Galileon inflation

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Galileon inflation is a radiatively stable higher derivative model of inflation. The model is determined by a finite number of relevant operators which are protected by a covariant generalization of the Galileon shift symmetry. We show that the nongaussianity of the primordial density perturbation generated during an epoch of Galileon inflation is a particularly powerful observational probe of these models and that, when the speed of sound is small, $f_{\rm NL}$ can be larger than the usual result $f_{NL} \propto c_s^{-2}$.

I. INTRODUCTION

Inflation is an era during which the cosmological scale factor $a(t)$ satisfies $\mathrm{d}^2a/\mathrm{d}t^2 > 0$ as a function of cosmic time t, growing by a factor e^N. It is a familiar idea that a successful implementation of inflation, by which we mean obtaining sufficiently large N, requires the inflationary Lagrangian density $\mathcal{L}$ to have an approximate shift symmetry—an invariance under the translation $\phi \to \delta_c\phi \equiv \phi + c$, where ϕ is the inflaton field and c is a constant.

Underlying this is the idea that the slow-roll conditions imply the shift symmetry is broken only mildly, both in the action and the equations of motion. In terms of the inflationary potential, $V(\phi)$, and the Planck mass, $M_{\rm P}$, these slow-roll conditions are typically expressed using the parameters ϵ and η, which satisfy $2\epsilon \equiv M_{\rm P}^2(V'/V)^2$ and $\eta \equiv M_{\rm P}^2 V''/V$.

How large a breaking can be acceptable? We expect any effective description to be valid only for a field excursion at most of order $M_{\rm P}$, before renormalization group flow introduces new physics which changes the description. Requiring the first- and second-order fractional variation in the potential, $V(\phi)$, to be small over an excursion of this magnitude yields

$$\delta_c \ln V \sim \sqrt{\epsilon} \ll 1, \quad \text{and} \quad \delta_c^2 \ln V \sim \eta - \epsilon \ll 1, \qquad (1)$$

where we have indicated approximate relations in terms of the slow-roll quantities ϵ and η. Therefore slow-roll inflation, defined by the conditions $\epsilon \ll 1$ and $|\eta| \ll 1$, entails tuning $V(\phi)$ so that only very mild breaking occurs even over large variations in field value.

This tuning has an important consequence. Creminelli observed that, once $V(\phi)$ has been adjusted to satisfy Eq. (1), we can add any operator invariant under the shift symmetry without spoiling the property of successful inflation [1]. There is a large class of such operators, constructed by applying any combination of derivatives to the inflaton field ϕ. This yields $\nabla\phi$, $\nabla\nabla\phi$ and higher gradients, with indices contracted in arbitrary combinations. It follows that the most general local, diffeomorphism-invariant action for ϕ coupled to Einstein gravity which is invariant under the shift symmetry can be written [2]

$$\mathcal{L} = \sqrt{-g}\left[\frac{M_{\rm P}^2}{2}R + \mathcal{L}_{\rm M}(\nabla\phi, \nabla\nabla\phi, \ldots)\right]. \qquad (2)$$

In this paper, we study how inflation can be realized in theories of the form (2). In comparison with slow-roll inflation using canonical kinetic terms there are new difficulties, associated with the appearance of unstable 'ghost' states and stability under radiative corrections. One class of ghost-free, radiatively stable models with an interesting inflationary phenomenology has been widely studied. These are the Dirac–Born–Infeld (DBI) models [3–5]. However, another class of ghost-free models has recently been constructed [6, 7], based on a so-called 'Galilean' symmetry to be defined in §II below. This Galileon field has been shown to arise naturally in theories of massive gravity without ghosts in their decoupling limit [8].

Galileon models describe an effective *short*-distance theory associated with a modification of gravity on large scales, and have mostly received attention as models of dark energy. However, it is equally possible that they

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can be used to describe cosmological evolution, and have been investigated for this purpose by numerous authors [9]. Ali *et al.* obtained constraints on the parameters of the model (to be discussed in §IV) using the evolution of perturbations during matter domination [10]. Later, Creminelli *et al.* used an effective theory of this type to propose an alternative to the usual inflationary mechanism for generating a primordial density perturbation [11].

Kobayashi *et al.* proposed a kinetically-driven model of inflation supported by a Galileon-like field, which they called "G-inflation." In this model, $\mathcal{L}_{\rm M}$ was taken to be of the form $\mathcal{L}_{\rm M} = P(X, \phi) + F(X, \phi)\Box\phi$, where $X = -\nabla_\mu\phi\nabla^\mu\phi$ [12]. Kobayashi *et al.* gave background solutions and studied the two-point correlation function of perturbations; three-point correlations in this model were later considered by Mizuno & Koyama [13]. As we will explain, although this scenario is qualitatively similar to a Galileon model, the "G-inflation" picture described in Ref. [12] relies on a hard breaking of the Galilean symmetry. This symmetry is essential to stabilize the theory against quantum corrections; if it is broken, there is no protection against the radiative generation of higher operators. These not only spoil the predictions of the theory, but lead to ghost-like instabilities.

In this paper we take a different approach. We argue that it is important to retain the Galilean symmetry, as far as possible. It will turn out that a non-trivial background geometry introduces soft breaking terms, which lead to an interesting phenomenology. However, Galileon models which preserve sufficient symmetry represent an alternative to the DBI Lagrangian as a ghost-free, radiatively stable higher-derivative inflationary model. In comparison with the studies of Kobayashi *et al.* and Mizuno & Koyama we work with the most general covariant completion of the Galileon-invariant Lagrangian.

We refer to an epoch of inflation driven by a Galilean-invariant field (perhaps broken by a nontrivial background) as 'Galileon inflation.' We calculate the bispectrum nongaussianity parameter $f_{\rm NL}$ [14] and show that Galileon models can have an observational signature which is distinct from both DBI and canonical inflation. Indeed, Galileon models occupy a previously unexplored regime in the effective field theory of inflationary perturbations, where observable nonlinearities may be much larger than those obtained from even the highly nonlinear DBI Lagrangian. This difference arises because the cubic interactions are described by a combination of dimension-six *and* dimension-seven operators, rather than the dimension-six operator alone, which would generically dominate.

In what follows we work with signature $(-, +, +, +)$ and allow Greek indices μ, ν, ..., to range over spacetime indices, whereas Latin indices i, j, ..., label purely spatial indices. We choose units in which the Dirac constant, $\hbar$, and the speed of light, c, are set to unity, and express the gravitational coupling in terms of the reduced Planck mass, $M_{\rm P}^2 = (8\pi G)^{-1/2}$. Spacetime indices in complementary upper and lower positions are contracted with the spacetime metric, and $\Box = \nabla^\mu\nabla_\mu$. We reserve ∂ to denote a derivative with respect to purely spatial indices. Spatial indices are contracted using a Kronecker delta, so that $\partial^2 = \partial_i\partial_i$.

II. INFLATION

In this section we briefly review the conditions $\mathcal{L}_{\rm M}$ must satisfy to constitute a well-defined, predictive description of an inflationary phase.

Unitarity. An arbitrary action of the form (2) contains derivatives of second-order and higher applied to the elementary fields, and therefore leads to equations of motion of third-order or higher. Unless an infinite number of derivatives are present such theories do not usually possess a well-defined Cauchy problem [15], which manifests itself in the appearance of ghost states. At the quantum level the result is a pernicious loss of unitarity. Accordingly we should restrict our attention to special choices of $\mathcal{L}_{\rm M}$ which lead to second-order equations of motion [16].

Quantum fluctuations. It has recently been argued that the choice $\mathcal{L}_{\rm M} = F(X, \phi)\Box\phi$ (used in Ref. [12]) leads to second-order equations of motion for any choice of F [17], and therefore unitary evolution as a quantum field theory. For this reason, one could contemplate such models as candidates for an inflationary action of the form (2).

However, unitarity is not the only condition for predictivity. A typical F can be represented as a power series in X and ϕ, with some specified coefficients. But we should recognize that in the absence of other symmetries there is no way to protect these coefficients from quantum corrections. An important example is the DBI Lagrangian $\mathcal{L}_{\rm M} \supseteq f(\phi)\sqrt{1 - X}$ for an arbitrary $f(\phi)$. In the limit $X \sim 1$, fluctuations around the background solution have small sound speeds $c_s \ll 1$ and acquire large nongaussianities of order $f_{\rm NL} \sim c_s^{-2}$ [5, 18]. But at $X \sim 1$ the square root formally receives contributions from all powers of X with coefficients in precisely defined ratios, which in principle are susceptible to disruption by renormalization. The prediction $f_{\rm NL} \sim c_s^{-2}$ is believed to be trustworthy only because a symmetry forbids large renormalizations of these coefficients [3, 19], forcing quantum corrections to involve the *two*-derivative combination $\nabla\nabla\phi$. A similar symmetry protects fluctuations around models that exhibit Galilean symmetries (to be defined below), such as those that arise in the decoupling limit of either massive gravity or the Dvali–Gabadadze–Porrati (DGP) model, and can even be extended to theories containing multiple fields [6, 7, 20–23]. We discuss these non-renormalization properties in §III.

The lesson we wish to draw from these examples is that our calculations are likely to be trustworthy only if the functional form of $\mathcal{L}_{\rm M}$ is protected from large renormalizations by the presence of a symmetry. In seeking to generalize the DBI and Galileon-type Lagrangians, there are essentially only two choices.

(a) The shift symmetry $\phi \to \phi + c$ is the only symmetry of the action. In this case we must take the leading relevant operators to be the quadratic mass and kinetic terms, ϕ^2 and $(\nabla\phi)^2$, and terms with more derivatives should be treated as irrelevant in the technical sense. Corrections to the Gaussian theory arise from at most a few non-renormalizable operators, leading to very small observable departures from purely Gaussian fluctuations [1, 24].

(b) The shift symmetry is combined with a second symmetry, which protects the form of any derivative interactions. Such a symmetry cannot commute with the generators of the Poincaré group, because it necessarily mixes derivatives of different orders. Therefore it must be nonlinearly realized as a nonfactorizable extension of the Poincaré group.

Choice (a) leads to canonical slow-roll inflation with very small corrections, and is well-understood. Interesting generalizations must therefore make use of (b) and can be classified according to their choice of nonfactorizable extension. Only a handful of such extensions are known, of which one is the conformal group. Others can be constructed beginning with the Poincaré, de Sitter or anti-de Sitter groups in dimension greater than four and applying Wigner–Inönü contractions. Following this reasoning one arrives at a short list of possible $\mathcal{L}_{\mathrm{M}}$ which are protected by symmetries. The choices are DBI, warped-DBI, Galileon and conformal-Galileon models, together with their higher-dimensional generalizations [6, 7] [25]. In this paper we focus on the general class of (conformal-) Galileon models. In such models the protecting 'Galilean' symmetry is an extension of the shift symmetry to include spacetime translations, $\phi \to \delta_{\mathrm{g}}\phi = \phi + b_\mu x^\mu + c$, where b_μ and c are constant.

Observables. Eq. (2) implies a large degeneracy among $\mathcal{L}_{\mathrm{M}}$ which realize given values of ϵ and η, and therefore the amplitude $\mathcal{P}^{1/2}$ and spectral index n_s of the primordial density perturbation [26]. Over the last decade, significant effort has been invested in developing observables which distinguish one choice of $\mathcal{L}_{\mathrm{M}}$ from another [1, 18, 24, 27]. As we now explain, the most powerful family of such observables is the amplitude of three-, four- and higher n-point correlations.

