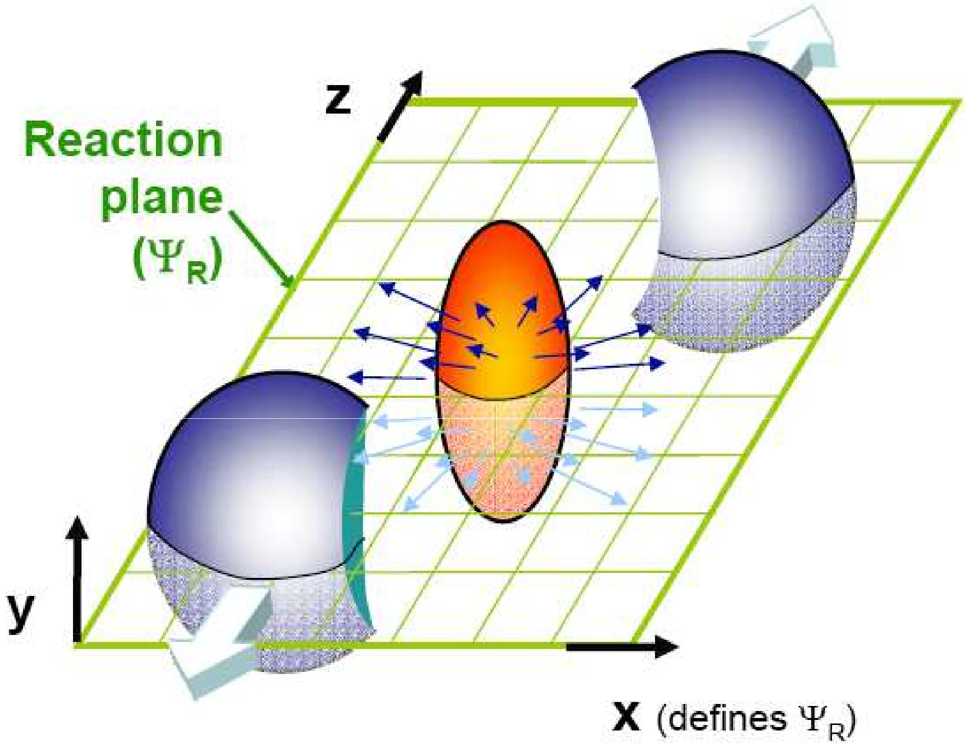


[1] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. D 78, 074033 (2008), URL http://arxiv.org/abs/0808.3382.

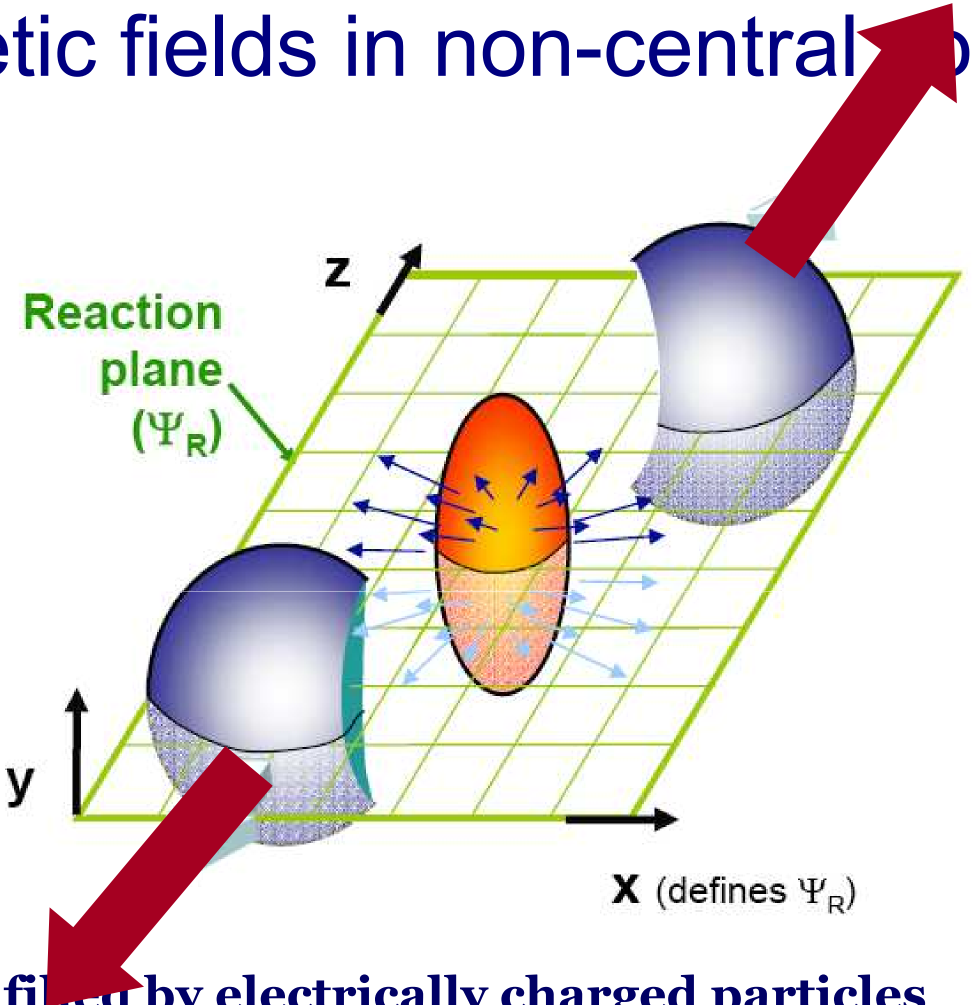
[2] D. Kharzeev, R. D. Pisarski, and M. H. G.Tytgat, Phys. Rev. Lett. 81, 512 (1998), URL http://arxiv.org/abs/hep-ph/9804221.

[3] D. Kharzeev, Phys. Lett. B 633, 260 (2006), URL http://arxiv.org/abs/hep-ph/0406125.

[4] D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, Nucl. Phys. A 803, 227 (2008), URL http://arxiv.org/abs/0711.0950.

Main picture by McLerran,Kharzeev,Fukushima

Magnetic fields in non-central collisions



Charge is large
Velosity is high

Thus we have two very big currents

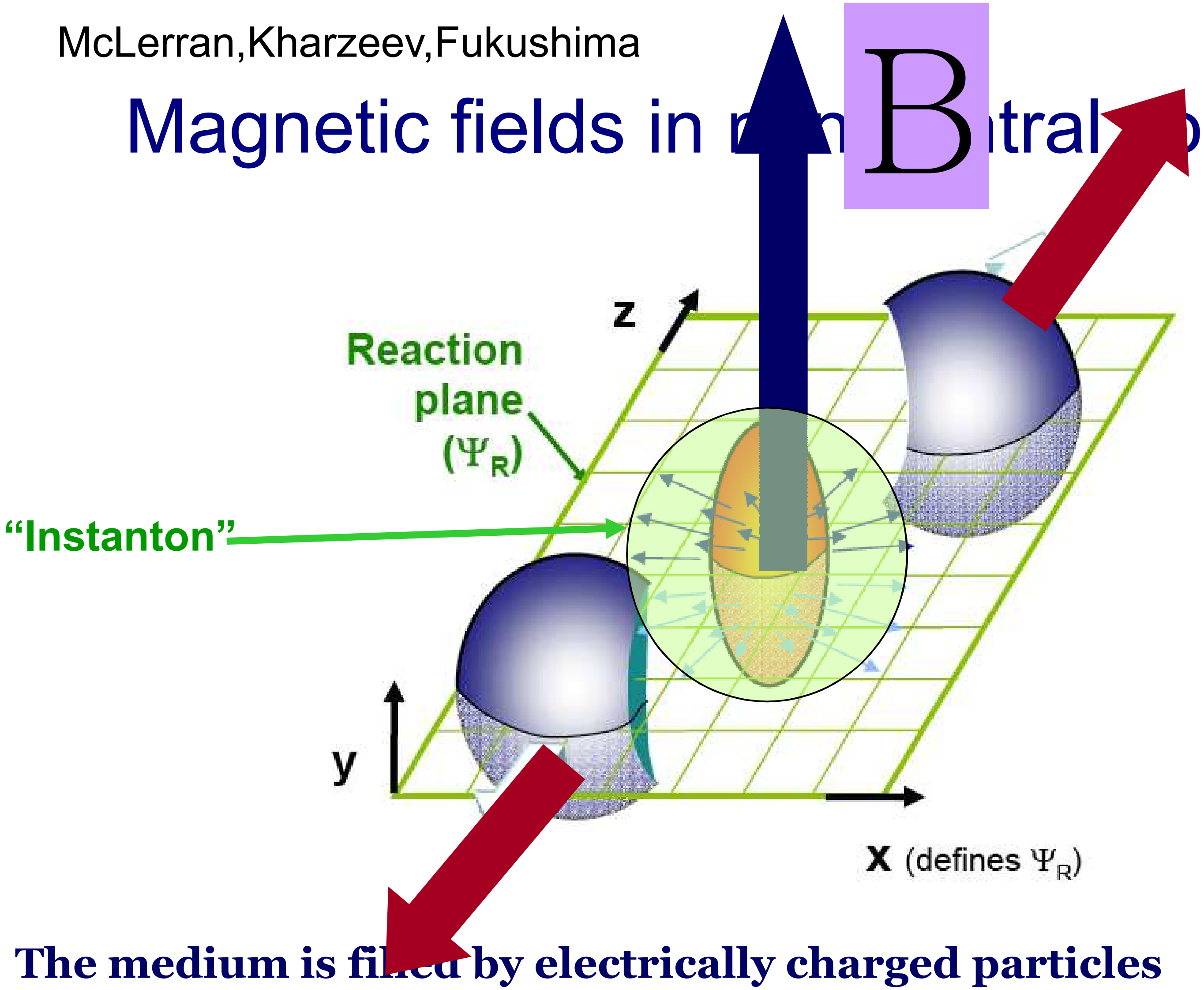
The medium is filled by electrically charged particles

Large orbital momentum, perpendicular to the reaction plane

Large magnetic field along the direction of the orbital momentum

McLerran,Kharzeev,Fukushima

Magnetic fields in non-central collisions



Two very big currents produce a very *big magnetic field*

The medium is filled by electrically charged particles

Large orbital momentum, perpendicular to the reaction plane

Large magnetic field along the direction of the orbital momentum

In heavy ion collisions magnetic forces are of the order of strong interaction forces