How are inflationary models to be distinguished? The renormalization programme has taught us that predictions extracted from quantum field theories, such as those governing inflation, express measurable quantities in terms of a finite number of experimental inputs—sometimes expressed as "observables in terms of observables." In canonical inflation there are two relevant operators, $(\nabla\phi)^2$ and ϕ^2, which each require a parameter to be extracted from experiment. Typically these are $(H/M_{\mathrm{P}})^2/\epsilon$ and η, extracted from $\mathcal{P}^{1/2}$ and n_s. The ratio $(H/M_{\mathrm{P}})^2$ alone can be determined from the tensor–scalar ratio r and fixes a particular de Sitter background geometry. In a noncanonical model of the form (2), the action for fluctuations may exhibit spontaneous breakdown of Lorentz invariance, giving $\dot{\phi}^2$ and $(\partial\phi)^2$ independent coefficients. This breaking is measured by the sound speed c_s^2. The gravitational coupling, M_{P}, is measured by terrestrial gravitational experiments.

We conclude that no measurement involving $\mathcal{P}^{1/2}$, n_s, and r alone can tell us about the operators present in $\mathcal{L}_{\mathrm{M}}$, although c_s can diagnose whether $\dot{\phi}^2$ and $(\partial\phi)^2$ have independent coefficients. To distinguish one choice of $\mathcal{L}_{\mathrm{M}}$ from another, we must ask whether further observables such as the bispectrum nongaussianity, f_{NL}, and the trispectrum observables τ_{NL} and g_{NL} [28] can be expressed in terms of the input parameter set. Maldacena showed that, in a single-field model, this is true for f_{NL} [29] unless an irrelevant operator associated with some higher slow-roll parameter is unexpectedly large. The same is true for the single-field trispectrum observables [30]. It is for this reason that nongaussianities are such a stringent test of the single-field framework, because measurements of f_{NL}, τ_{NL} and g_{NL} cannot simply be absorbed into a new parameter of the model.

Similar reasoning applies to noncanonical models described by (2). There is a simplification in theories which are not approximately canonical inflation, to be discussed in §IV C below, because the influence of gravity becomes negligible [5, 27, 31–33]. Therefore observables involve the parameters of $\mathcal{L}_{\mathrm{M}}$ alone, and do not mix with M_{P}. Different choices of $\mathcal{L}_{\mathrm{M}}$ may require a larger or smaller set of input parameters to express their predictions. However, once these are measured, the remaining observables are uniquely determined. Therefore these express genuine differences between one $\mathcal{L}_{\mathrm{M}}$ and another. For example, in DBI inflation there is a firm prediction for f_{NL} once c_s has been determined, perhaps using the tilt of the tensor power spectrum [34]. We will show in §IV E below that there is a different prediction for f_{NL} in Galileon models, making the nongaussianity of the primordial density perturbation a particularly powerful observational probe of Galileon inflation. The situation is less favourable if the form of $\mathcal{L}_{\mathrm{M}}$ is not protected by a symmetry as with $\mathcal{L}_{\mathrm{M}} \supseteq F(X,\phi)\Box\phi$ for general F. In that case, an infinite number of experimental inputs are required to determine F and the theory loses predictivity.

Experimental limits. Measurements of the basic input parameters $\mathcal{P}^{1/2}$ and n_s are now quite precise [35], and will improve further once data arrive from the *Planck* satellite. Limits on the bispectrum vary depending on the momentum dependence, or 'shape' [36]. Present-day limits are moderately constraining [32, 35, 37], and data from the Planck satellite is expected to furnish very stringent constraints. For this reason it is realistic to anticipate the role of f_{NL} as a constraint on inflationary models. Limits on the trispectrum parameters are presently weak [38]. Smidt *et al.* estimate that an optimistic future experiment may constrain f_{NL} to ± 3, τ_{NL} to ± 225 and g_{NL} to $\pm 6 \times 10^4$. It is not yet clear whether the amplitude of higher correlations can ever be significantly constrained, so these parameters are likely to exhaust the available input parameters for at least the near future.

If $\mathcal{L}_{\mathrm{M}}$ requires more input parameters than can be measured, the model again becomes unpredictive. This contrasts with accelerator experiments where—in principle—there is typically no limit to the number of parameters which can be taken from experiment. In this paper we wish to emphasize the distinction of *principle* between a predictive theory, where a symmetry picks out a finite number of invariant operators whose coefficients fix the other observables, and unpredictive theories where quantum effects imply that an infinite number of coefficients must be retained. In practice, however, a failure of predictivity for whatever cause is problematic.

III. BREAKING THE SHIFT SYMMETRY

An exact shift symmetry in Eq. (2) makes the value of ϕ unphysical. To proceed we softly break the symmetry by hand, giving ϕ a dynamical vacuum expectation value. We first argue that this breaking is necessary because we would otherwise obtain $\dot{\phi} = 0$, preventing the existence of a conserved curvature perturbation ζ at long wavelengths.

The mildest possible breaking is to introduce a potential $V = V_0 - \lambda^3\phi$, where V_0 is a constant and λ has engineering dimension [mass]. Taking the decoupling limit $M_{\mathrm{P}} \to \infty$ while keeping the de Sitter background fixed, so that $3H^2M_{\mathrm{P}}^2 = V_0$, we recover a scalar field theory on exact de Sitter space. In this limit, a term linear in ϕ does not break the shift symmetry because the action transforms as a total derivative,

$$\delta_c \left[\sqrt{-g}\;\phi\right] = a(t)^3 c = \frac{1}{3}\partial_i \left[a(t)^3 c\, x_i\right]. \tag{3}$$

We conclude that with dynamical gravity the symmetry must be broken softly in the sense that its effects are suppressed by powers of $1/M_{\mathrm{P}}$.

Because the shift symmetry is exact in the decoupling limit, we conclude that the equation of motion for ϕ can be written as a current conservation equation [39]

$$\nabla_\mu J^\mu = \dot{J}^t + 3HJ^t = \lambda^3, \tag{4}$$

where J^μ is the Noether current,

$$J^\mu = \frac{\partial \mathcal{L}_{\mathrm{M}}}{\partial(\nabla_\mu\phi)} - \nabla_\nu\left[\frac{\partial\mathcal{L}_{\mathrm{M}}}{\partial(\nabla_\nu\nabla_\mu\phi)}\right] + \cdots. \tag{5}$$

Eq. (4) admits the exact solution $J^t = \lambda^3/3H$, which could equally have been obtained by passing to the overdamped limit $\dot{J}^t/J^t \to 0$. Since J^t is constant, the shift symmetry allows us to infer that $\dot{\phi}$ is also constant and proportional to λ^3. (To reach this conclusion we require the theory to have a well-posed initial value formulation, which is guaranteed by our restriction to second-order equations of motion.) In the unbroken case where $\lambda = 0$ we would obtain $\dot{\phi} = 0$.

At first, it might seem surprising that inflation in this scenario is possible only in the presence of a potential $\lambda \neq 0$. It is now well-understood that models of "k-inflation" type [27] can achieve an accelerated regime in purely kinetic scenarios, where $\mathcal{L}_{\mathrm{M}} = \mathcal{L}_{\mathrm{M}}(X)$. The resolution is that the equations of motion admit solutions of two types: a transient decaying contribution—which, in perturbation theory, is the decaying mode; and a dominant growing-mode solution, which is the one we consider above. Purely kinetic k-inflation models make use of the normally transient decaying mode to source the background expansion. In what follows we shall work with the more conventional solution.

Nonrenormalization of mass. The necessity of including a nontrivial potential is problematic. Since potential terms are not invariant under the shift symmetry, they must be treated as irrelevant operators which induce small corrections. But operators of mass dimension less than four would typically receive large quantum corrections and, if present, such large renormalizations destroy the radiative stability of $\mathcal{L}_{\mathrm{M}}$, making the model of limited interest. In practice we cannot avoid this issue because a potential term of at least quadratic order is required to describe a graceful exit from the inflationary phase.

Remarkably, the operators ϕ and ϕ^2 are protected by a nonrenormalization theorem and therefore can be treated consistently as irrelevant deformations of the shift-invariant Lagrangian [21]. This conclusion may be reached most straightforwardly by considering the one-particle irreducible effective action, $\Gamma[\phi]$,

$$\exp \mathrm{i}\Gamma[\phi] = \int \mathrm{D}\psi\; \exp\left\{\mathrm{i}S[\phi+\psi] - \mathrm{i}\psi\frac{\delta\Gamma[\phi]}{\delta\phi}\right\}. \tag{6}$$

We separate $\Gamma[\phi]$ into its classical part, which coincides with the classical action $S[\phi]$, and a quantum part Γ_{q}, writing $\Gamma[\phi] = S[\phi] + \Gamma_{\mathrm{q}}[\phi]$. Therefore

$$\begin{aligned}\exp \mathrm{i}\Gamma_{\mathrm{q}}[\phi] = \int \mathrm{D}\psi\; \exp\Bigg\{&\mathrm{i}S[\phi+\psi] - \mathrm{i}S[\phi] \\ &- \mathrm{i}\psi\frac{\delta S[\phi]}{\delta\phi} - \mathrm{i}\psi\frac{\delta\Gamma_{\mathrm{q}}[\phi]}{\delta\phi}\Bigg\}.\end{aligned} \tag{7}$$

First, consider an action of the form

$$S[\phi] = S_0[\phi] + \int \mathrm{d}^4x\; \left(\lambda\phi + \frac{1}{2}m^2\phi^2\right). \tag{8}$$

We assume that $S_0[\phi]$ is invariant under the Galilean symmetry, so that $S_0[\delta_{\mathrm{g}}\phi] = S_0[\phi]$, where $\delta_{\mathrm{g}}\phi = \phi + b_\mu x^\mu + c$ is the Galileon shift operator defined in §II. It follows that

$$\begin{aligned}\exp \mathrm{i}\Gamma_{\mathrm{q}}[\phi] = \int \mathrm{D}\psi\; \exp\Bigg\{&\mathrm{i}S_0[\phi+\psi] - \mathrm{i}S_0[\phi] \\ &+ \int \mathrm{d}^4x\; \frac{1}{2}m^2\psi^2 - \mathrm{i}\psi\frac{\delta S_0[\phi]}{\delta\phi} - \mathrm{i}\psi\frac{\delta\Gamma_{\mathrm{q}}[\phi]}{\delta\phi}\Bigg\}.\end{aligned} \tag{9}$$

Therefore we conclude $\Gamma_{\rm q}[\delta_{\rm g}\phi] = \Gamma_{\rm q}[\phi]$. Hence, even though a mass term *explicitly* breaks the Galileon symmetry, this does not induce any further operators which violate the symmetry at the quantum level.

The same conclusion can be reached by analysing the properties of Feynman diagrams. Consider any nontrivial diagram with two external lines, which in principle could contribute to a renormalization of ϕ^2. Such a diagram includes two field operators where the external lines attach to the interior of the diagram, and the structure of $\mathcal{L}_{\rm M}$ implies that each of these operators carries at least *one* derivative. We conclude that the net effect of these field operators must introduce an overall factor of at least two powers of the external momenta. After expanding into a series of operator products, each term must contain at least *two* derivatives and cannot include the operator ϕ^2, contrary to our original supposition. As a trivial special case we recover the obvious fact that a ϕ^2 operator, considered as a deformation of the Gaussian kinetic term $\mathcal{L}_{\rm M} = X/2$, is not renormalized.

The same argument does not apply if we extend (8) to include higher powers of ϕ. In particular, inclusion of a ϕ^3 operator or higher would typically renormalize the coefficient of ϕ^2 to $\sim \Lambda^2$, where Λ is the cutoff of the theory. In such circumstances we would be obliged to take ϕ^2 as a relevant operator, leading to strong radiative breaking of the supposed shift symmetry exhibited by $\mathcal{L}_{\rm M}$. Therefore inclusion of such operators is inconsistent.

The conclusion of these arguments is that it is possible to construct an inflationary model based on a Galileon field, in which the shift and Galilean symmetry protects the form of the Lagrangian. Inflation can end, despite the presence of these symmetries, at least in the decoupling limit $M_{\rm P} \to \infty$. Although the Galileon symmetry is broken when coupled to gravity, the breaking terms will be parameterically suppressed by powers of $\Lambda/M_{\rm P}$, which we assume to be small. In what follows we will allow a little extra freedom and work with an arbitrary potential $V(\phi)$. Although this will generally break the Galilean symmetry explicitly, the foregoing argument demonstrates that any operators generated in this way will be suppressed by powers of three or more derivatives of $V(\phi)$. We will suppose that the models used to obtain Galileon inflation all break the Galilean symmetry mildly in this above sense.

IV. GALILEON INFLATION

On a curved background, such as de Sitter space, Deffayet *et al.* [40] remarked that the Galileon action constructed by Nicolis *et al.* [6] leads to unwanted higher-derivative equations of motion, spoiling the expected construction of a ghost-free, unitary theory. This can be cured using a nonminimal coupling to gravity, which Deffayet *et al.* described as 'covariantization.' The covariant Galileon action, which can also be obtained from the five-dimensional covering theory [7], is

$$S = \int \mathrm{d}^4x\, \sqrt{-g}\Bigg[-\frac{c_2}{2}(\nabla\phi)^2 + \frac{c_3}{\Lambda^3}\Box\phi(\nabla\phi)^2 - \frac{c_4}{\Lambda^6}(\nabla\phi)^2\Big\{(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) - \frac{1}{4}R(\nabla\phi)^2\Big\} + \frac{c_5}{\Lambda^9}(\nabla\phi)^2\Big\{(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2(\nabla_\mu\nabla_\nu\phi)(\nabla^\nu\nabla^\alpha\phi)(\nabla_\alpha\nabla^\mu\phi) - 6G_{\mu\nu}\nabla^\mu\nabla^\alpha\phi\nabla^\nu\phi\nabla_\alpha\phi\Big\}\Bigg], \tag{10}$$

where and $G_{\mu\nu}$ and R are respectively the Einstein tensor and scalar curvature of the background. Typically, both covariantization and inclusion of a non-Minkowski background metric will softly break the Galilean symmetry. It will emerge that this has important consequences for the phenomenology of the model. The coefficients c_i are dimensionless, and—as above—Λ is a mass scale which determines the naïve cutoff of the theory. In practice, fluctuations around a nontrivial background can be valid up to energies somewhat larger than Λ if a Vainshtein effect is operative (see Ref. [41]), discussed in more detail in Ref. [7]. Had we included nonminimal coupling to the geometry in Eq. (2), the curvature terms involving $G_{\mu\nu}$ and R would have been accompanied by other geometric invariants, but all such terms are suppressed by powers of H/Λ (*cf.* Eqs. (20)–(25), where such suppressed contributions can be clearly identified). In the inflationary regime of interest, where nonlinearities are dominated by the Galileon self-interactions, it will transpire that such terms are negligible, but for completeness we continue to retain the nonminmal curvature couplings required by covariantization.

The action for fluctuations in the decoupling limit of the DGP model has $c_4 = c_5 = 0$. Constraints on the c_i obtained from short-distance gravitational effects were studied in Refs. [6, 42] for the case that Eq. (10) describes the short-distance effects of a modification of gravity today. If the scalar ϕ is taken to be relevant only during inflation, however, the c_i are unrestricted and must be determined independently from cosmological probes. Cosmological constraints on c_2, c_3 and c_4 are quoted by Ali *et al.* [10].

A. Infrared completion of the Galileon

Models of this type are unusual, because they must be viewed as effective field theories both in the usual sense of being valid below some energy scale Λ, and also in the sense that they may require a non-trivial infrared completion. For instance, as described in §I, they typically arise as intermediate-scale effective theories corresponding to massive gravities, and are therefore modified at an infrared scale set by the Compton wavelength of the graviton. Such models were discussed in Refs. [7, 8]. The consistency of our present analysis only requires that the infrared cutoff is much larger than the Hubble scale during inflation. However, in a full cosmological model accounting for the subsequent expansion of the universe, it is likely that this scale must be many orders of magnitude larger. We do not address this issue in this paper.

B. Inflation in the de Sitter decoupling limit

To get a sense of the background solutions and new features in Galileon models, consider the background field evolution in a de Sitter decoupling limit, defined by the limit $M_{\rm P} \to \infty$ where H is kept fixed. (We caution that this should not be confused with the decoupling limit considered in the next section, which allows for a more general FRW background). This limit is applicable if the variation ΔV in the inflationary potential over the duration of inflation satisfies $|\Delta V/V| \ll 1$. In this limit we have a Galileon model living on the background of de Sitter spacetime, with scale factor $a(t) = {\rm e}^{Ht}$.

After integrating by parts to obtain the action in first-order form, and cancelling any boundary terms generated by this process [43], the action for a homogeneous field configuration $\phi(t)$ can be written

$$S_0 = \int {\rm d}^4x \, a^3 \left\{ \frac{c_2}{2}\dot{\phi}^2 + \frac{2c_3 H}{\Lambda^3}\dot{\phi}^3 + \frac{9c_4 H^2}{2\Lambda^6}\dot{\phi}^4 + \frac{6c_5 H^3}{\Lambda^9}\dot{\phi}^5 + \lambda^3\phi \right\}. \tag{11}$$

According to the discussion of Eqs. (4)–(5), the current J^t can be written

$$J^t = c_2\dot{\phi} + \frac{6c_3 H^2}{\Lambda^3}\dot{\phi}^2 + \frac{18c_4 H^2}{\Lambda^6}\dot{\phi}^3 + \frac{30 c_5 H^3}{\Lambda^9}\dot{\phi}^4 = \frac{\lambda^3}{3H}. \tag{12}$$

There are two regimes. First, a weakly coupled solution in which the linear term $\dot{\phi}$ dominates,

$$\dot{\phi} \sim \frac{\lambda^3}{3H} \, . \qquad \textsf{(weak coupling)} \tag{13}$$

In this regime the outcome is very close to canonical slow-roll inflation. Second, there is a strongly coupled solution for which $\dot{\phi}^2$ dominates. Consider the Galileon theory corresponding to the DGP model [20], for which only c_2 and c_3 are nonzero. The strongly coupled regime implies

$$\dot{\phi} \sim \sqrt{\frac{\Lambda^3\lambda^3}{18c_3 H^2}} \, . \qquad \textsf{(strong coupling)} \tag{14}$$

The smooth solution interpolating between Eqs. (13) and (14) is

$$\dot{\phi} = \frac{\Lambda^3}{12H}\left(-1 + \sqrt{1 + \frac{8c_3\lambda^3}{\Lambda^3}} \right). \tag{15}$$

More generally, when the other Galilean interactions are present, the field configuration interpolates between the weak coupling regime (13) and the strongly coupled solution $\dot{\phi}H \sim (\lambda\Lambda^2)$ if $c_4 \neq 0$ or $\dot{\phi}H \sim (\lambda^3\Lambda^9)^{1/4}$ if $c_5 \neq 0$.

What is the relevance of corrections to (11) from non-minimal coupling to the geometry, which were briefly discussed below Eq. (10)? To be concrete we consider a coupling of the form ϕG, where G is the Gauss-Bonnet invariant, which would retain the important feature of second-order equations of motion. Since G involves the square of contractions of the Riemann tensor, the leading contribution must be of the form $\sim H^3\dot{\phi}/\Lambda$. This will be of comparable importance to the c_3-dependent term $\sim H\dot{\phi}^3/\Lambda^3$ only if

$$\frac{1}{Z^2}\frac{H^4}{\Lambda^4} \gtrsim 1, \tag{16}$$

where Z is the dimensionless combination

$$Z \equiv \frac{H\dot{\phi}}{\Lambda^3}. \tag{17}$$

We shall see that Z plays the role of coupling constant in Galileon theories. Unless $Z < (H/\Lambda)^2 \ll 1$, Eq. (16) and similar conditions for the c_4 and c_5-terms show that the Gauss–Bonnet invariant is negligible compared to the terms proportional to c_i, arising from Galileon operators. On the other hand, the relative importance of the higher-derivative Galileon operators to the lower-derivative operators is controlled by positive powers of Z.

This has a simple interpretation. When $Z \gtrsim 1$ the nonlinearities of the Galileon sector are important, as in Eq. (14). In this limit the mixing with gravity can be neglected, and non-minimal couplings such as ϕG are irrelevant. It is in this regime that an interesting Vainshtein effect can emerge. In addition, as we will explain below, the inflationary phenomenology is rather different to canonical slow-roll models. On the other hand, the limit $Z \ll 1$, as in Eq. (13), describes weakly coupled perturbations in the Galileon sector, giving a theory almost equivalent to canonical slow-roll inflation. In this limit the strongest interactions come from mixing with gravity, and a coupling such as ϕG cannot be neglected.

C. Effective field theory for inflation

How does an era of Galileon inflation differ from a canonical inflationary phase? We have argued in §II that two Lagrangians are inequivalent only if they make different predictions for observables. Therefore we must study perturbations generated by the action (2), which are the appropriate measurable quantities.

Unitary gauge action. Cheung *et al.* [31] argued that certain properties of the perturbations generated in an inflationary model were fixed by the background, and were therefore model independent, whereas others varied between theories and could be used to probe different choices of $\mathcal{L}_{\mathrm{M}}$. This conclusion was obtained by constructing the most general action for small fluctuations on a quasi-de Sitter background, subject to the condition of unbroken spatial diffeomorphisms and nonlinearly realized Lorentz invariance. In §IV D below we will construct the action for small fluctuations in Galileon inflation using a more direct approach. However, this action could equally have been obtained by specializing the result of Ref. [31] to a scenario with Galilean symmetries. Therefore, before proceeding with a detailed calculation, it is of interest to determine what constraints are placed on the model by the construction of Cheung *et al.*

The authors of Ref. [31] worked in a model with a single scalar field, ϕ, and constructed their action in a gauge where slices of constant time coincided with slices of uniform ϕ. In this gauge there are no explicit scalar fluctuations, but only perturbations of the metric. The unit vector normal to slices of constant time is n^μ, and constitutes a preferred vector field breaking manifest Lorentz invariance. The Lagrangian for inflationary perturbations can be built only out of operators which are invariant under spatial diffeomorphisims associated with reparametrizations of the induced three-dimensional spatial metric $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$. Cheung *et al.* showed that it was sufficient to take the Lagrangian to comprise a general scalar combination of the Riemann tensor, $R^\mu{}_{\nu\rho\sigma}$, together with the time–time component of the metric, g^{00}, and the extrinsic curvature, $K_{\mu\nu} = -h_{(\mu}{}^\rho h_{\nu)}{}^\sigma \nabla_\rho n_\sigma$, associated with slices of constant time [31]. Bartolo *et al.* argued that the most general Lagrangian including terms of up to cubic order in small fluctuations can be written [33]

$$\begin{aligned} S = \int \mathrm{d}^4x\, \sqrt{-g} \Bigg[& \frac{1}{2}M_{\mathrm{P}}^2 R - c(t) g^{00} - \Lambda(t) + \frac{1}{2}M_2(t)^4 (g^{00}+1)^2 + \frac{1}{3}M_3(t)^4(g^{00}+1)^3 \\ & - \frac{\bar{M}_1(t)^3}{2}(g^{00}+1)\delta K^\mu{}_\mu - \frac{\bar{M}_2(t)^2}{2}(\delta K^\mu{}_\mu)^2 - \frac{\bar{M}_3(t)^2}{2}\delta K^{\mu\nu}\delta K_{\mu\nu} \\ & - \frac{\bar{M}_4(t)^3}{2}(g^{00}+1)^2\delta K^\mu{}_\mu - \frac{\bar{M}_5(t)^2}{2}(g^{00}+1)(\delta K^\mu{}_\mu)^2 - \frac{\bar{M}_6(t)^2}{2}(g^{00}+1)\delta K^{\mu\nu}\delta K_{\mu\nu} \\ & - \frac{\bar{M}_7(t)}{2}(\delta K^\mu{}_\mu)^3 - \frac{\bar{M}_8(t)}{2}(\delta K^\mu{}_\mu)(\delta K^{\rho\sigma}\delta K_{\rho\sigma}) - \frac{\bar{M}_9(t)}{2}\delta K^{\mu\nu}\delta K_{\nu\sigma}\delta K^\sigma{}_\mu \Bigg], \end{aligned} \tag{18}$$

where we have used the Riemann tensor only in the form of the Ricci scalar, to match low-energy gravitational experiments which probe the Einstein action.