$$eB \approx \Lambda^2_{QCD}$$

Magnetic forces are of the order of strong interaction forces

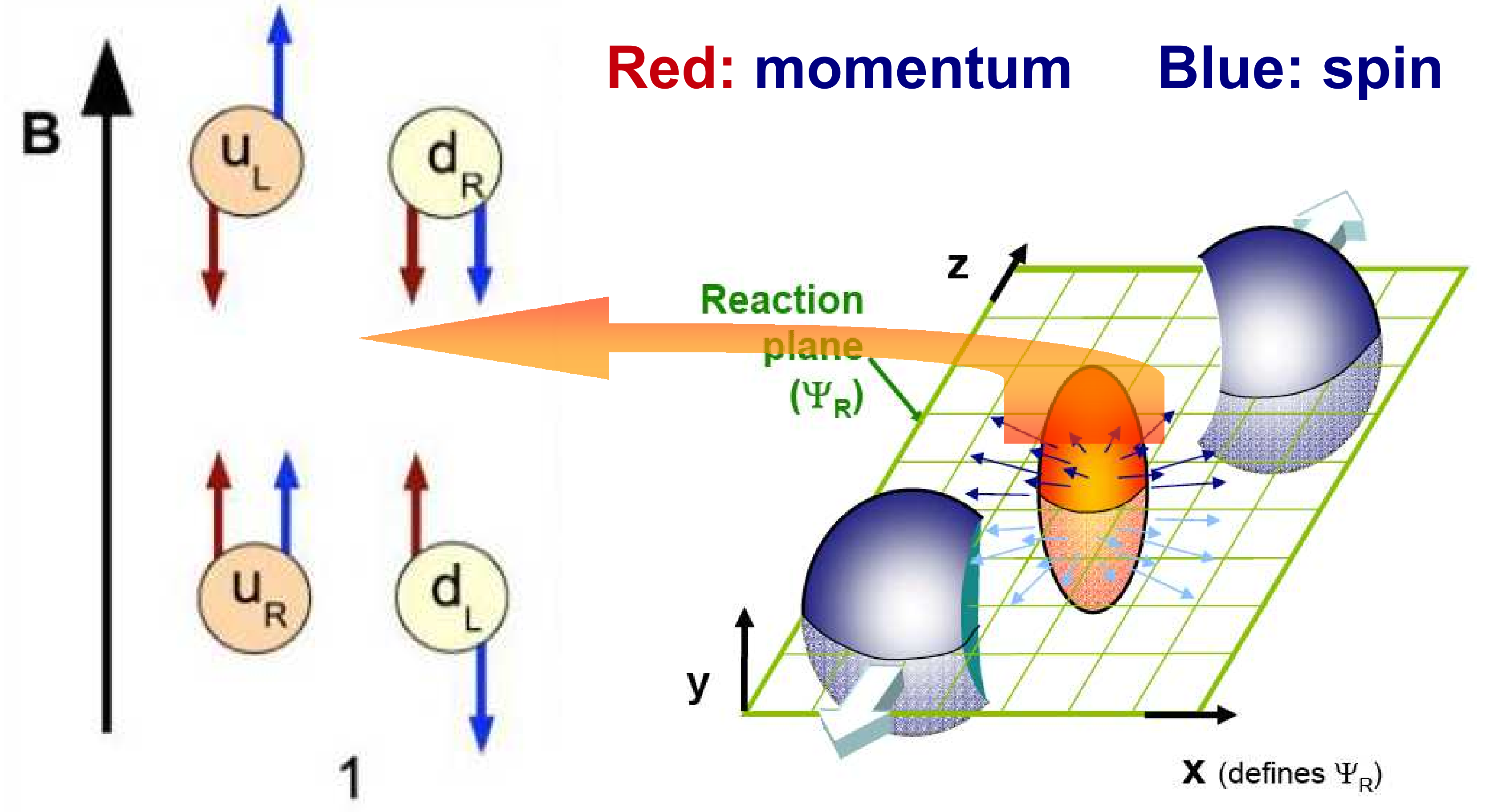
$$eB \approx \Lambda^2_{QCD}$$

We expect the influence of magnetic field on strong interaction physics

Chiral Magnetic Effect by
Fukushima, Kharzeev, Warringa, McLerran

1. Massless quarks in external magnetic field.

Red: momentum **Blue: spin**



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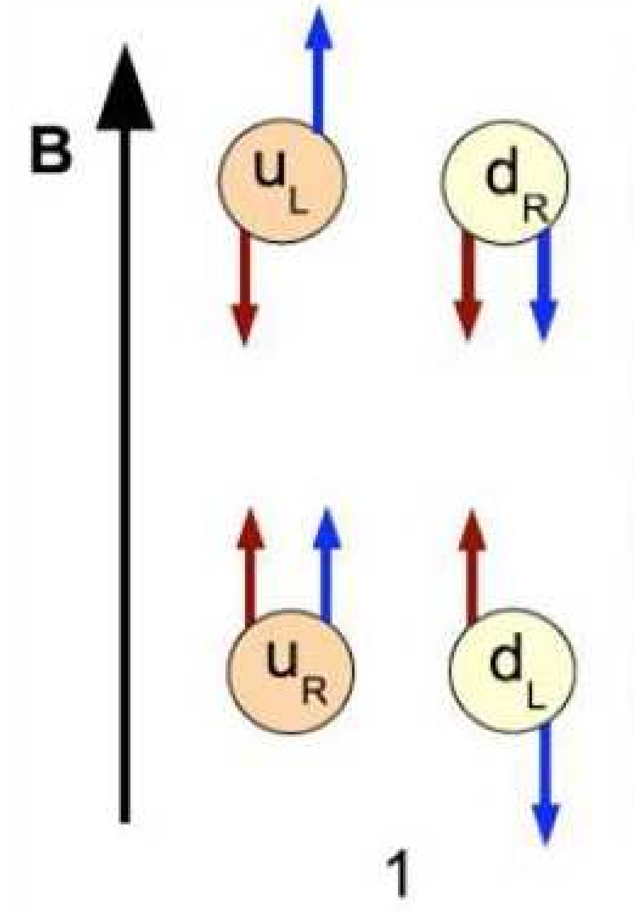
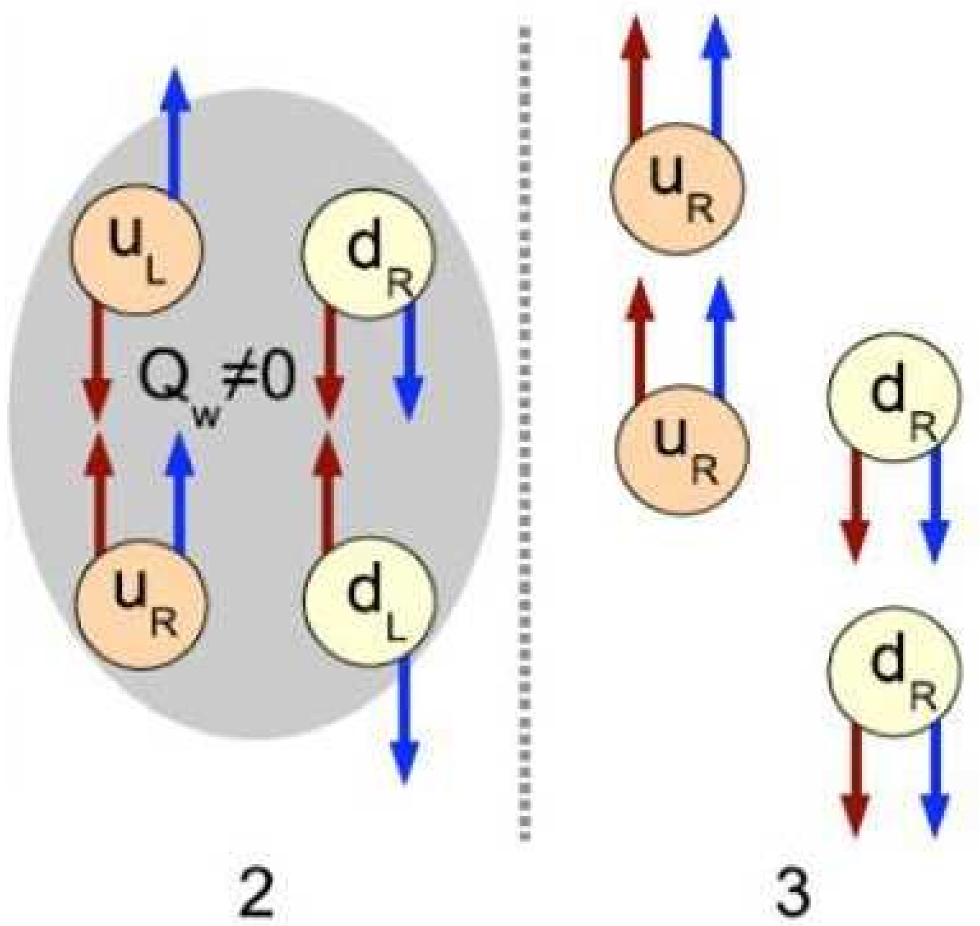


Red: momentum **Blue: spin**

Chiral Magnetic Effect by
Fukushima, Kharzeev, Warringa, McLerran

2. Quarks in the instatnton field.



Red: momentum
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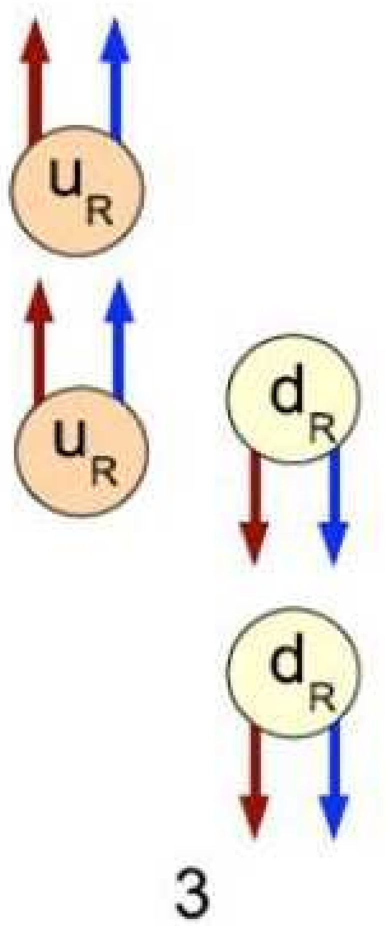
Effect of topology:

$u_L \rightarrow u_R$

$d_L \rightarrow d_R$

Chiral Magnetic Effect by
Fukushima, Kharzeev, Warringa, McLerran

3. Electric current along magnetic field



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Effect of topology:

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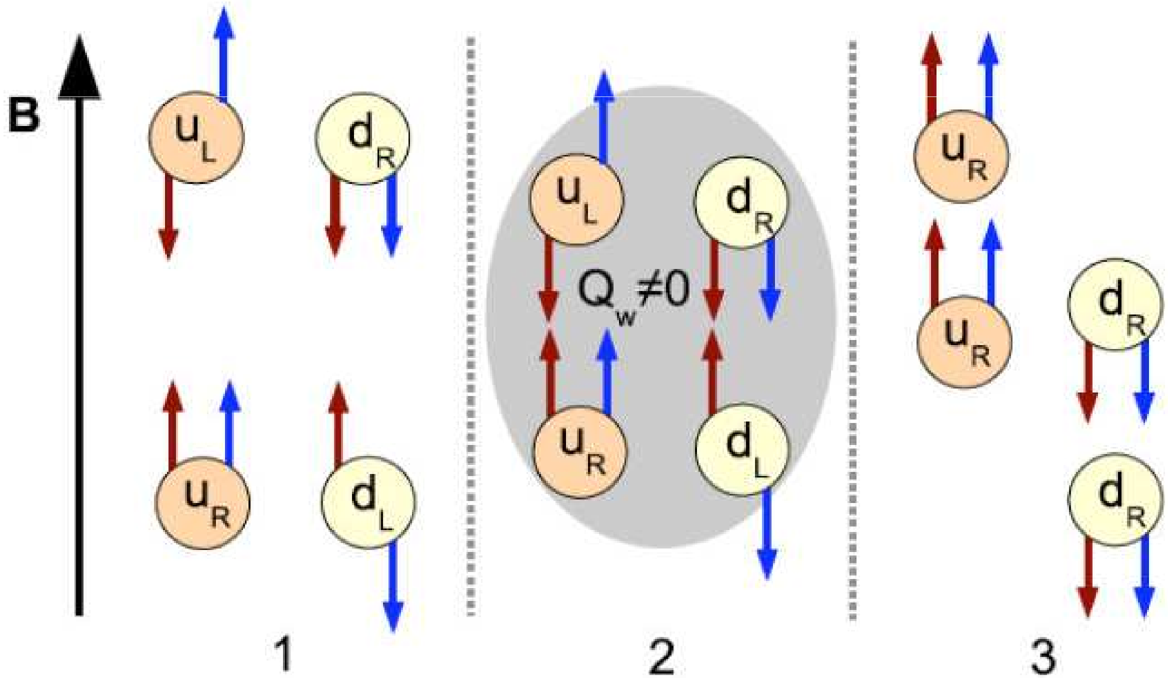
u-quark: q=+2/3
d-quark: q= - 1/3