Following Refs. [31, 33], in writing Eq. (18) we have organized the expansion in powers of $\delta g^{00} = g^{00} + 1$ and $\delta K_{\mu\nu}$, which are fluctuations around an unperturbed Friedmann–Robertson–Walker geometry. Therefore only the operators multiplying $c(t)$ and $\Lambda(t)$ are non-zero on the background, which fixes these coefficients in terms of the expansion history $H(t)$. The $M_i(t)$ and $\bar{M}_i(t)$ are not fixed by the background evolution and encode differences between models determined by our choice of $\mathcal{L}_{\mathrm{M}}$. It is in the effects generated by these operators that we should look for distinctive signatures of Galileon inflation.

To convert Eq. (18) into a form suitable for computation it is helpful to make a gauge transformation, $t \to \tilde{t} = t - \pi(\mathbf{x}, t)$. After this transformation, the equal-time hypersurfaces of the uniform-field ('unitary') gauge are deformed by π, which Cheung *et al.* argued could be considered as the Goldstone boson associated with broken time translation invariance. Each term in Eq. (18) generates a series in powers of π, with each copy of π carrying least one gradient in time or space. Each copy of $\delta K_{\mu\nu}$ may additionally contribute a single power of π carrying two gradients. It follows that Eq. (18) generates cubic interactions involving three copies of π with between three and six derivatives. In this way a description of the system is constructed in terms of the lowest dimension operators compatible with the underlying symmetries—the effective field theory approach.

Decoupling limit. Eq. (18) is quite generally applicable and accounts for the mixing between π fluctuations and the metric. However, when large nonlinearities are associated with the self-interactions of π there exists a decoupling limit in which reliable predictions can be extracted while neglecting gravity [5, 27, 31, 32]. For this limit to be a reasonable approximation, the M_{P}-suppressed terms which are neglected must be smaller than terms which do not vanish when $M_{\mathrm{P}} \to \infty$.

We briefly recapitulate the argument of Ref. [31]. The most relevant kinetic term for the metric fluctuation δg^{00} will come from the Ricci scalar. Therefore we can pass to canonical normalization by the rescaling $\delta g^{00} \to \delta g^{00}_c = M_{\rm P}\delta g^{00}$. The most relevant kinetic term for π will arise from some nonlinear operator in Eq. (18). If it is minimal in derivatives it will be of the form $M^4\dot{\pi}^2$, where M is some combination of the scales M_i or $\bar{M}_i$. The canonically normalized field is $\pi_c = M^2\pi$. At quadratic level, a mixing term such as $M^4\dot{\pi}\delta g^{00}$ is negligible in comparison with the π kinetic term for wavenumbers k which satisfy $k \gtrsim E_{\rm mix} = M^2/M_{\rm P}$. The same applies for cubic terms, where the leading mixing term $M^4\dot{\pi}^2\delta g^{00}$ is negligible in comparison with $M^4\dot{\pi}^3$ under the same condition. A similar argument can be given if the most relevant kinetic term for π contains higher derivatives [31], or if the leading cubic terms enter with a mass scale different from M. In the decoupling limit the scale $E_{\rm mix}$ must be smaller than the Hubble scale during inflation, making our predictions accurate to a relative error of order $E_{\rm mix}/H$. In what follows we work in this limit, in which the metric can be taken to be unperturbed. Therefore it is most convenient to work in the uniform curvature gauge, where the unperturbed metric is spatially flat and can be taken as the background de Sitter geometry.

D. Fluctuations

On the basis of the foregoing discussion, we should study the effect of fluctuations around a cosmological background solution in the decoupling limit by constructing small fluctuations $t \mapsto t + \xi(\mathbf{x}, t)$ on a hypersurface of constant time, working in a gauge where such hypersurfaces are spatially flat. Note that this gives ξ an engineering dimension $[\text{mass}]^{-1}$.

The comoving curvature perturbation, ζ, satisfies $\zeta = H\xi$ and will be conserved on superhorizon scales, where $kc_s/aH \to 0$. Working to cubic order in ξ, the action can be written

$$S \supseteq \int \mathrm{d}^4x\, a^3 \bigg[\alpha\dot{\xi}^2 - \frac{\beta}{a^2}(\partial\xi)^2 + f_1\dot{\xi}^3 + \frac{f_2}{a^2}\dot{\xi}^2\partial^2\xi + \frac{f_3}{a^2}\dot{\xi}(\partial\xi)^2 + \frac{f_4}{a^4}(\partial\xi)^2\partial^2\xi \bigg], \tag{19}$$

where the symbol '$\supseteq$' is used to denote that S contains these contribution among other higher-order ones, and the time-dependent coefficients α, β and f_i satisfy

$$\alpha = \frac{\dot{\phi}^2}{2}\left(c_2 + 12c_3Z + 54c_4Z^2 + 120c_5Z^3\right), \tag{20}$$

$$\beta = \frac{\dot{\phi}^2}{2}\bigg\{ c_2 + 4c_3(2Z + \frac{\ddot{\phi}}{\Lambda^3}) + 2c_4\left[13Z^2 + \frac{6}{\Lambda^6}\left(\dot{H}\dot{\phi}^2 + 2H\dot{\phi}\ddot{\phi}\right)\right] + \frac{24c_5}{\Lambda^9}H\dot{\phi}^2\left[2\dot{\phi}(H^2 + \dot{H}) + 3H\ddot{\phi}\right] \bigg\}, \tag{21}$$

$$f_1 = \frac{2H\dot{\phi}^3}{\Lambda^3}\left(c_3 + 9c_4Z + 30c_5Z^2\right), \tag{22}$$

$$f_2 = -\frac{2\dot{\phi}^3}{\Lambda^3}\left(c_3 + 6c_4Z + 18c_5Z^2\right), \tag{23}$$

$$f_3 = -\frac{2H\dot{\phi}^3}{\Lambda^3}(c_3 + 7c_4Z + 18c_5Z^2) + \frac{2\dot{\phi}^2\ddot{\phi}}{\Lambda^3}(c_3 + 6c_4Z + 18c_5Z^2), \tag{24}$$

$$f_4 = \frac{\dot{\phi}^3}{\Lambda^3}\left\{c_3 + 3c_4Z + 6c_5\left[Z^2 + \frac{\dot{H}\dot{\phi}^2}{\Lambda^6}\right]\right\} + \frac{3\dot{\phi}^3\ddot{\phi}}{\Lambda^6}(c_4 + 4c_5Z). \tag{25}$$

The quantity Z was defined in Eq. (17). In order that Galileon self-coupling dominate the interactions, and mixing with gravity can be neglected, we must have have $Z \gtrsim 1$. Although one can contemplate the limit $Z \gg 1$, there is some risk that this would spoil inflation unless renormalized by a Vainshtein effect. In this paper we restrict our attention to the case $Z \sim 1$ where nonlinearities are significant but not problematic.

The contribution proportional to $\dot{\xi}^2\partial^2\xi$ can be removed after a field redefinition. Making the transformation $\xi \to \pi = \xi + f_2\dot{\xi}^2/2\beta$, it follows that Eq. (19) can be written

$$S \supseteq \int \mathrm{d}^4x\, a^3 \bigg[\alpha\left\{\dot{\pi}^2 - \frac{c_s^2}{a^2}(\partial\pi)^2\right\} + g_1\dot{\pi}^3 + \frac{g_3}{a^2}\dot{\pi}(\partial\pi)^2 + \frac{g_4}{a^4}(\partial\pi)^2\partial^2\pi \bigg], \tag{26}$$

where $g_3 = f_3$, $g_4 = f_4$, we have defined $c_s^2 = \beta/\alpha$, and g_1 is defined by

$$g_1 = f_1 + \frac{2H}{c_s^2}f_2 + \frac{2}{3c_s^2}\frac{\dot{\alpha}}{\alpha}f_2 - \frac{\beta}{3c_s^2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{f_2}{\beta}\right). \tag{27}$$

Had we worked from the uniform-field gauge action of Cheung *et al.*, Eq. (18), we would have obtained α, c_s and the g_i in terms of $c(t)$, $\Lambda(t)$ and the theory-dependent scales $M_i(t)$ and $\bar{M}_i(t)$.

Although ξ and π will differ on small scales, they become equal whenever $\dot{\xi} = 0$ and are therefore equal in any epoch when ξ is conserved. In particular, they coincide on superhorizon scales. Therefore, to obtain the correlation functions of ζ it suffices to obtain the correlation functions of π.

Relation to EFT action. The Galileon Lagrangian, constructed above, could have been recovered from Eq. (18) by imposition of the Galileon symmetry, after taking advantage of possible field redefinitions and integration by parts. Therefore, one might have some reservations that Eq. (26) is in conflict with the conclusions of Cheung *et al.*, who found that the coefficient of the operator $\dot{\pi}(\partial\pi)^2$ was fixed by $M_2(t)$. In a generic model this coefficient is also responsible for a nontrivial speed of sound, $c_s < 1$. Therefore the coefficient of $\dot{\pi}(\partial\pi)^2$ is fixed once the background evolution and c_s have been specified. On the other hand, Eqs. (26)–(27) show that the coefficient of $\dot{\pi}(\partial\pi)^2$ in the Galileon theory is independent.

This apparent discrepancy disappears if one accounts for all terms in Eq. (18), which in principle contains fourteen free coefficient functions. Of these, the Planck mass is measured by terrestrial experiments and the pair $\{c(t), \Lambda(t)\}$ must be chosen to match the expansion history $H(t)$, leaving eleven free coefficients overall. Contributions to $\dot{\pi}(\partial\pi)^2$ arise from many of these operators, which are made relevant owing to the symmetries of the Galileon theory. (See §IV E, where the relative magnitude of each term is clearly expressed by their contributions to the observable parameter f_{NL}.) It is these additional terms which break the expected correlation between g_3 and c_s^2.

E. Primordial density perturbations

We now proceed to compute the form of the bispectrum for Galileon inflation. As we have explained above, we focus on nongaussianities because we expect them to parametrize the difference between inequivalent choices of the inflationary Lagrangian. Although it is also important to study the properties of two-point statistics, these are effectively constrained to match observation by the closeness of the background solution to a de Sitter era with slowly varying H.

When computed using Eq. (26), the correlation properties of the primordial density perturbation can be expressed in terms of the coefficients of the relevant operators, which are α, c_s, g_1, g_3 and g_4. These represent the largest contribution to each observable. We have argued that there are two irrelevant operators, $\lambda^3\phi$ and $m^2\phi^2$, which although not invariant under the Galilean symmetry can self-consistently be made small, but play an essential role in ending inflation. These correct the predictions of Eq. (26). Their influence can be accommodated by inclusion of the first subleading slow-roll corrections. As remarked by Kobayashi *et al.* [12], another reason to calculate these corrections is that c_s can exhibit a modest but non-negligible variation over the duration of inflation, which manifests itself as a correction which is formally of first subleading order.

Corrections of this kind were calculated for the two-point function in canonical inflation by Stewart & Lyth [44]. The two-point function describing correlations induced by the quadratic part of Eq. (26) was given at leading order by Garriga & Mukhanov [27]. Subleading slow-roll corrections were later obtained for the case $\mathcal{L}_{\mathrm{M}} = P(X, \phi)$ by Chen *et al.* [18], who also gave expressions for slow-roll corrections to the bispectrum. These involved quadratures of the sine and cosine integrals $\mathrm{Si}\, x$ and $\mathrm{Ci}\, x$ which could not be evaluated in closed form. Very recently, Kobayashi *et al.* [12] obtained slow-roll corrections to the two-point function in models of the form $\mathcal{L}_{\mathrm{M}} = P(X, \phi) + F(X, \phi)\Box\phi$, which includes the term involving c_3 in Eq. (10) as a special case but not the terms containing c_4 and c_5.

Two-point correlations. We define $s = H\dot{c}_s/c_s$ [24], which measures the time dependence of the speed of sound. Similarly we shall require the rate of variation per e-fold of each time-dependent coefficient in Eq. (26). We define

$$v = \frac{\dot{\alpha}}{H\alpha} \tag{28}$$

$$h_i = \frac{\dot{g}_i}{Hg_i} \tag{29}$$

for $i = 1$, 3 and 4, and treat all these combinations as the same order of magnitude as ϵ and η. Certain combinations of these parameters occur frequently, for which it is convenient to define abbreviations,

$$\lambda = \epsilon + \frac{v}{2} + \frac{3s}{2} \tag{30}$$

$$\mu_0 = \epsilon + v + 2s + \mathrm{i}\frac{\pi}{2}\lambda \tag{31}$$

$$\mu_1 = \epsilon + s - \mathrm{i}\frac{\pi}{2}\lambda. \tag{32}$$

We note that the combination $\mu_0 + \mu_1 = 2\lambda$ is purely real.

Translating to conformal time, defined by $\tau = \int_\infty^t \mathrm{d}t/a(t)$, the two-point function can be written

$$\langle \pi(\mathbf{k}_1, \tau)\pi(\mathbf{k}_2, \tau')\rangle = (2\pi)^3\delta(\mathbf{k}_1 + \mathbf{k}_2)G_{k_1}(\tau, \tau') \tag{33}$$

in which G_k is defined by

$$G_k = \begin{cases} u_k^*(\tau')u_k(\tau) & \tau < \tau' \\ u_k^*(\tau)u_k(\tau') & \tau' < \tau \end{cases} . \tag{34}$$

The elementary wavefunction u_k out of which this two-point function is built satisfies

$$u_k(\tau) = \frac{\sqrt{\pi}}{2\sqrt{2}}\frac{(1+s)^{1/2}}{a(\tau)}\sqrt{\frac{\tau}{\alpha(\tau)}}H_\nu^{(2)}[-kc_s(1+s)\tau], \tag{35}$$

where $H_\nu^{(2)}$ is the Hankel function of the second kind. The order can be written $\nu = 3/2 + \lambda$ [45].

The power spectrum, $P(k,\tau)$, is defined by the equal-time correlation function,

$$P(k,\tau) = G_k(\tau,\tau). \tag{36}$$

On superhorizon scales, where $|k\tau| \ll 1$, Eqs. (33)–(36) imply that the power spectrum achieves a time-independent value

$$P(k,\tau) \to P(k) = \frac{H_k^2}{2(kc_{s|k})^3}\frac{1+2E_k}{2\alpha_k}, \tag{37}$$

where E is the combination

$$E = -\epsilon - s + (2 - \gamma_{\mathrm{E}} - \ln 2)\lambda, \tag{38}$$

in which γ_{E} is the Euler–Mascheroni constant, and a subscript k denotes evaluation at the time $kc_s\tau = -1$. Eq. (37) has engineering dimension $[\text{mass}]^{-5}$, showing that the two-point function, Eq. (33), has engineering dimension $[\text{mass}]^{-8}$ which is correct given our definition of π. The time-independence of $P(k)$ outside the horizon can be understood as a consequence of the shift symmetry of Eq. (2), which guaranteed the conservation of ζ outside the horizon [46].

In writing expressions valid to subleading order in slow-roll parameters we must be cautious when specifying the time at which background quantities such as H, c_s and the slow-roll parameters are to be evaluated. A small shift in the evaluation point translates to a difference of $\mathrm{O}(1)$ in the slow-roll suppressed terms. We therefore have some freedom to rearrange coefficients by judicious choice of the time of evaluation.

Wavefunction corrections. At the level of the three-point function, slow-roll corrections arise from several sources. These were identified in Ref. [18]. There are corrections due to the presence of time-dependent, slow-roll suppressed factors at each vertex. In addition there are corrections involving the combination E which arise from taking the $|k\tau| \ll 1$ limit of each external line. A third class of corrections arise from modifications to the wavefunctions carried by internal lines. These were obtained explicitly by Chen *et al.* [18], but in this section we give a slightly different treatment which will enable us to evaluate the final three-point function in closed form. Further details are given in Appendix A.

The background wavefunctions can be written

$$u(k,\tau) = \frac{\mathrm{i}H_k}{2\sqrt{\alpha_k}}\frac{1}{(kc_{s|k})^{3/2}}(1 - \mathrm{i}kc_{s|k}\tau)\mathrm{e}^{\mathrm{i}kc_{s|k}\tau}. \tag{39}$$

The $\mathrm{O}(\epsilon)$ correction, $\delta u(k,\tau)$, is obtained by expanding Eq. (35) uniformly to first order in small quantities. The variation with respect to the order, ν, of each Hankel function can be evaluated using expressions (B.42)–(B.46) of Ref. [18]. Assembling sine and cosine integral terms, these expressions can be rewritten to find

$$\frac{\partial H_\nu^{(2)}(x)}{\partial \nu} = -\frac{\mathrm{i}}{x^{3/2}}\sqrt{\frac{2}{\pi}} \times \left[\mathrm{e}^{\mathrm{i}x}(1-\mathrm{i}x)\,\mathrm{Ei}(-2\mathrm{i}x) - 2\mathrm{e}^{-\mathrm{i}x} - \mathrm{i}\frac{\pi}{2}\mathrm{e}^{-\mathrm{i}x}(1+\mathrm{i}x)\right]. \tag{40}$$

Using Eq. (40) and Eq. (35), and rotating the contour of integration of Ei, we finally obtain

$$\delta u(k,\tau) = \frac{\mathrm{i}H_k}{2\sqrt{\alpha_k}}\frac{1}{(kc_{s|k})^{3/2}}\Bigg\{ -\lambda_k \mathrm{e}^{-\mathrm{i}kc_{s|k}\tau}(1+\mathrm{i}kc_{s|k}\tau)\int_{-\infty}^{\tau}\frac{\mathrm{d}\xi}{\xi}\,\mathrm{e}^{2\mathrm{i}kc_{s|k}\xi} + \mathrm{e}^{\mathrm{i}kc_{s|k}\tau}\left[\mu_{0|k} + \mathrm{i}\mu_{1|k}kc_{s|k}\tau + s_k k^2 c_{s|k}^2\tau^2 + \lambda_k N_k - \mathrm{i}\lambda_k kc_{s|k}N_k\tau - s_k k^2 c_{s|k}^2 N\tau^2\right]\Bigg\} \tag{41}$$

We have defined $N_k = \ln|kc_{s|k}\tau|$. The remaining integral is to be taken over a contour displaced slightly above the negative real axis for large $|\xi|$, which renders it finite. Similar integrals are generated in the Schwinger (or "in–in") formulation of quantum field theory, where the same contour prescription is obtained after accounting for $\mathrm{i}\epsilon$ terms which project onto the vacuum at past infinity [29, 47]. Differentiating with respect to τ and using Eqs. (30)–(32) one can confirm that, despite the appearance of an apparent logarithmic singularity, $\delta u(k,\tau)' \to 0$ in the limit $\tau \to 0$. This is the same behaviour as $u(k,\tau)$ itself, and guarantees that the introduction of slow-roll corrections does not cause a convergent time integral to become divergent.

Log-divergent integrals of the form appearing in Eq. (41) have previously been obtained in Refs. [30, 48, 49], which discussed the possibility of singularities for certain kinematic configurations of the k_i, including the 'squeezed' configurations where one k_i becomes much smaller than the other two. The behaviour of the three-point function in this limit is not trivial, but it can be determined nonperturbatively in the single-field framework following an argument due to Maldacena [29, 50], and is known to be regular. Therefore any singularities arising from this log-divergent integral must cancel. We have confirmed that our final expressions contain no singularities, but we discuss the significance of these potential divergences in Appendix A.

Three-point correlations. We give technical details of the calculation of the three-point functions arising from each cubic operator in Eq. (26) in Appendix B. In this section we report the final values of f_{NL}, specialized to the equilateral limit where all k_i have a common magnitude. We would typically expect the bispectrum to be maximized on a configuration close to equilateral, and this limit should give a good estimate of the magnitude of the bispectrum on this peak configuration.

We adopt the convention that background quantities are to be evaluated at the horizon-crossing time corresponding to the symmetric point $k_t = k_1 + k_2 + k_3$, and denote evaluation at this time by a subscript '$\star$'. This is somewhat larger than any individual k_i. In the equilateral case this moves the point of evaluation to $\ln 3 \approx 1$ e-folds after the common time of horizon exit.