Chiral Magnetic Effect by
Fukushima, Kharzeev, Warringa, McLerran

3. Electric current is along magnetic field
In the *instanton* field



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Effect of topology:
$u_L \rightarrow u_R$
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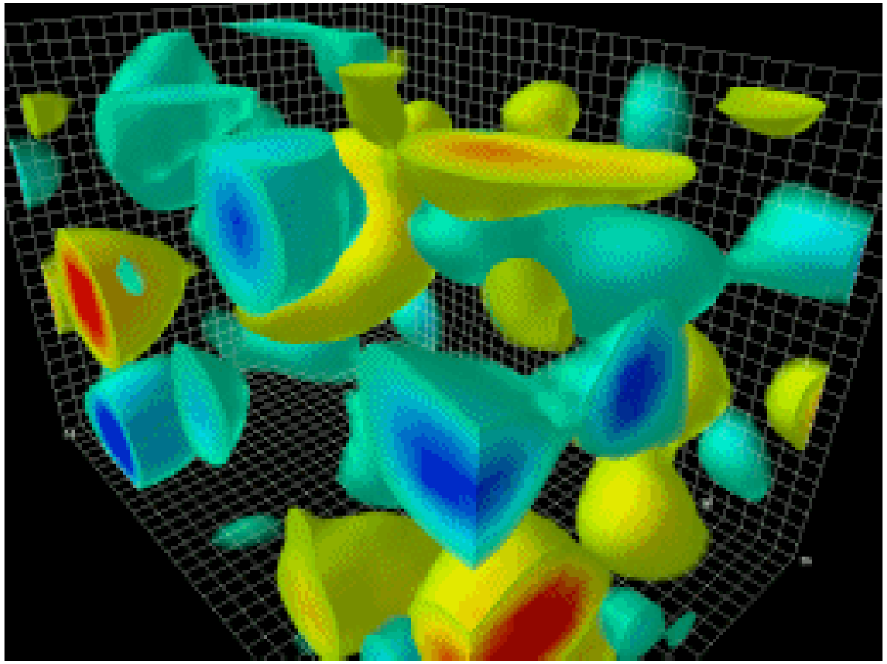
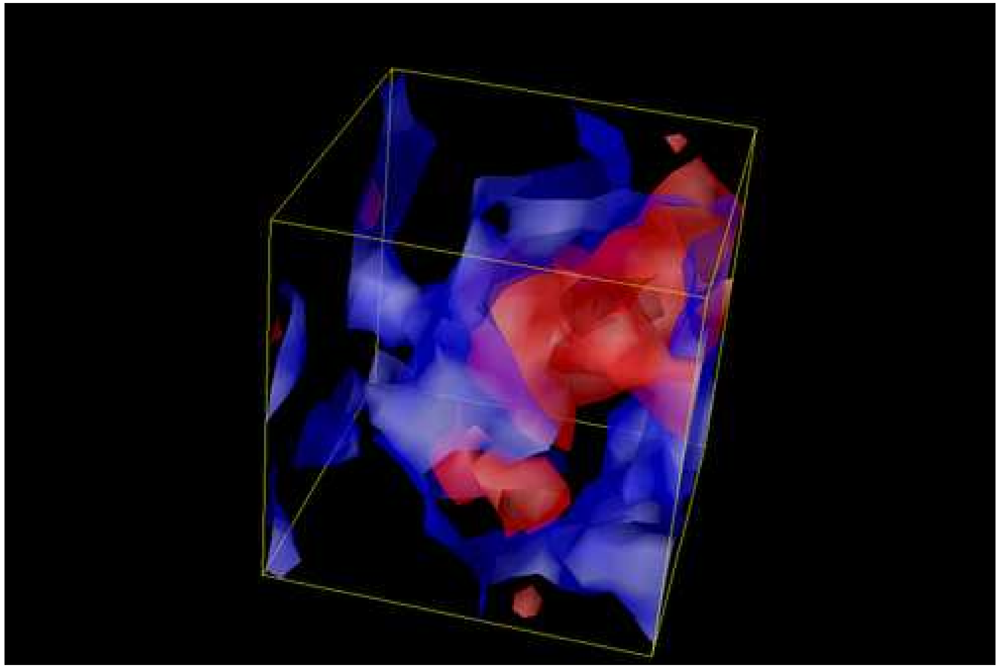
3D time slices of topological charge density, lattice calculations



D. Leinweber

Topological charge density after
vacuum cooling



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T.K. Kalaydzhyan, M.I. Polikarpov

Fractal topological charge density
without vacuum cooling

Magnetic forces are of the order of strong interaction forces

$$eB \approx \Lambda^2_{QCD}$$

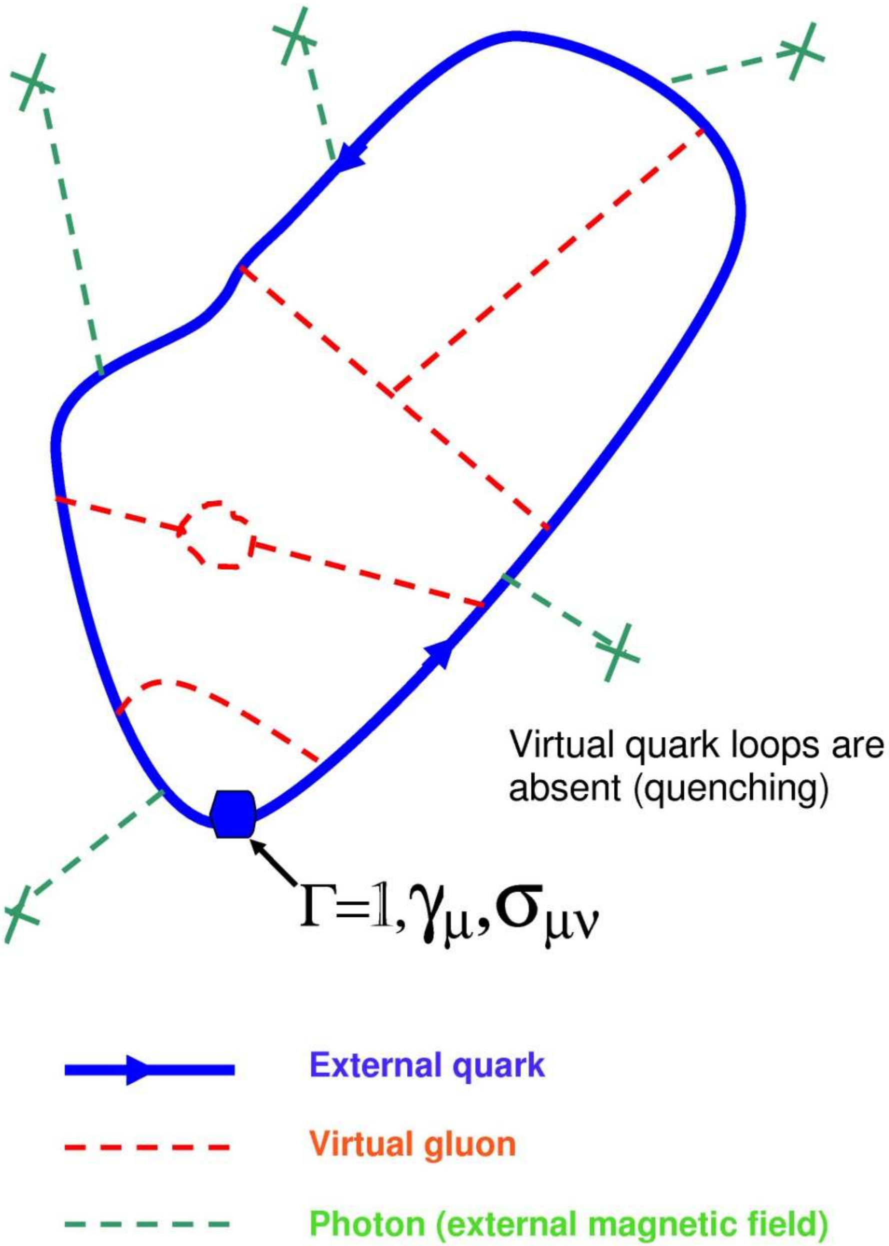
We expect the influence of magnetic field on strong interaction physics

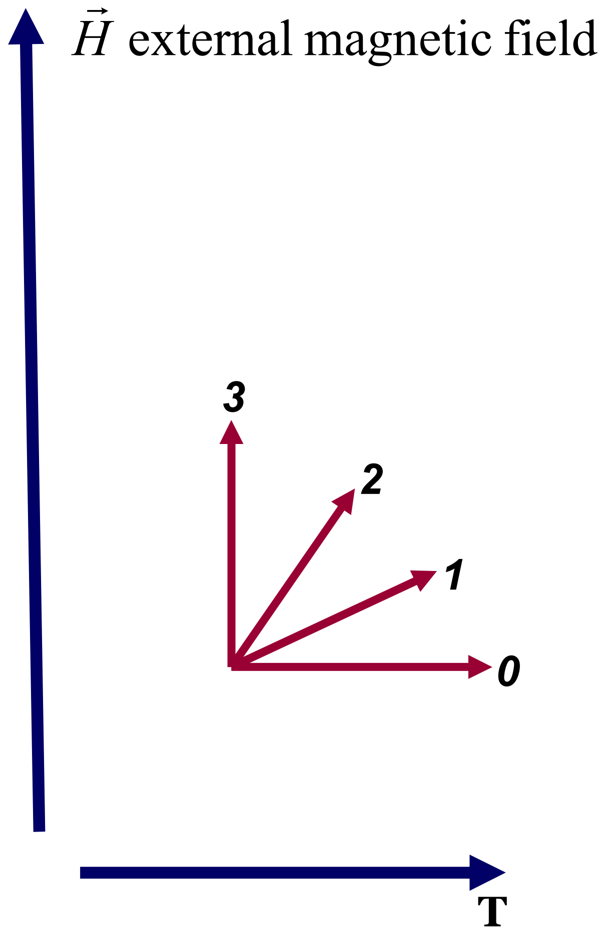
The effects are nonperturbative,

and we use

Lattice Calculations

We calculate $< \overline{\psi}\, \Gamma \psi >;\ \Gamma = 1, \gamma_{\mu}, \sigma_{\mu\nu}$ **in the external magnetic field and in the presence of the vacuum gluon fields We consider SU(2) gauge fields and quenched approximation**



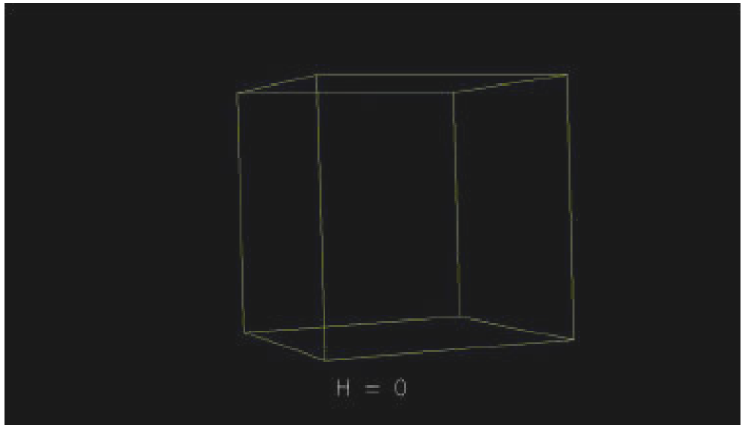


Quenched vacuum, overlap Dirac operator, external magnetic field

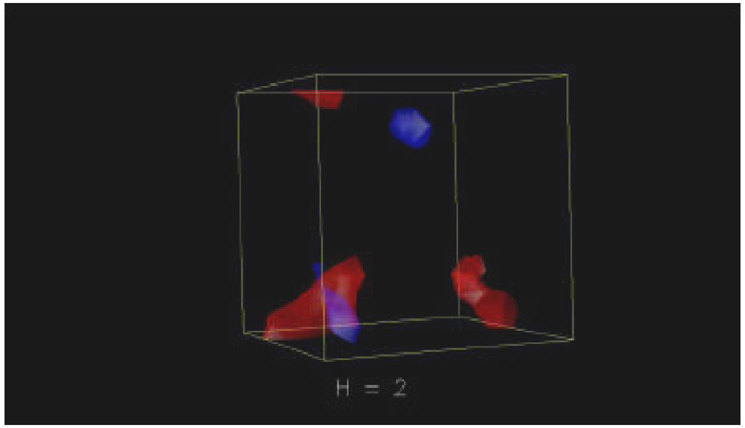
$$eB = \frac{2\pi qk}{L^2};\, eB \geq (250\ Mev)^2$$