We define the bispectrum $B_\tau(k_1, k_2, k_3)$ by

$$\langle \pi(\mathbf{k}_1, \tau)\pi(\mathbf{k}_2, \tau)\pi(\mathbf{k}_3, \tau)\rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\tau(k_1, k_2, k_3) \tag{42}$$

where the momentum-conservation condition allows us to make B a function of the magnitudes k_i alone, independent of the relative orientation among the $\mathbf{k}_i$. We define f_{NL} to be the reduced bispectrum

$$B(k_1, k_2, k_3) = \frac{6}{5} f_{\mathrm{NL}} \times \Big[P(k_1)P(k_2) + P(k_1)P(k_3) + P(k_2)P(k_3) \Big], \tag{43}$$

where all quantities are evaluated at time τ. Our convention that the background quantities in each copy of the power spectrum $P(k)$ are to be evaluated at $\tau_\star$ implies that there is a logarithmic correction to Eq. (37) proportional to $\ln k/k_t$. Combining Eqs. (37) and (43) we obtain

$$\frac{6}{5} f_{\mathrm{NL}} = \frac{\prod_i k_i^3}{\sum_i k_i^3 (1 + 4E_\star - 2\lambda_\star \ln[k_i^{-1} k_t^{-2} \prod_j k_j])} \times \left(\frac{H_\star^2}{4\alpha_\star c_{s\star}^3} \right)^{-2} B(k_1, k_2, k_3). \tag{44}$$

To obtain our final answers we expand this expression uniformly to first order in quantities of $\mathrm{O}(\epsilon)$. We define a numerical constant ω, satisfying $\coth \omega = 5$. In the equilateral limit $k_i = k$ for all i, each f_{NL} becomes independent of k for dimensional reasons and we find

$$\begin{aligned}
\frac{6}{5} f_{\mathrm{NL}}^{\dot\pi^3} &= \frac{2}{27} \frac{g_{1\star} H_\star}{\alpha_\star} \left(1 + \frac{2\gamma_{\mathrm{E}} - 3}{2} h_{1\star} + \frac{160\omega - 2\gamma_{\mathrm{E}} - 29}{2} v_\star + \frac{480\omega - 98}{2} s_\star + \frac{320\omega - 2\gamma_{\mathrm{E}} - 63}{2} \epsilon_\star \right) \\
&= \frac{2}{27} \frac{g_{1\star} H_\star}{\alpha_\star} (1 - 0.923 h_{1\star} + 1.141 v_\star - 0.344 s_\star + 0.360 \epsilon_\star)
\end{aligned} \tag{45}$$

$$\begin{aligned}
\frac{6}{5} f_{\mathrm{NL}}^{\dot\pi(\partial\pi)^2} &= -\frac{17}{54 c_{s\star}^2} \frac{g_{3\star} H_\star}{\alpha_\star} \left(1 + \frac{17\gamma_{\mathrm{E}} - 9}{17} h_{3\star} + \frac{32 \ln \frac{3}{2} - 17\gamma_{\mathrm{E}} + 22}{17} v_\star + \frac{96 \ln \frac{3}{2} - 34\gamma_{\mathrm{E}} + 40}{17} s_\star \right. \\
&\qquad \left. + \frac{64 \ln \frac{3}{2} - 17\gamma_{\mathrm{E}} + 18}{17} \epsilon_\star \right) \\
&= -\frac{17}{54 c_{s\star}^2} \frac{g_{3\star} H_\star}{\alpha_\star} (1 + 0.048 h_{3\star} + 1.480 v_\star + 3.489 s_\star + 2.008 \epsilon_\star)
\end{aligned} \tag{46}$$

$$\begin{aligned}
\frac{6}{5} f_{\mathrm{NL}}^{\partial^2\pi(\partial\pi)^2} &= -\frac{13}{27 c_{s\star}^4} \frac{g_{4\star} H_\star^2}{\alpha_\star} \left(1 + \frac{6\gamma_{\mathrm{E}} + 5}{6} h_{4\star} + \frac{173 - 26\gamma_{\mathrm{E}} - 256 \ln \frac{3}{2}}{26} v_\star + \frac{733 - 156\gamma_{\mathrm{E}} - 1152 \ln \frac{3}{2}}{39} s_\star \right. \\
&\qquad \left. + \frac{147 - 26\gamma_{\mathrm{E}} - 256 \ln \frac{3}{2}}{13} \epsilon_\star \right) \\
&= -\frac{13}{27 c_{s\star}^4} \frac{g_{4\star} H_\star^2}{\alpha_\star} (1 + 1.411 h_{4\star} + 2.084 v_\star + 4.509 s_\star + 2.169 \epsilon_\star) .
\end{aligned} \tag{47}$$

On the other hand, in the 'squeezed' limit where one k_i becomes much less than the other two, each f_{NL} decays to zero. Expressions for f_{NL} which describe the complete momentum dependence can be extracted from the three-point functions given in Appendix B, but because they are lengthy and ultimately not illuminating we do not write them explicitly.

The leading-order contributions from each of these operators were recently computed by Mizuno & Koyama [13]. We have verified that the leading terms of Eqs. (B1), (B9) and (B25), which generate the leading-order terms of Eqs. (45)–(47), correspond with Eqs. (32)–(36) of Mizuno & Koyama.

Note that, although we are using the conventional notation '$f_{\rm NL}$' to denote the reduced bispectrum, Eqs. (45)–(47) are not directly measurable quantities, because (42)–(43) define them in terms of π. Indeed, according to these definitions, Eqs. (45)–(47) express $\langle\pi^3\rangle/\langle\pi^2\rangle^2$, after removal of the momentum-conservation δ-function from each correlator, for different choices of $\langle\pi^3\rangle$. Therefore they possess engineering dimension [mass]. The observable quantity is the ratio $\langle\zeta^3\rangle/\langle\zeta^2\rangle^2$. The appropriate $f_{\rm NL}$ which measure this ratio are obtained from Eqs. (45)–(47) after division by $H_\star$, and are dimensionless.

Dependence on c_s^2. Eqs. (45)–(47) express predictions for the three-point correlations generated by the Galileon Lagrangian, assuming that the coefficient of each relevant operator can be determined by measurement.

This pattern of three-point correlations gives rise to an interesting phenomenology, considerably broader than has previously been encountered using noncanonical models. In theories such as DBI and k-inflation, it is a familiar result that $f_{\rm NL} \sim c_{s\star}^{-2}$ [18]. Eq. (47) already shows that this is not guaranteed in a model exhibiting Galilean invariance, unless the background conspires to require g_4/α proportional to c_s^2, and as we will argue below this is not automatically the case. This effect is visible in the calculation of Mizuno & Koyama, although these authors did not discuss its significance.

The possibility that $f_{\rm NL}$ is *not* proportional to $c_{s\star}^{-2}$ in the limit of small sound speed has not previously been noticed. Why is this? Cheung *et al.* observed that the leading relevant operator contributing to the three-point function would be $\dot{\pi}(\partial\pi)^2$, since this is suppressed by fewest gradients. In a generic theory where each operator enters with an approximate common mass scale M, the operator $\dot{\pi}(\partial\pi)^2$ is dimension six after canonical normalization, whereas $\partial^2\pi(\partial\pi)^2$ is dimension seven. Accordingly we would expect $\dot{\pi}(\partial\pi)^2$ to dominate $f_{\rm NL}$ at wavenumbers $k \lesssim M$. The crucial point is that, if $\dot{\pi}(\partial\pi)^2$ is the only relevant cubic operator, then it arises from a term which also fixes the speed of sound [31]. As a result, the nonlinearly realized Lorentz invariance requires $f_{\rm NL} \sim c_{s\star}^{-2}$.

The important point we wish to emphasize is that this is not forced to occur in Eq. (47), because it is a dimension-seven operator rather than $\dot{\pi}(\partial\pi)^2$ which gives this contribution. We will discuss this in more detail below. One might wonder whether even more powers of $c_{s\star}^2$ can accumulate in the denominator. However, this is not possible. The factor $c_{s\star}^{-4}$ arises from an operator which is not suppressed by any temporal gradients, each of which contributes a factor $c_{s\star}^2$. No matter how many spatial gradients are added, no greater enhancement is possible at small $c_{s\star}$. Before drawing any conclusions about the scaling of $f_{\rm NL}$ with $c_{s\star}$, however, it is necessary to specify the unknown coefficients $g_{i\star}$ and $\alpha_\star$, and for this one must work with a concrete model.

Mizuno & Koyama discussed two such models, each of which could be written in the generic form $\mathcal{L}_{\rm M} = P(X,\phi) + F(X,\phi)\Box\phi$, including an approximation to the DBI Galileon example introduced in Ref. [7]. They studied this example in two limits, differentiated by $b_D \ll 1$ and $b_D \gg 1$ in their notation, respectively corresponding to the dimension-six and dimension-seven operators dominating the cubic interactions. In each of these extreme limits they found $f_{\rm NL} \sim c_{s\star}^{-2}$, because in these limits the coefficient of the dominant operator also determines c_s^2. In the limit where the dimension-six operator is dominant, the dimension-seven operator gives a contribution to $f_{\rm NL}$ of order $c_{s\star}^{-5}$ which is reproduced by our formulas. However, on its own this is of limited interest because this contribution must be subdominant overall.

The full, covariant Galileon result extends this in an interesting way. First, it shows clearly that it is not necessary to assume that one operator or the other is dominant. As we will explain, the Galileon symmetry makes it natural for them to be of equal importance.

Second, the most general Galileon model has three higher-derivative interactions, each of which contributes to the speed of sound, c_s. If any one of the c_3, c_4 or c_5 operators dominates α and β, then c_s^2 reduces to a simple rational fraction which is not especially small. If the same operator dominates the g_i, then each $f_{\rm NL}$ in Eqs. (45)–(47) reduces to $\sim H$, giving an $f_{\rm NL}$ for ζ of order unity. To obtain a very small speed of sound, one must suppose that the c_i are arranged in such a way that β becomes small relative to α. Having done so, in the absence of other constraints, there is still sufficient freedom to balance the g_i in such a way that the dominant contribution to $f_{\rm NL}$ scales parametrically faster than $c_{s\star}^{-2}$ in the limit of small sound speed. This is one of the key results of this paper.

Relevance of dimension-seven operator. Let us explain in more detail how the dimension-six and dimension-seven operators are naturally of comparable importance.

First consider a generic theory, without a Galilean symmetry. If we tune $k \sim M$, then a window exists in which the contribution from both operators may be competitive, but in this window we are close to the cutoff of the effective theory unless it is forced to higher scales by a Vainshtein-like effect [41]. In practice, if c_s is very small then the contribution from $\partial^2\pi(\partial\pi)^2$ could receive an additional enhancement, which might make it competitive with $\dot{\pi}(\partial\pi)^2$. However, c_s cannot be made too small without encountering undesirable stability problems [31, 51]. Therefore the dimension-seven operator seems effectively irrelevant.

The situation changes if the Lagrangian exhibits a Galilean invariance. In this case, the dimension-seven operator gives a significant contribution because the dimension-six operators explicitly break the Galilean symmetry. Thus, they arise only because there is a nontrivial cosmological background which breaks the symmetry, as discussed below Eq. (10). Because this breaking is done by the background, it is suppressed by powers of H/Λ. This makes the dimension-six operators formally of the same order as the dimension-seven ones. This possibility has not been covered in previous discussions of the effective field theory of inflationary perturbations.

To see this in detail, Eqs. (22)–(25) show that $\dot{\pi}(\nabla\pi)^2$ and $\partial^2\pi(\partial\pi)^2$ do not enter with a common mass scale. Instead, the coefficient of $\dot{\pi}(\partial\pi)^2$ is suppressed by H, which is the same order of magnitude as the extra gradient carried by $\partial^2\pi(\partial\pi)^2$. Therefore, the covariant Galileon model (specialized to de Sitter space) tunes these operators to give precisely competitive contributions when $c_s \sim 1$. In terms of observables, the same conclusion is easy to obtain from Eqs. (45)–(47), in which the dimension-seven contribution from Eq. (47) is naïvely suppressed by $\mathrm{O}(H)$ in comparison with the dimension-six contributions from Eqs. (45)–(46). However, the Galileon symmetry forces $g_{1\star}$ and $g_{3\star}$ to contain an extra power of $H_\star$ [*cf.* Eqs. (22) and (24)], making the contribution to f_{NL} from each of Eqs. (45)–(47) precisely comparable.

From the point of view of an arbitrary effective field theory, such tuning would be highly unnatural. It is remarkable that in a Galileon theory this tuning is automatic, stable and technically natural: this also follows from the dimension-six operators' violation of the Galilean symmetry. Therefore we may choose them to be small while preserving technical naturalness, in the precise sense of 't Hooft [52].

V. CONCLUSIONS

Successful inflationary models must obey an approximate shift symmetry in order to generate sufficient e-folds of inflation. This shift symmetry allows us to add any scalar constructed from gradients of the inflaton field to the inflationary Lagrangian. However, when adding these gradient terms we must be wary of a number of pitfalls. Adding arbitrary higher derivative operators to the inflationary Lagrangian can lead to a loss of unitarity, due to the appearance of ghost states. The functional form of the Lagrangian need not be protected from large renormalizations, and if the Lagrangian requires more input parameters than can be measured then the theory loses all predictivity.

In this article we have presented a model, termed Galileon inflation, which avoids these pitfalls. Building on the success of the Galileon models of dark energy, we have constructed an inflationary Lagrangian containing noncanonical derivative operators whose form is protected by the covariant generalisation of the Galileon shift symmetry. It contains a finite number of operators and gives rise to second order field equations, implying the absence of ghosts.

We have constructed the action which describes fluctuations about the Galileon inflationary solution to third order in perturbations. In contrast to previous claims in the literature we find that none of the coefficients of the terms at third order in the inflationary fluctuation are fixed by matching to the background evolution and the two-point statistics. Therefore Galileon inflation is an explicit example of a new class of higher derivative inflationary models where the nongaussianity is not constrained to obey $f_{\mathrm{NL}} \sim 1/c_s^2$.

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Appendix A: Integrals of the Ei function

To obtain expressions for f_{NL} in closed form, we are obliged to compute several integrals over the exponential integral function Ei,

$$\mathrm{Ei}\, x = \int_{-\infty}^{x} \frac{\mathrm{e}^t}{t}\, \mathrm{d}t, \tag{A1}$$

defined for real nonzero x. If $x < 0$ the integral is manifestly well-defined; if $x > 0$ it must be understood as a Cauchy principal value. If $x = 0$ there is a logarithmic singularity which cannot be removed by the principal value technique, so the origin must explicitly be excluded from the domain of Ei. The exponential integral is related to the sine and cosine integrals by analogues of Euler's formula,

$$\mathrm{Ci}\, x + \mathrm{i}\,\mathrm{Si}\, x = \mathrm{Ei}(\mathrm{i}x) + \frac{\mathrm{i}\pi}{2} \tag{A2}$$

$$\mathrm{Ci}\, x - \mathrm{i}\,\mathrm{Si}\, x = \mathrm{Ei}(-\mathrm{i}x) - \frac{\mathrm{i}\pi}{2}. \tag{A3}$$

In constructing the $\mathrm{O}(\epsilon)$ perturbed wavefunctions in §IV D we must evaluate $\mathrm{Ei}(2\mathrm{i}kc_s\tau)$, which can be defined by complexifying t in Eq. (A1) and interpreting the integral over a suitable contour. This contour passes from the region where $|t| \to \infty$ with $\pi/2 < \arg t < 3\pi/2$ and terminates at x. Because $\tau < 0$, for $x = 2\mathrm{i}kc_s\tau$ the terminal point lies on the negative imaginary axis. Application of Cauchy's theorem allows the contour of integration to be rotated, obtaining the result quoted in Eq. (41).

Convergent series representation. In calculating f_{NL} we require integrals of the form

$$I_0(k_3) = \int_{-\infty}^{\tau} \mathrm{d}\zeta \, \mathrm{e}^{\mathrm{i}(k_1+k_2-k_3)c_s\zeta} \int_{-\infty}^{\zeta} \frac{\mathrm{d}\xi}{\xi} \, \mathrm{e}^{2\mathrm{i}k_3 c_s \xi}, \quad \text{(A4)}$$

where although I_0 is a function of k_1, k_2 and k_3 we have adopted the convention that only the momentum which occurs asymmetrically is indicated explicitly, in this case k_3, leaving the symmetric dependence on k_1 and k_2 implicit. It is convenient to define variables ϑ_3 and θ_3, satisfying

$$\vartheta_3 = 1 - \frac{2k_3}{k_t} \quad \text{and} \quad \theta_3 = \frac{1-\vartheta_3}{2\vartheta_3}, \quad \text{(A5)}$$

in terms of which, after further contour rotations, Eq. (A4) can be rewritten

$$I_0(k_3) = \frac{\mathrm{i}}{\vartheta_3 k_t c_s} \int_{\infty}^{0} \mathrm{d}u \, \mathrm{e}^{-u} \int_{\infty}^{\theta_3 u} \frac{\mathrm{d}v}{v} \, \mathrm{e}^{-2v}. \quad \text{(A6)}$$

We have taken the limit $\tau \to 0$, which incurs an exponentially small error if τ is sufficiently late that the wavenumber k_t is a few e-folds outside the horizon [29]. In principle there may be an obstruction to carrying out the contour rotation if $\vartheta_3 = 0$, which occurs in the squeezed limit where k_1 or k_2 approaches zero. In this limit, the ζ-integral in Eq. (A4) diverges. As discussed in Refs. [30, 49] one should define this limit by analytic continuation, first carrying out the integral with $\vartheta_3 > 0$ and only subsequently squeezing one of the momenta to zero. For this reason we need not worry about subtleties associated with rotation of either integral to the imaginary axis. There is also a logarithmic singularity when $\theta_3 = 0$, causing the interior v-integral to diverge.

The v-integral has a convergent series representation for all $\theta_3 u > 0$, which is guaranteed to be the case in virtue of our assumptions and the domain of integration for u. This convergent series representation is inherited from e^{-2v}, which is an entire function. We find

$$\int_{\infty}^{\theta_3 u} \frac{\mathrm{d}v}{v} \, \mathrm{e}^{-2v} = \gamma_{\mathrm{E}} + \ln 2\theta_3 u + \sum_{n=1}^{\infty} \frac{(-2\theta_3 u)^n}{n \cdot n!}. \quad \text{(A7)}$$

Because Eq. (A7) is valid for all $u > 0$ we may substitute in Eq. (A6) and integrate term-by-term, which yields

$$I_0(k_3) = \frac{\mathrm{i}}{\vartheta_3 k_t c_s} \left(\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(2\theta_3)^n}{n} - \ln 2\theta_3 \right). \quad \text{(A8)}$$

Analytic continuation. The sum converges for $|\theta_3| \leqslant 1/2$. Since I_0 is defined by an integral it is an analytic function of θ_3 on an open neighbourhood of the positive real axis, although excluding the origin $\theta_3 = 0$ where we have noted that Eq. (A6) exhibits a logarithmic singularity. Therefore I_0 may be determined by analytic continuation of Eq. (A8) from any open set where the sum converges. We conclude that I_0 has the compact representation

$$I_0(k_3) = \frac{\mathrm{i}}{\vartheta_3 k_t c_s} \ln \frac{1+2\theta_3}{2\theta_3} = -\frac{\mathrm{i}}{\vartheta_3 k_t c_s} \ln(1-\vartheta_3). \quad \text{(A9)}$$

Eq. (A9) correctly reproduces the expected pole as $\vartheta_3 \to 0$, associated with a failure of convergence in the ζ-integral of Eq. (A4). It also correctly reproduces the logarithmic singularity as $\theta_3 \to 0$, which corresponds to the limit $\vartheta_3 \to 1$. This is *also* a squeezed limit, occurring when $k_3 \to 0$, and forces $k_1 = k_2$. Away from these singular points, Eq. (A9) expresses $I_0(k_3)$ as a well-defined analytic function of θ_3. In practice these singularities cancel among themselves in our final expressions for f_{NL}, which serves as a consistency check on the calculation. Note that the equilateral limit is $\vartheta_3 = 1/3$ or $\theta_3 = 1$.

Repeating these steps enables us to find representations for integrals analogous to Eq. (A4), with insertions of arbitrary polynomials in the ζ-integral,

$$I_m(k_3) = \int_{-\infty}^{\tau} \mathrm{d}\zeta \, \zeta^m \mathrm{e}^{\mathrm{i}(k_1+k_2-k_3)c_s\zeta} \int_{-\infty}^{\zeta} \frac{\mathrm{d}\xi}{\xi} \, \mathrm{e}^{2\mathrm{i}k_3 c_s \xi}. \quad \text{(A10)}$$

We find

$$I_1(k_3) = \frac{1}{(\vartheta_3 k_t c_s)^2} \big[\vartheta_3 + \ln(1-\vartheta_3) \big] \quad \text{(A11)}$$

$$I_2(k_3) = \frac{\mathrm{i}}{(\vartheta_3 k_t c_s)^3} \big[\vartheta_3(2+\vartheta_3) + 2\ln(1-\vartheta_3) \big] \quad \text{(A12)}$$

$$I_3(k_3) = -\frac{1}{(\vartheta_3 k_t c_s)^4} \big[\vartheta_3(6+3\vartheta_3+2\vartheta_3^2) + 6\ln(1-\vartheta_3) \big] \quad \text{(A13)}$$

$$I_m(k_3) = \frac{\mathrm{i}^{m+1}}{(\vartheta_3 k_t c_s)^{m+1}} \left[2\theta_3 (m+1)! \, F\left(\begin{matrix} 1 \;\; 1 \;\; 2+m \\ 2 \;\; 2 \end{matrix} \,\middle|\, -2\theta_3 \right) - m! \Big\{ \gamma_{\mathrm{E}} + \ln 2\theta_3 + \psi^{(m+1)}(0) \Big\} \right], \quad \text{(A14)}$$

where we have retained the convention of denoting dependence on the single asymmetric momenta alone. The transcendental functions appearing here are the generalized hypergeometric function, F, and the polygamma function, $\psi^{(m)}$. For the purposes of this paper, we require only I_m with $m \leqslant 3$.

Appendix B: Three-point functions

The rules used to obtain correlation functions from a Lagrangian such as (26) are discussed at several places in the literature, to which we refer for calculational details [29, 47, 53]. We calculate the contribution to $\langle \pi^3 \rangle$ induced in turn by each operator in Eq. (26). The possible operators are $\dot{\pi}^3$, $\dot{\pi}(\partial\pi)^2$ and $\partial^2\pi(\partial\pi)^2$. In a model containing more than one of these, their contributions add linearly to the total three-point correlation function.