Density of the electric charge vs. magnetic field, 3D time slices

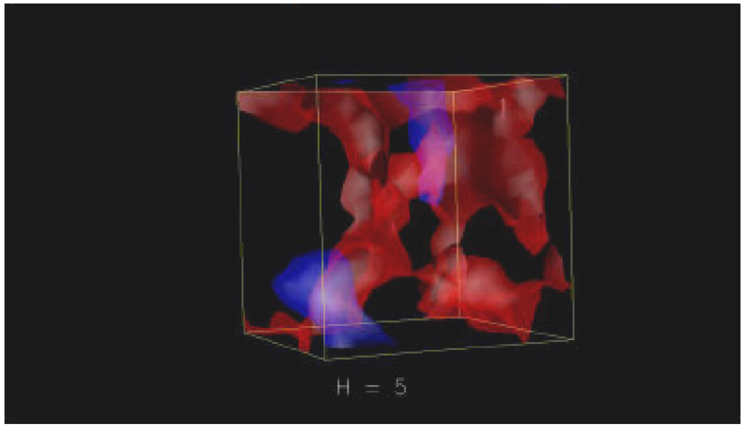
$B = 0$



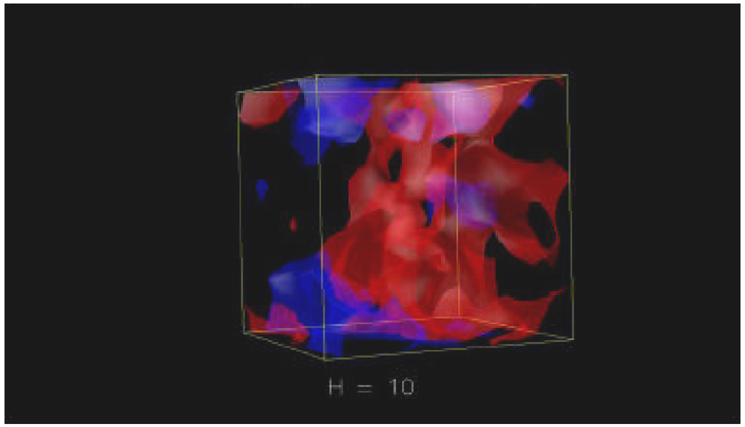
$B = (500\,\mathrm{MeV})^2$



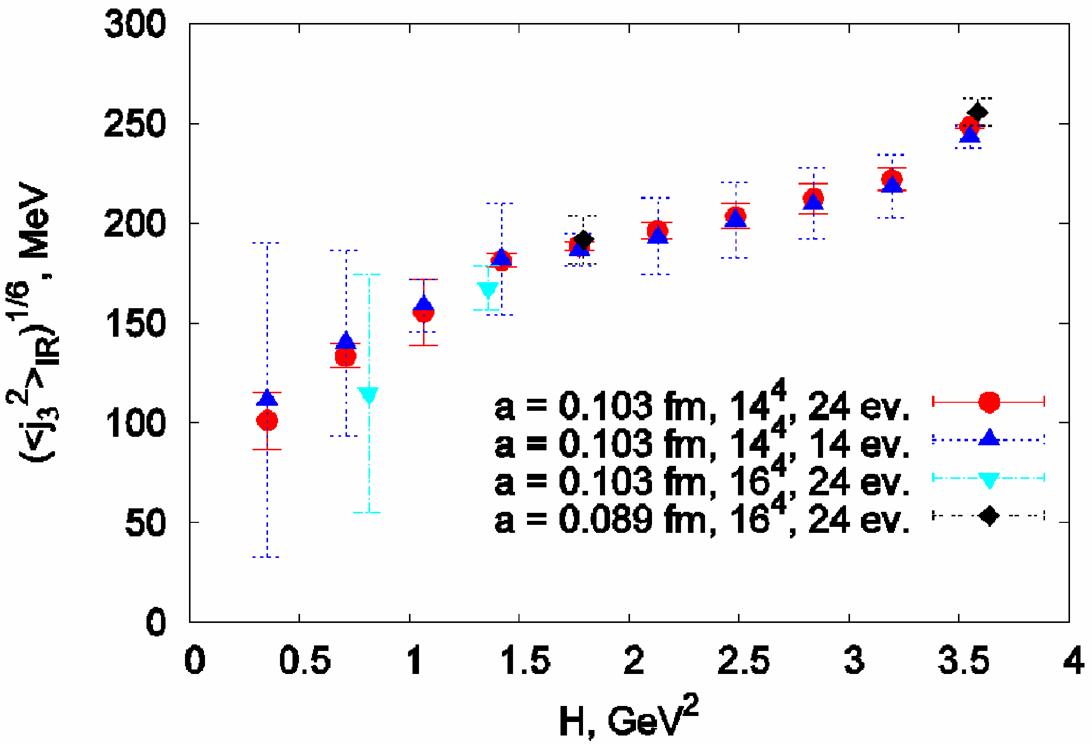
$B = (780\,\mathrm{MeV})^2$



$B = (1.1\,\mathrm{GeV})^2$



1. Chiral Magnetic Effect on the lattice, numerical results T=0



Regularized electric current:

$$< j_3^2 >_{IR} = < j_3^2(H,T) > - < j_3^2(0,0) >, \quad j_3 = \overline{\psi}\gamma_3\psi$$

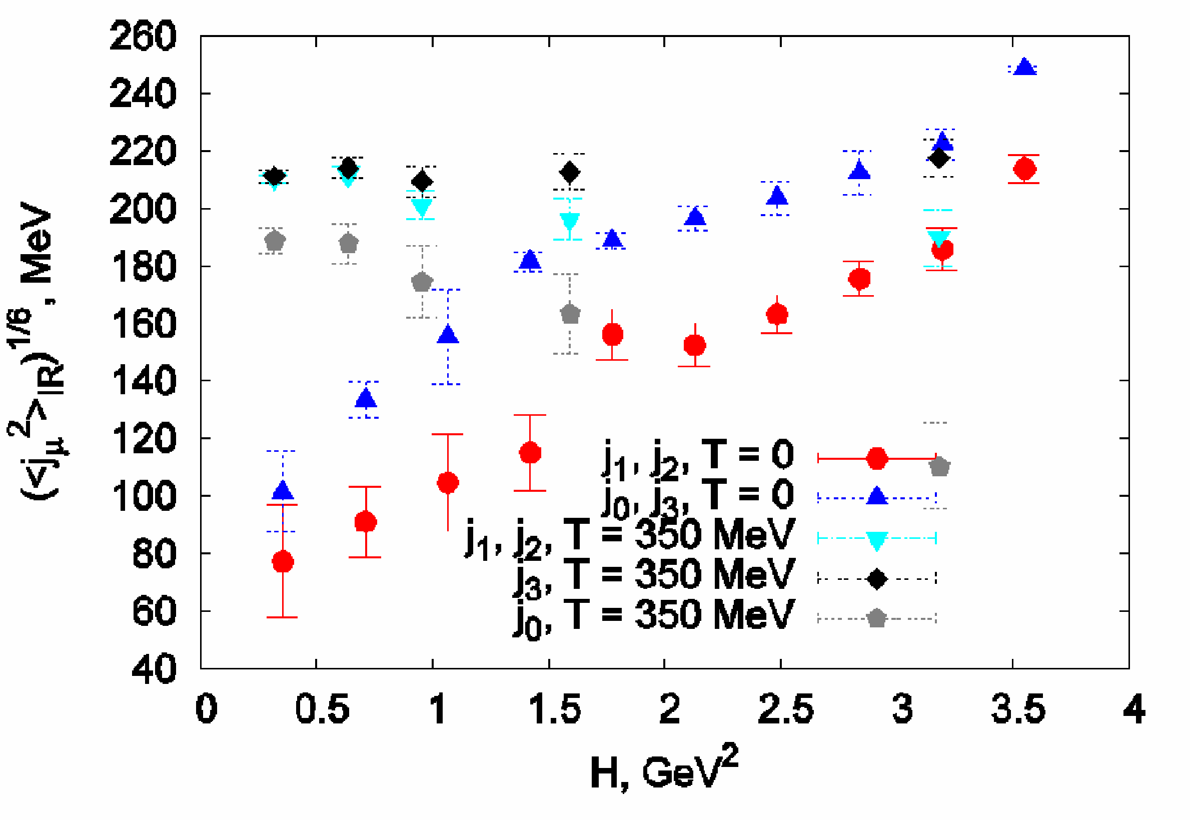
Chiral Magnetic Effect on the lattice, numerical comparison of results nearTc and nearzero

$T=0$

$F_{12}\neq 0$

$<j_1^2>=<j_2^2>$

$<j_3^2>=<j_0^2>$



$T>0$

$F_{12}\neq 0$

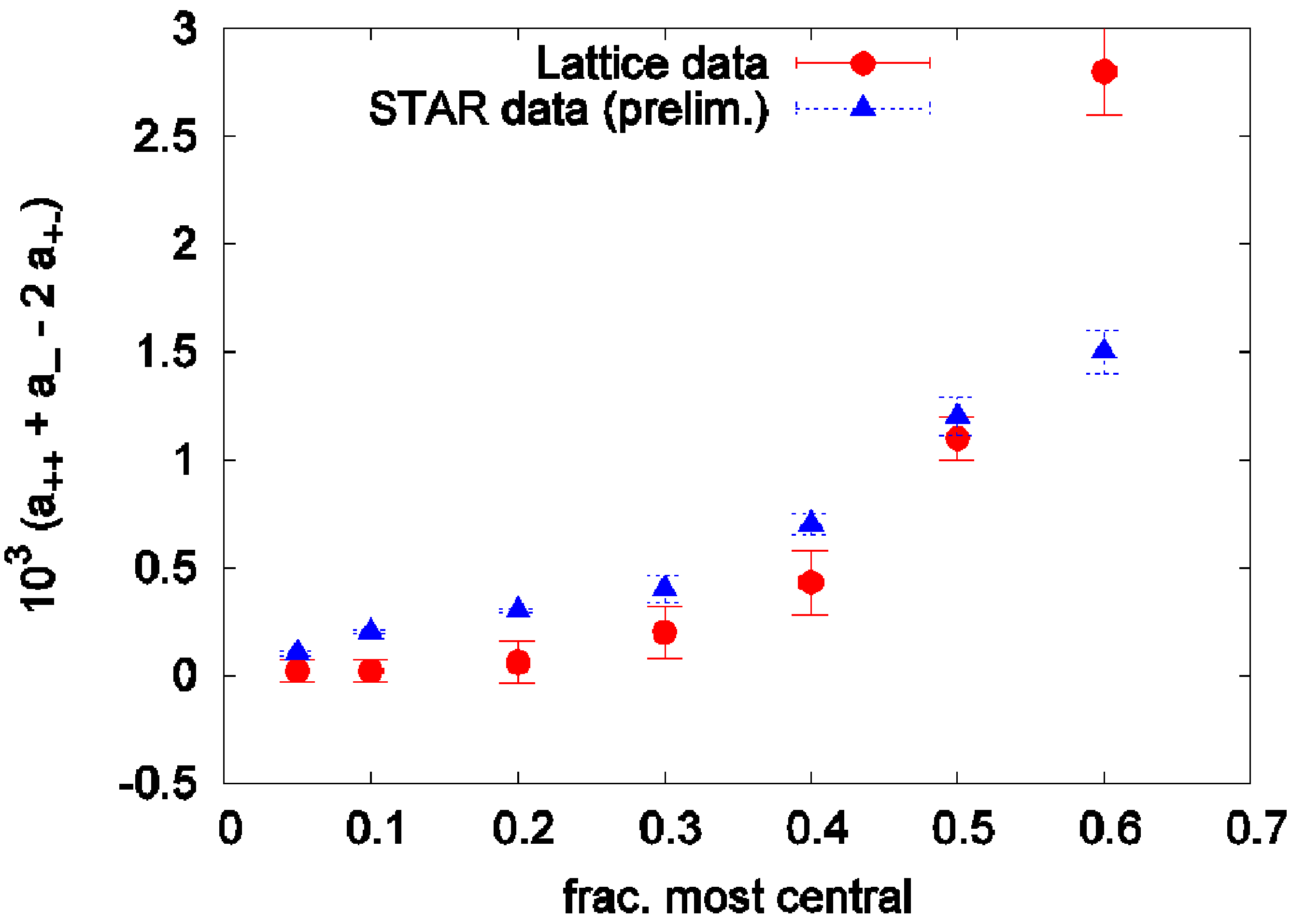
$<j_1^2>=<j_2^2>$

$<j_3^2>\neq<j_0^2>$

Regularized electric current:

$$<j_i^2>_{IR}=<j_i^2(H,T)>-<j_i^2(0,0)>,\quad j_i=\overline{\psi}\gamma_i\psi$$

Chiral Magnetic Effect,
EXPERIMENT VS LATTICE DATA (Au+Au)



Chiral Magnetic Effect,
EXPERIMENT VS LATTICE DATA

$$a_{ab} = \frac{1}{N_e}\sum_{e=1}^{N_e}\frac{1}{N_a N_b}\sum_{i=1}^{N_a}\sum_{j=1}^{N_b}\cos\left(\phi_{ia}+\phi_{jb}\right)$$

experiment

$R \approx 5\ fm$

$\rho \approx 0.2\ fm$

$\tau \approx 1\ fm$

$$\frac{\langle (\Delta Q)^2 \rangle}{N_q^2} = a_{++} + a_{--} - 2a_{+-}$$

our fit

D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, Nucl. Phys. A 803, 227 (2008),

$$= \frac{4\pi\tau^2\rho^2R^2}{3N_q^2}\left(\langle j_\parallel^2\rangle + 2\langle j_\perp^2\rangle\right)$$

our lattice data at *T*=350 *Mev*

1.2 Magnetic Field Induced Conductivity of the Vacuum

Qualitative definition of conductivity, σ

$$< j_\mu(x) j_\nu(y) > = C + A \cdot \frac{\exp\{-m|x-y|\}}{r^\alpha}$$

$$j_\mu(x) = \bar{q}(x)\gamma_\mu q(x)$$

$$\boxed{\sigma \propto C}$$

Magnetic Field Induced Conductivity of the Vacuum

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$ **- Conductivity (Kubo formula)**

$$G_{ij}(\tau) = \int\limits_0^{+\infty} \frac{dw}{2\pi} K(w,\tau)\, \rho_{ij}(w),$$ **Maximal entropy method**

$$K(w,\tau) = \frac{w}{2T} \frac{\cosh\left(w\left(\tau - \frac{1}{2T}\right)\right)}{\sinh\left(\frac{w}{2T}\right)},$$

$$G_{ij}(\tau) = \int d^3\vec{x} \langle j_i\left(\vec{0},0\right) j_j\left(\vec{x},\tau\right)\rangle$$

Magnetic Field Induced Conductivity of the Vacuum

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$ **- Conductivity (Kubo formula)**

For weak constant *electric* field

$$< j_i > = \sigma_{ik} E_k$$

Magnetic Field Induced Conductivity of the Vacuum
Calculations in SU(2) gluodynamics

$$\langle \bar{q}(x)\gamma_i q(x)\ \bar{q}(y)\gamma_j q(y)\rangle$$

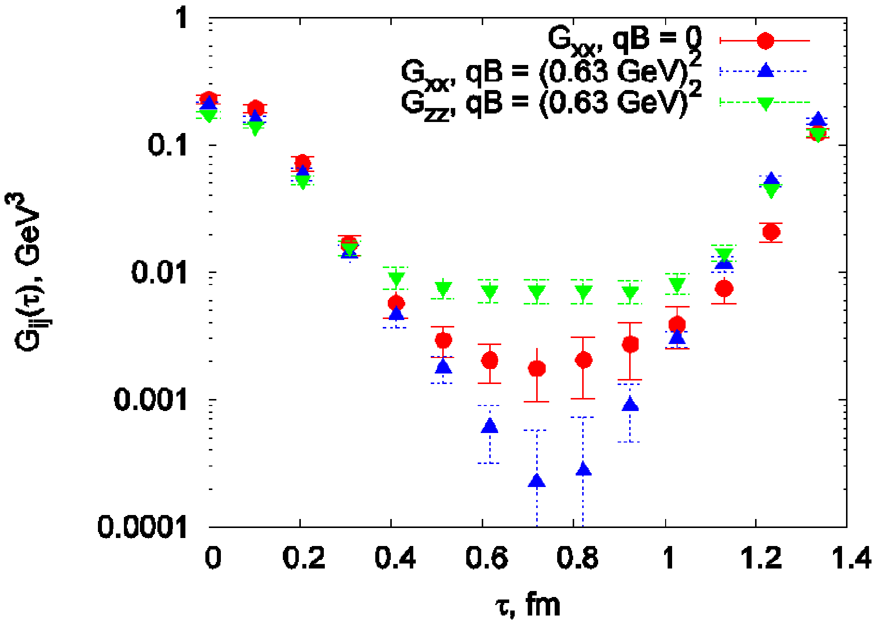
$$= \int \mathcal{D}A_\mu\, e^{-S_{YM}[A_\mu]}\, \mathrm{Tr}\left(\frac{1}{\mathcal{D}+m}\gamma_i \frac{1}{\mathcal{D}+m}\gamma_j\right)$$

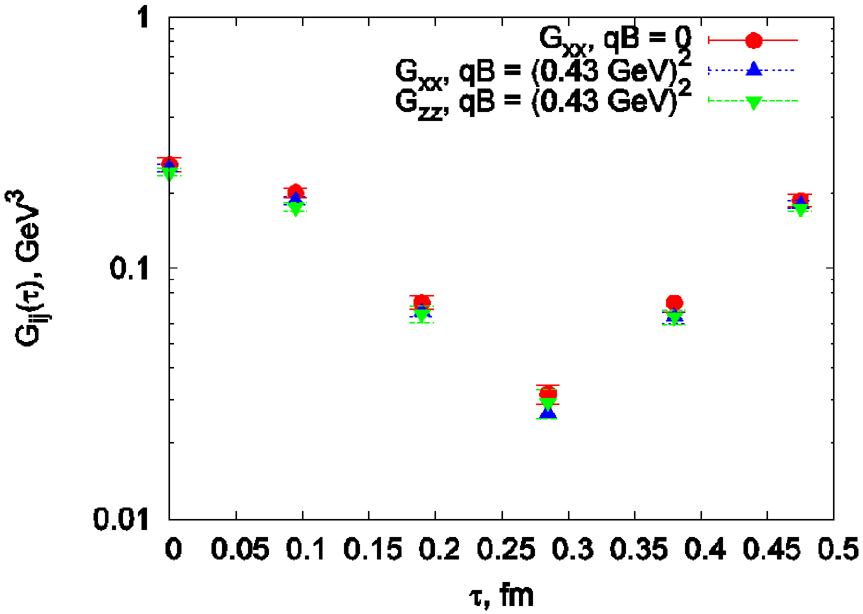
We use overlap operator + Shifted Unitary Minimal Residue Method *(Borici and Allcoci (2006))* to obtain fermion propagator

$$G_{ij}(\tau) = \int d^3\vec{x}\langle j_i\left(\vec{0},0\right) j_j(\vec{x},\tau)\rangle$$

Magnetic Field Induced Conductivity of the Vacuum
Calculations in SU(2) gluodynamics

$$G_{ij}(\tau) = \int d^3\vec{x} \langle j_i\left(\vec{0},0\right) j_j(\vec{x},\tau)\rangle$$