$\dot{\pi}^3$ operator. It is convenient to divide the calculation into (a) a part containing the O(1) contribution and O(ϵ) contributions from external lines and the vertex; and (b) a part containing the O(ϵ) contributions from internal lines. We use the notation of Eqs. (30)–(32), (38), and (42) and label the bispectrum generated by parts (a) and (b) as $B^{(a)}$ and $B^{(b)}$, respectively. For part (a) we find

$$B^{(a)} = \frac{g_{1\star}}{H_\star} \frac{H_\star^6}{4^3 \alpha_\star^3} \frac{24}{c_{s\star}^6} \frac{1}{k_t^3 \prod_i k_i} \left[1 + 3E_\star - \lambda_\star \ln \frac{k_1 k_2 k_3}{k_t^3} + (\epsilon_\star + h_{1\star})\gamma_{\mathrm{E}} - \frac{1}{2}(\epsilon_\star + 3h_{1\star}) \right] \tag{B1}$$

To evaluate (b) it is first convenient to obtain an expression for the integral

$$J = \frac{1}{k_t c_{s\star}} \int_{-\infty}^{\tau} \mathrm{d}\left(\mathrm{e}^{\mathrm{i}k_t c_s \xi}\right) \Big[\gamma_0 + \mathrm{i}\gamma_1 c_{s\star}\xi + \gamma_2 c_{s\star}^2 \xi^2 + \mathrm{i}\gamma_3 c_{s\star}^3 \xi^3 + \gamma_4 c_{s\star}^4 \xi^4 + \delta_0 N_\star(\xi) + \mathrm{i}\delta_1 c_{s\star} N_\star(\xi)\xi + \delta_2 c_{s\star}^2 N_\star(\xi)\xi^2 + \mathrm{i}\delta_3 c_{s\star}^3 N_\star(\xi)\xi^3 + \delta_4 c_{s\star}^4 N_\star(\xi)\xi^4 \Big], \tag{B2}$$

where $N_\star(\xi) = \ln|k_t c_{s\star} \xi|$. This is sufficiently general to encompass the τ-integration required for each operator, not including integrations of the exponential integral function Ei which we treat separately and are discussed in Appendix A. Carrying out the ξ integral, we find

$$J = \frac{1}{k_t c_{s\star}} \left[\gamma_0 - \frac{\gamma_1 + \delta_1}{k_t} - \frac{2\gamma_2 + 3\delta_2}{k_t^2} + \frac{6\gamma_3 + 11\delta_3}{k_t^3} + \frac{24\gamma_4 + 50\delta_4}{k_t^4} - \left(\gamma_{\mathrm{E}} + \frac{\mathrm{i}\pi}{2}\right) \left(\delta_0 - \frac{\delta_1}{k_t} - \frac{2\delta_2}{k_t^2} + \frac{6\delta_3}{k_t^3} + \frac{24\delta_4}{k_t^4} \right) \right]. \tag{B3}$$

In terms of J and the integral I_2 define by Eq. (A12) of Appendix A, the (b) contribution can be written

$$B^{(b)} = \frac{g_{1\star}}{H_\star} \frac{H_\star^6}{4^3 \alpha_\star^3} \frac{6}{c_{s\star}^5} \frac{1}{\prod_i k_i} \left[- J_3^{\dot{\pi}^3} + \mathrm{i}\lambda_\star c_{s\star}^2 I_2(k_3) \right] + \text{c.c.} + (k_3 \to k_2 \to k_1), \tag{B4}$$

where $J_3^{\dot{\pi}^3}$ denotes (B3) with the assignments $\gamma_0 = \gamma = \delta_0 = \delta_1 = 0$, and

$$\gamma_2 = s_\star - \mu_{1\star} \tag{B5}$$
$$\gamma_3 = k_3 s_\star \tag{B6}$$
$$\delta_2 = \lambda_\star - 2s_\star \tag{B7}$$
$$\delta_3 = -k_3 s_\star. \tag{B8}$$

The notation '+c.c' denotes addition of the complex conjugate of the preceding term, and $k_3 \to k_2 \to k_1$ indicates that Eq. (B4) is to be symmetrized over the exchanges $k_3 \leftrightarrow k_2$ and $k_3 \leftrightarrow k_1$, yielding a final expression which is symmetric between the labels 1, 2 and 3.

$\dot{\pi}(\partial\pi)^2$ operator. We divide the calculation into (a) and (b) parts, as above. The O(1) contribution and O(ϵ) contributions from external lines and the vertex give

$$B^{(a)} = \frac{g_{3\star}}{H_\star} \frac{H_\star^6}{4^3 \alpha_\star^3} \frac{2}{c_{s\star}^7} \frac{k_3^2(\mathbf{k}_1 \cdot \mathbf{k}_2)}{\prod_i k_i^3} \left[\frac{1}{k_t c_{s\star}} \left(1 + 3E_\star - \lambda_\star \ln \frac{k_1 k_2 k_3}{k_t^3} \right) \left(1 + \frac{k_t(k_1 + k_2) + 2k_1 k_2}{k_t^2} \right) + J_{3(a)}^{\dot{\pi}(\partial\pi)^3} \right] + \text{c.c} + (k_3 \to k_2 \to k_1), \tag{B9}$$

where $J_{3(a)}^{\dot{\pi}(\partial\pi)^3}$ is defined by the assignments $\gamma_3 = \gamma_4 = \delta_3 = \delta_4 = 0$, and

$$\gamma_0 = \epsilon_\star \tag{B10}$$
$$\gamma_1 = -\epsilon_\star(k_1 + k_2) \tag{B11}$$
$$\gamma_2 = -\epsilon_\star k_1 k_2 \tag{B12}$$
$$\delta_0 = -\epsilon_\star - h_{3\star} \tag{B13}$$
$$\delta_1 = (\epsilon_\star + h_{3\star})(k_1 + k_2) \tag{B14}$$
$$\delta_2 = (\epsilon_\star + h_{3\star})k_1 k_2. \tag{B15}$$

The O(ϵ) corrections from internal lines contribute

$$\begin{aligned} B^{(b)} = \frac{g_{3\star}}{H_\star}\frac{H_\star^6}{4^3\alpha_\star^3}\frac{2}{c_{s\star}^7}\frac{k_3^2(\mathbf{k}_1\cdot\mathbf{k}_2)}{\prod_i k_i^3}\Bigg[-\mathrm{i}\lambda_\star\Big\{ & I_0(k_3) - \mathrm{i}(k_1+k_2)c_{s\star}I_1(k_3) - k_1k_2c_{s\star}^2 I_2(k_3) \\ & + I_0(k_1) - \mathrm{i}(k_2-k_1)c_{s\star}I_1(k_1) + k_1k_2c_{s\star}^2I_2(k_1) \\ & + I_0(k_2) - \mathrm{i}(k_1-k_2)c_{s\star}I_1(k_2) + k_1k_2c_{s\star}^2I_2(k_2)\Big\} + J_{3(b)}^{\dot{\pi}(\partial\pi)^2}\Bigg] \\ & + \text{c.c.} + (k_3 \to k_2 \to k_1), \end{aligned} \tag{B16}$$

where $J_{3(b)}^{\dot{\pi}(\partial\pi)^2}$ is defined by the assignments $\gamma_4 = \delta_4 = 0$, and

$$\gamma_0 = 2\mu_{0\star} - \mu_{1\star} + s_\star \tag{B17}$$
$$\gamma_1 = s_\star k_3 + (k_1+k_2)(2\mu_{1\star} - \mu_{0\star} - s_\star) \tag{B18}$$
$$\gamma_2 = k_1k_2(3\mu_{1\star} - s_\star) + s_\star(k_1^2+k_2^2) + s_\star k_3(k_1+k_2) \tag{B19}$$
$$\gamma_3 = -s_\star k_1k_2k_t \tag{B20}$$
$$\delta_0 = 3\lambda_\star - 2s_\star \tag{B21}$$
$$\delta_1 = (2s_\star - 3\lambda_\star)(k_1+k_2) - s_\star k_3 \tag{B22}$$
$$\delta_2 = k_1k_2(2s_\star - 3\lambda_\star) - s_\star(k_1^2+k_2^2) - s_\star k_3(k_1+k_2) \tag{B23}$$
$$\delta_3 = s_\star k_1k_2k_t. \tag{B24}$$

$\partial^2\pi(\partial\pi)^2$ operator. Applying the same procedure to the final operator, $\partial^2\pi(\partial\pi)^2$, gives a contribution at O(1) and including O(ϵ) terms from the external lines and vertex:

$$\begin{aligned} B^{(a)} = g_{4\star}\frac{H_\star^6}{4^3\alpha_\star^3}\frac{4}{c_{s\star}^{10}}\frac{k_3^2(\mathbf{k}_1\cdot\mathbf{k}_2)}{k_t\prod_i k_i^3}\Bigg[& \left(1+3E_\star - \lambda_\star\ln\frac{k_1k_2k_3}{k_t^3}\right)\left(1+\frac{3k_1k_2k_3+k_tK^2}{k_t^3}\right) \\ & + \frac{h_{4\star}}{2}\left\{1+\frac{3K^2}{k_t^2} + 11\frac{k_1k_2k_3}{k_t^3} + \left(\gamma_{\mathrm{E}} + \frac{\mathrm{i}\pi}{2}\right)\left(2+\frac{2K^2}{k_t^2}+\frac{6k_1k_2k_3}{k_t^3}\right)\right\}\Bigg] \\ & + \text{c.c} + (k_3 \to k_2 \to k_1), \end{aligned} \tag{B25}$$

where we have defined $K^2 = k_1k_2 + k_1k_3 + k_2k_3 = \sum_{i<j} k_ik_j$. Finally we also require the (b) contribution, from O(ϵ) corrections to each internal line, which yields

$$\begin{aligned} B^{(b)} = g_{4\star}\frac{H_\star^6}{4^3\alpha_\star^3}\frac{2}{c_{s\star}^9}\frac{k_3^2(\mathbf{k}_1\cdot\mathbf{k}_2)}{\prod_i k_i^3} & \\ \Bigg[-\mathrm{i}\lambda_\star\Big\{ & I_0(k_3) - \mathrm{i}(k_1+k_2-k_3)c_{s\star}I_1(k_3) + (k_3k_1+k_3k_2-k_1k_2)c_{s\star}^2I_2(k_3) - \mathrm{i}k_1k_2k_3c_{s\star}^3I_3(k_3) \\ & + I_0(k_1) - \mathrm{i}(k_2+k_3-k_1)c_{s\star}I_1(k_1) + (k_1k_2+k_1k_3-k_2k_3)c_{s\star}^2I_2(k_1) - \mathrm{i}k_1k_2k_3c_{s\star}^3I_3(k_1) \\ & + I_0(k_2) - \mathrm{i}(k_1+k_3-k_2)c_{s\star}I_1(k_2) + (k_2k_1+k_2k_3-k_1k_3)c_{s\star}^2I_2(k_2) - \mathrm{i}k_1k_2k_3c_{s\star}^3I_3(k_2)\Big\} + J_3^{\partial^2\pi(\partial\pi)^2}\Bigg] \\ & + \text{c.c.} + (k_3 \to k_2 \to k_1), \end{aligned} \tag{B26}$$

where $J_3^{\partial^2\pi(\partial\pi)^2}$ is defined by the assignments

$$\gamma_0 = 3\mu_{0\star} \tag{B27}$$
$$\gamma_1 = k_t(\mu_{1\star} - 2\mu_{0\star}) \tag{B28}$$
$$\gamma_2 = s_\star(k_1^2 + k_2^2 + k_3^2) + K^2(2\mu_{1\star} - \mu_{0\star}) \tag{B29}$$
$$\gamma_3 = -s_\star\left[k_1^2(k_2+k_3) + k_2^2(k_1+k_3) + k_3^2(k_1+k_2)\right] - 3\mu_{1\star}k_1k_2k_3 \tag{B30}$$
$$\gamma_4 = -s_\star k_1k_2k_3k_t \tag{B31}$$
$$\delta_0 = 3\lambda_\star \tag{B32}$$
$$\delta_1 = -3\lambda_\star k_t \tag{B33}$$
$$\delta_2 = -s_\star(k_1^2 + k_2^2 + k_3^2) - 3\lambda_\star K^2 \tag{B34}$$
$$\delta_3 = s_\star\left[k_1^2(k_2+k_3) + k_2^2(k_1+k_3) + k_3^2(k_1+k_2)\right] + 3\lambda_\star k_1k_2k_3 \tag{B35}$$
$$\delta_4 = s_\star k_1k_2k_3k_t. \tag{B36}$$

Note that, because this interaction is symmetric apart from the arrangement of spatial gradients, Eqs. (B25) and (B26) are in fact symmetric under permutation of the labels 1, 2 and 3 except for the overall factor $k_3^2(\mathbf{k}_1\cdot\mathbf{k}_2)$, which arises from the specific gradient combination appearing in $\partial^2\pi(\partial\pi)^2$.

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