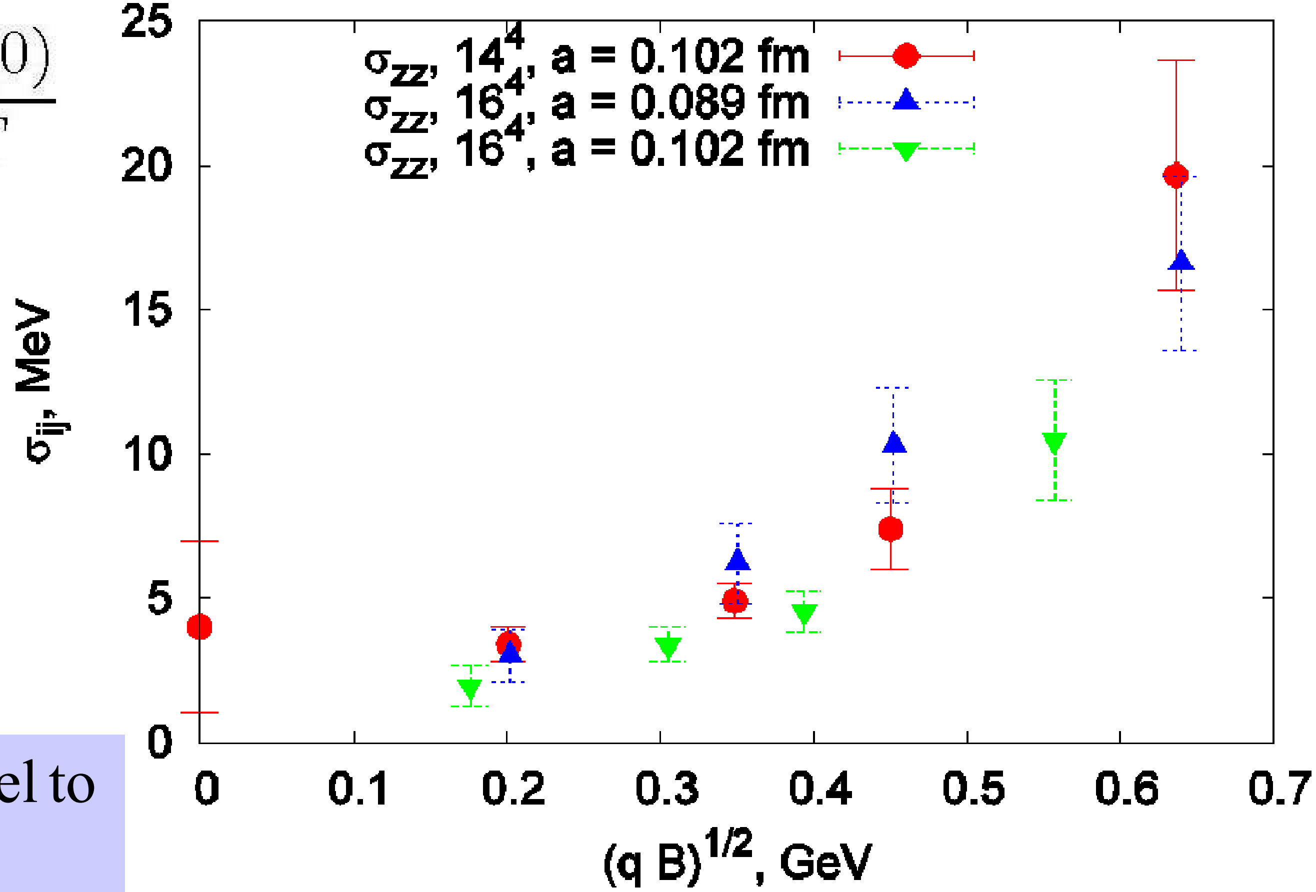


T = 0

T/Tc = 1.12

Calculations in SU(2) gluodynamics, conductivity along magnetic field at *T=0*

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$

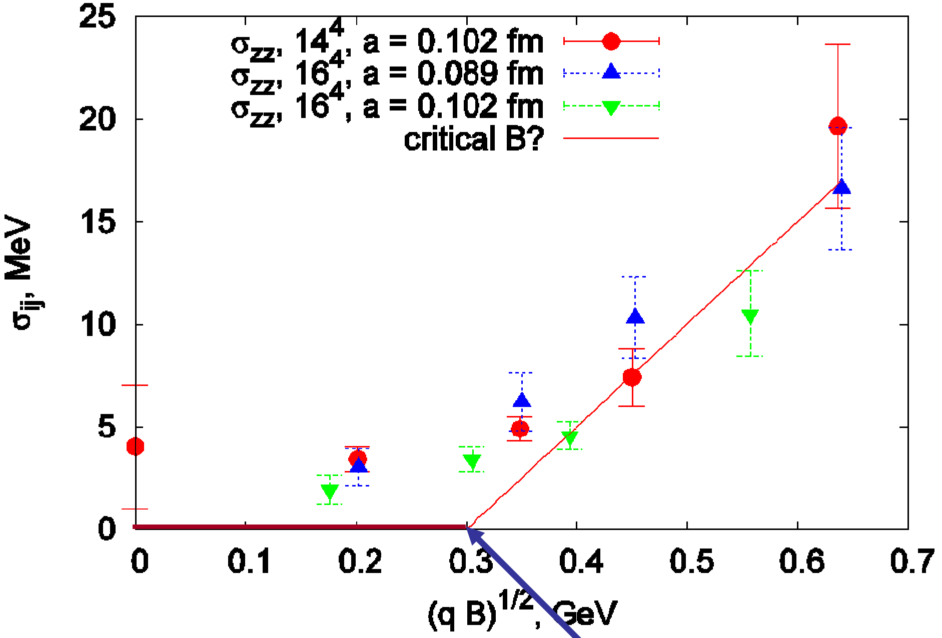


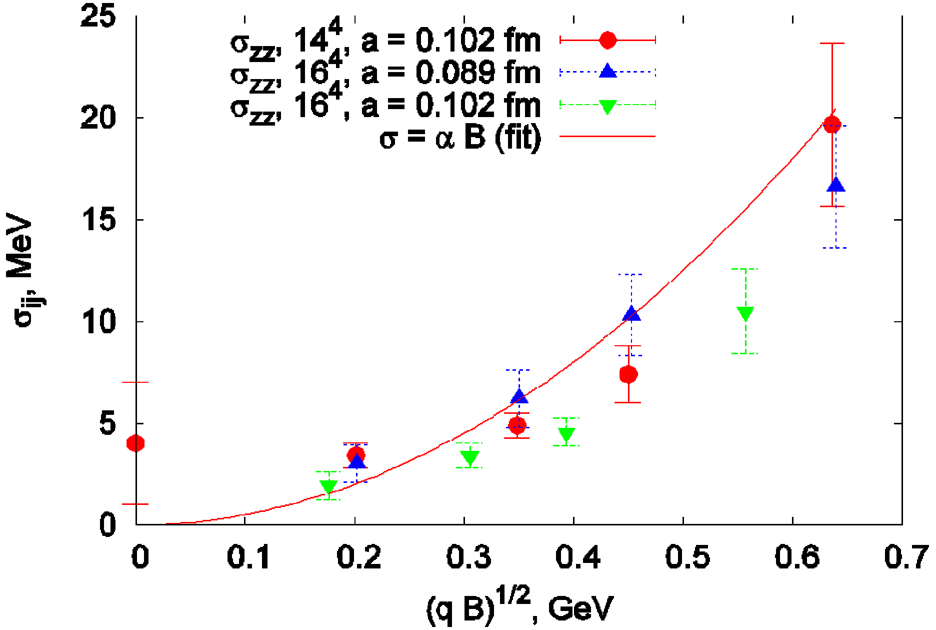
$\vec{H}$ is parallel to $0Z$ axis

Calculations in SU(2) gluodynamics, conductivity along magnetic field at *T=0*

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$

At T=0, B=0 vacuum is insulator

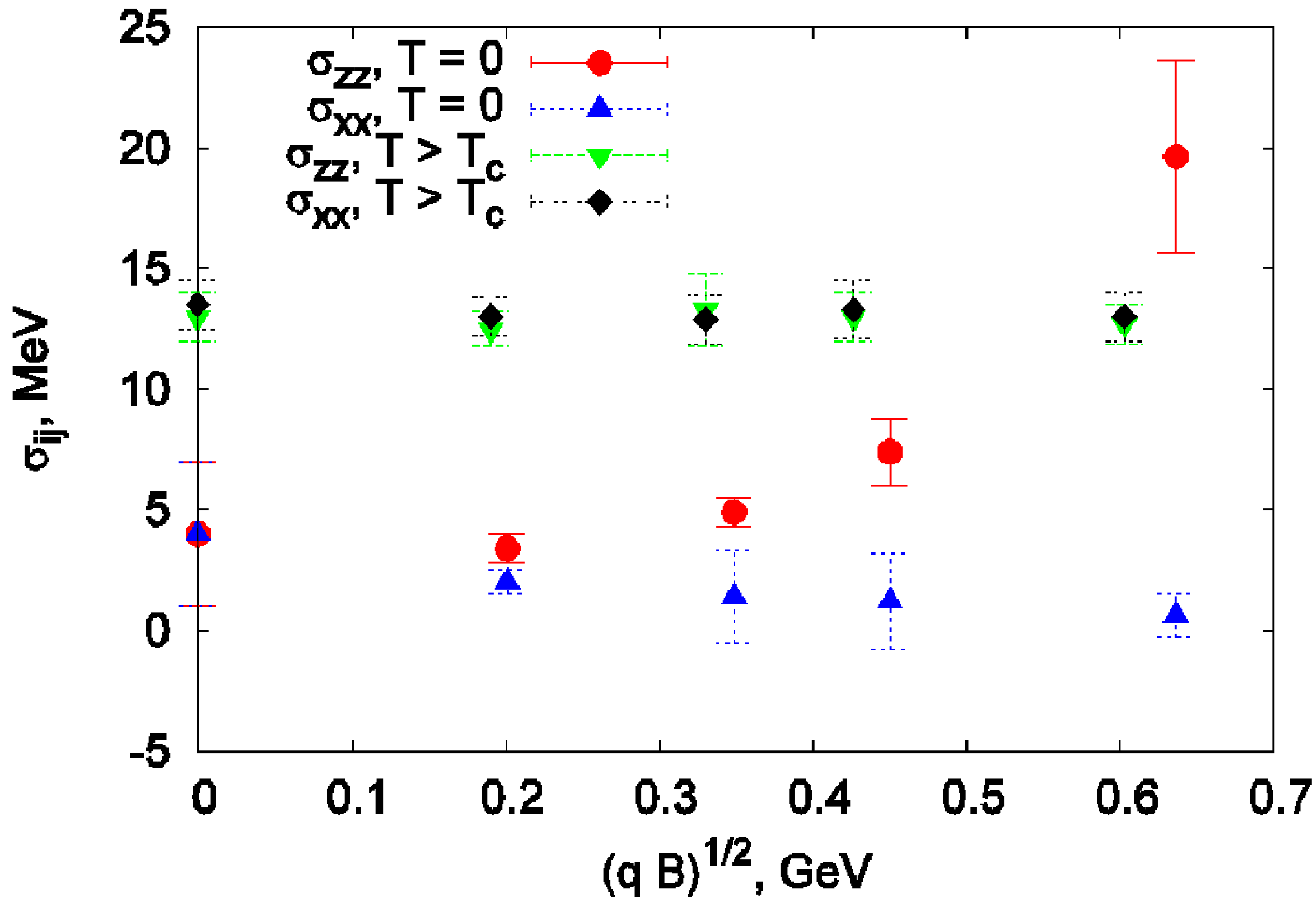




Critical value of magnetic field?

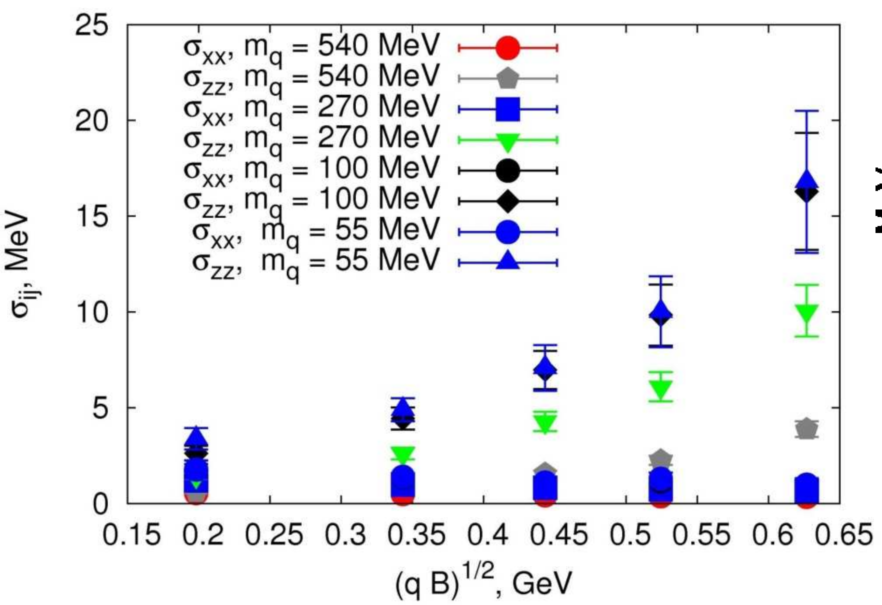
Calculations in SU(2) gluodynamics, conductivity along magnetic field at *T=0, T>0*

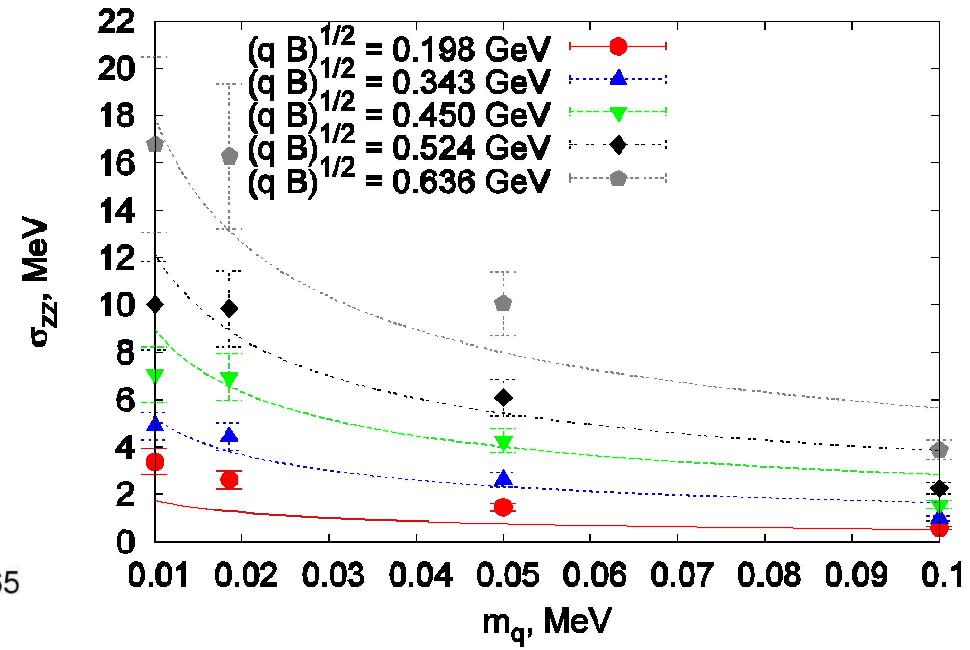
$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$



$\vec{H}$ is parallel to $0Z$ axis

Calculations in SU(2) gluodynamics, conductivity at *T=0*, variation of the quark mass



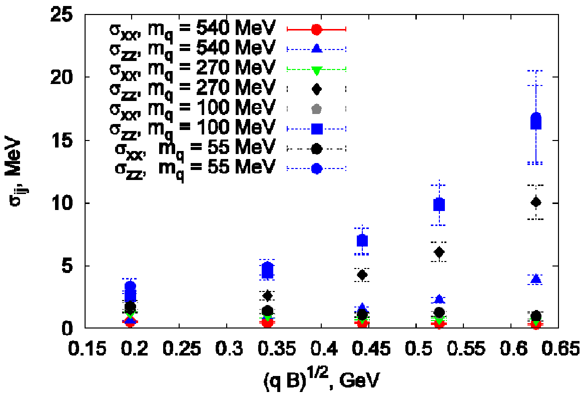


$$\sigma_{zz}(m_q, qB) \sim m_q^{-\alpha} (|qB|)^{\beta} \qquad \alpha = (0.45 \pm 0.06)$$

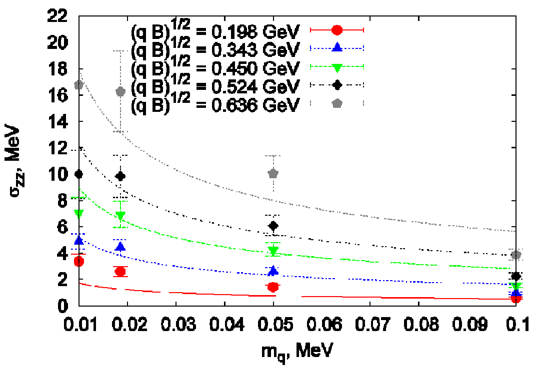
$$\beta = (1.1 \pm 0.2)$$

Calculations in SU(2) gluodynamics, conductivity at *T=0*, variation of the quark mass and magnetic field

$$\sigma_{ij} \propto \frac{B_i B_j}{B\sqrt{m_q}}$$



Why?



1.3 Dilepton emission rate

L. D. McLerran and T. Toimela, Phys. Rev. D 31, 545 (1985),
E. L. Bratkovskaya, O. V. Teryaev, and V. D.Toneev, Phys. Lett. B 348, 283 (1995)

$$\frac{R}{V} = -4e^4 \int \frac{d^3 p_1}{(2\pi)^3\, 2E_1} \frac{d^3 p_1}{(2\pi)^3\, 2E_1} L^{\mu\nu}(p_1, p_2) \frac{\rho_{\mu\nu}(q)}{q^4}, (7)$$

where p_1 and p_2 are the momenta of the leptons, $q = p_1 + p_2$, m is their mass and $L^{\mu\nu} = \left((p_1 \cdot p_2 + m^2)\,\eta^{\mu\nu} - p_1^\mu p_2^\nu - p_2^\mu p_1^\nu\right)$ is the dilepton tensor. If the electric conductivity is nonzero in the direction of the magnetic field, for sufficiently small p_1, p_2 one has $\rho_{ij}(q) \approx \rho_{ij}(0) \sim \sigma_{ij} \sim B_i B_j / |B|$, and hence

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$

$$\frac{R}{V} \sim \int \frac{d^3 p_1}{(2\pi)^3\, 2E_1} \frac{d^3 p_1}{(2\pi)^3\, 2E_1} \frac{(p_1 \cdot B)(p_2 \cdot B)}{|B|}. \qquad (8)$$

- There should be more soft dileptons in the direction of magnetic field

$$\frac{d\sigma}{dp_1 dp_2} \propto \frac{(\vec{p}_1 \cdot \vec{B})(\vec{p}_2 \cdot \vec{B})}{|B|\sqrt{m_q}}$$

2. Other effects induced by magnetic field

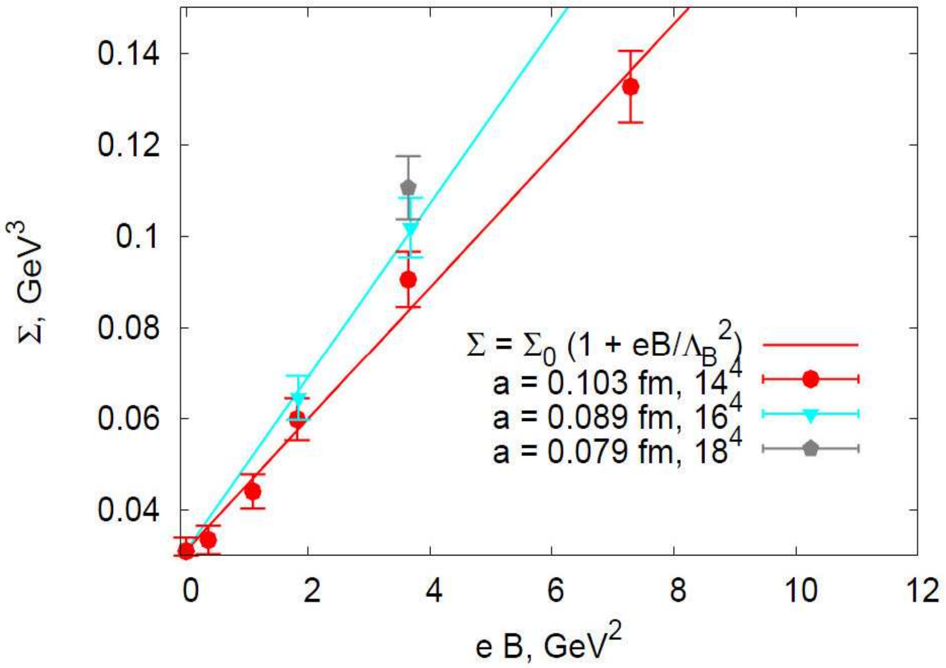
2.1 Chiral symmetry breaking
2.2 Magnetization of the vacuum
2.3 Electric dipole moment of quark along the direction of the magnetic field

3. Chiral condensate in QCD

$$\Sigma = -<\overline{\psi}\psi>$$

$$m_\pi^2 f_\pi^2 = m_q <\overline{\psi}\psi>$$

Chiral condensate vs. field strength, SU(2) gluodynamics



$$\Sigma = \Sigma_0 \left(1 + \frac{eB}{\Lambda_B^2}\right)$$

- Our value for Λ_B:

$$\Lambda_B^{\rm fit} = (1.41 \pm 0.14 \pm 0.20)\,\mathrm{GeV}$$

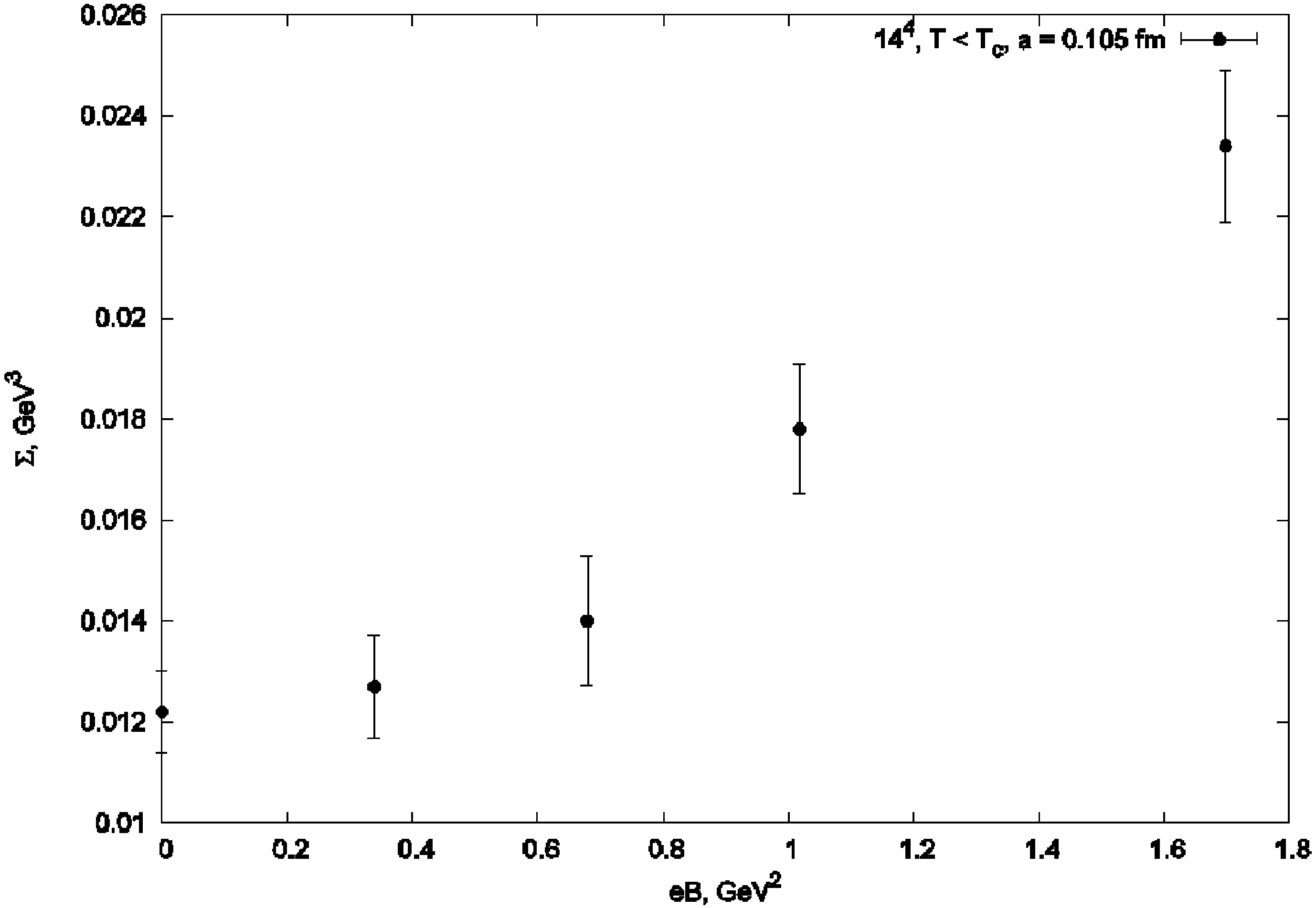
- χPT result:

$$\Lambda_B^{\chi PT} = 1.96\,GeV \quad (F_\pi = 130\,MeV - real\ world)$$
$$\Lambda_B^{\chi PT} = 1.36\,GeV \quad (F_\pi = 90\,MeV - quenched)$$

- Chiral condensate at $B = 0$: $\Sigma_0^{\rm fit} = [(310 \pm 6)\ \mathrm{MeV}]^3$

We are in agreement with the chiral perturbation theory: the chiral condensate is a linear function of the strength of the magnetic field!

Chiral condensate vs. field strength, SU(3) gluodynamics

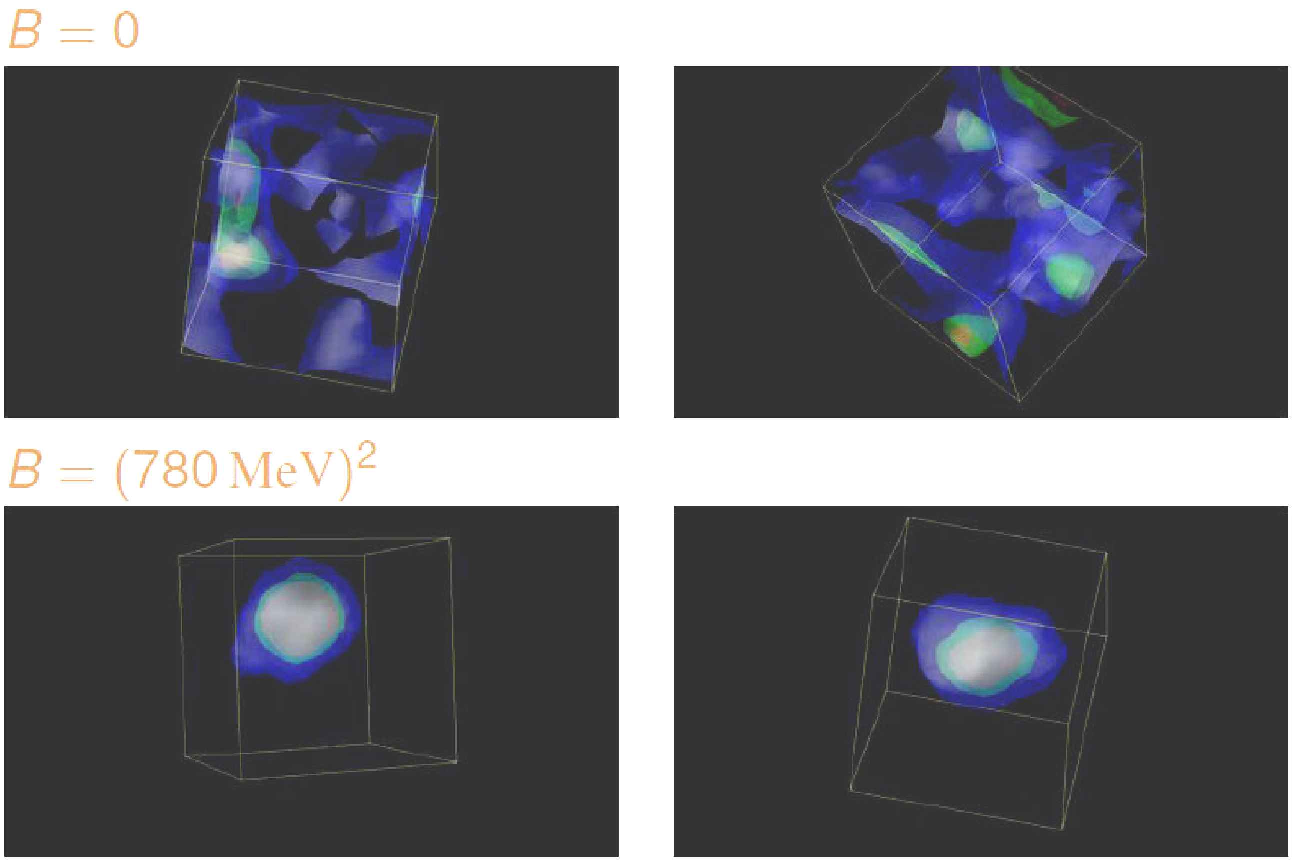


Localization of Dirac Eigenmodes

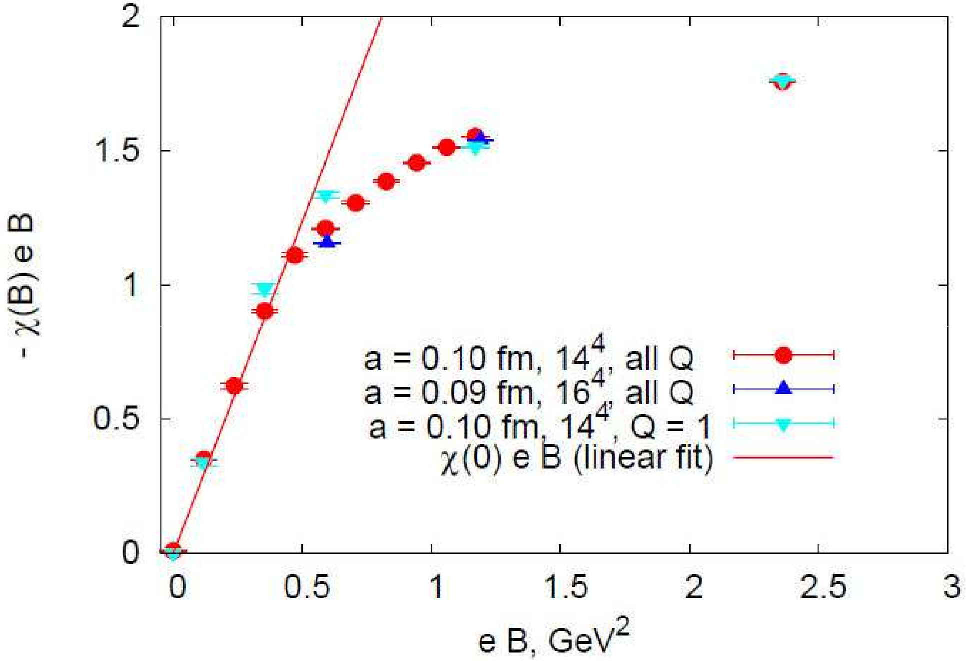
Typical densities of the nearzero eigenmodes vs. the strength of the external magnetic field

$B=0$

$B=(780 Mev)^2$



4. Magnetization of the vacuum as a function of the magnetic field



Spins of virtual quarks turn parallel to the magnetic field

$$<\bar{\psi}\sigma_{\alpha\beta}\psi> = \chi <\bar{\psi}\psi> F_{\alpha\beta}$$

$$\sigma_{\alpha\beta} = \frac{1}{2i}[\gamma_\alpha, \gamma_\beta]$$

$<\bar{\psi}\psi>\chi = -46(3)Mev \leftrightarrow$ our result

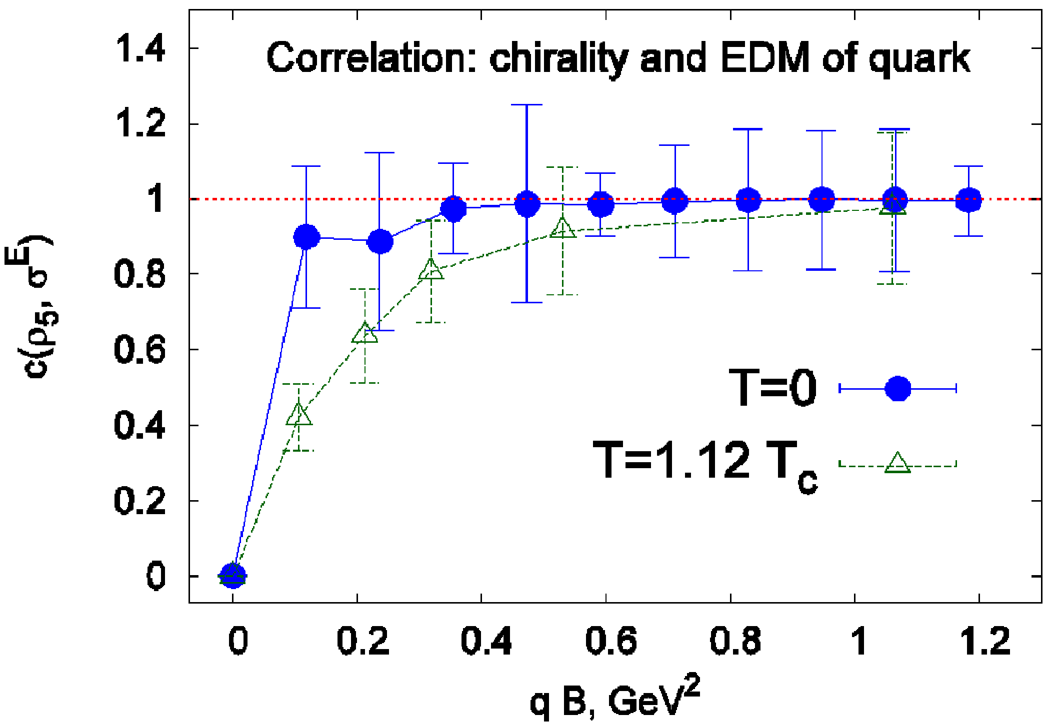
$<\bar{\psi}\psi>\chi \approx -50Mev \leftrightarrow$ QCD sum rules

(I. I. Balitsky, 1985, P. Ball, 2003.)

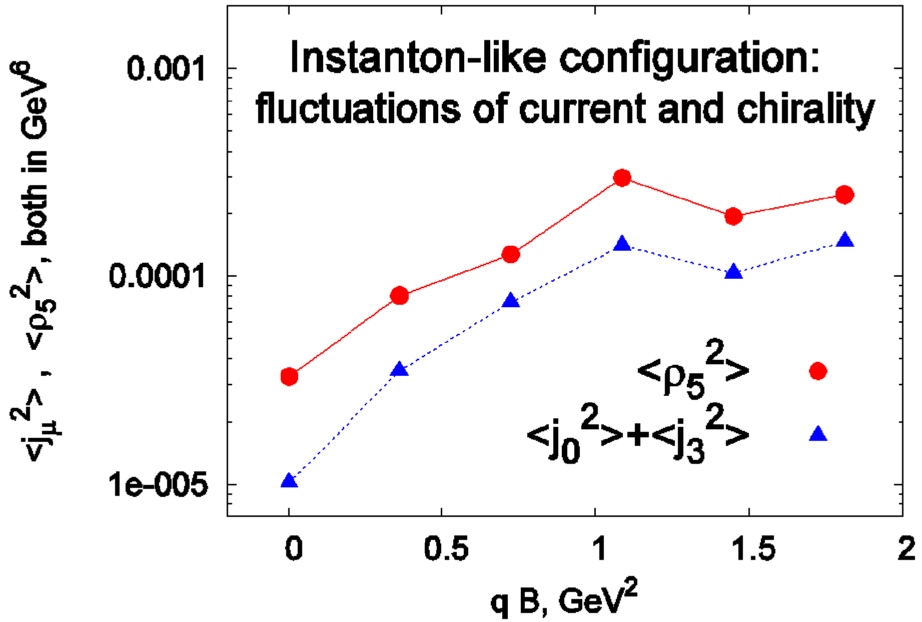
5. Generation of the anomalous quark electric dipole moment along the axis of magnetic field

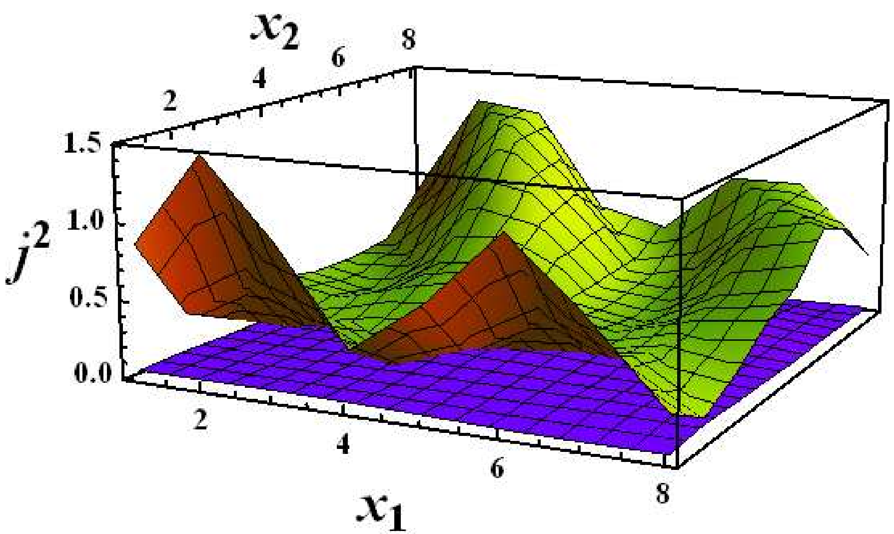
- **Large correlation between square of the electric dipole moment**

$\sigma_{0i} = i\overline{\psi}[\gamma_0, \gamma_i]\psi$ **and chirality** $\rho_5 = \overline{\psi}\gamma_5\psi$



6. Electric currents in instanton field+magnetic field (CME)





The fluctuations of the chirality $\rho_5 = \overline{\psi}\gamma_5\psi$ and the fluctuations of the longitudinal electric current as a function of the magnetic field.

The squared components of the electric current in a 12-plane. The upper sheet represents the spatial distribution of the longitudinal current, the lower sheet corresponds to the transverse current.

$$< j_i^2 >_{IR} = < j_i^2(H,T) > - < j_i^2(0,0) >, \quad j_i = \overline{\psi}\gamma_i\psi$$

Conclusions

- 1. We obsreve signatures of the Chiral Magnetic Effect, but the physics may differ from the model of Kharzeev, McLerran and Warringa (arXiv:0907.0494, Phys.Rev.D79: 106003,2009)
- 2. We observe that in the confinement phase the external magnetic field induces nonzero electric conductivity along the direction of the magnetic field, transforming the system from an insulator into an anisotropic conductor. In the deconfinement phase the conductivity does not exhibit any sizable dependence on the magnetic field (arXiv:1003.2180).
- 3. The conductivity is weaker for heavy quarks, thus it is interesting to measure experimentally the charge asymmetry for S and C quarks.

Conclusions

- 4. We observe that the chiral condensate is proportional to the strength of the magnetic field, the coefficient of the proportionality agrees with Chiral Perturbation Theory. Microscopic mechanism for the chiral enhancement is the localization of fermion modes in the vacuum (arXiv:0812.1740, Phys.Lett. B 682:484-489,2010).
- 5. The calculated vacuum magnetization is in a qualitative agreement with model calculations (arXiv:0906.0488, Nucl.Phys. B 826 (2010) 313).
- 6. We observe very large correlation between electric dipole moment of quark and chirality (arXiv:0909.2350 Phys.Rev.D81:036007,2010).

Systematic errors

- SU(2) gluodynamics instead of QCD
- Moderate lattice volumes
- Not large number of gauge field configurations
- In some cases we calculate the overlap propagator using summation over eigenfunctions:

$$\langle \bar{\Psi} \Sigma_{\alpha\beta} \Psi \rangle = 2m \langle \sum_{\lambda_k > 0} \frac{\psi_k^\dagger(x)\, \Sigma_{\alpha\beta}\, \psi_k(x)}{\lambda_k^2 + m^2} \rangle$$