GUT scale extra dimensions and light moduli in supergravity and cosmology

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Zusammenfassung

Ich untersuche die dynamischen Eigenschaften geometrischer Moduli in Orbifold-Kompaktifizierungen von fünf- und sechs-dimensionalen Supergravitations-Theorien. Das Hauptaugenmerk liegt hierbei auf dem Einfluss des Kähler-Potentials, das in beiden Fällen in führender Ordnung von sogenannter 'no-scale' Struktur ist. Im fünfdimensionalen Fall kann der Volumen-Modulus, das Radion-Feld, stabilisiert werden durch perturbative Korrekturen am Kähler-Potential. In sechs Dimensionen gilt dasselbe für die Größe und die komplexe Struktur der beiden Zusatzdimensionen, aber nur wenn zuvor das Dilaton durch nicht-perturbative Effekte fixiert werden kann, und zwar mit verschwindender potentieller Energie im Vakuum.

Ich gebe eine systematische Beschreibung von Modellen mit 'beinahe no-scale' Struktur und leite eine modell-unabhängige Formel für die Radion-Masse ab. Die Masse des Radions ist parametrisch unterdrückt im Vergleich zur Masse des Gravitinos. Das Massenverhältnis reflektiert die Hierarchie zwischen der Planck-Skala und der Kompaktifizierungs-Skala. In einem konkreten Beispiel wird die Kompaktifizierungs-Skala bestimmt durch Fayet-Iliopoulos-Terme, die zusammen mit einer lokal anomalen U(1) Eichgruppe auftreten und von der Größenordnung der GUT-Skala sind. Für den Fall, dass das Gravitino gleichzeitig für die dunkle Materie im Universum verantwortlich ist, resultiert eine Radion-Masse von 1 - 10 MeV. In diesem Energiebereich ist das Radion kosmologisch stabil und trägt einen kleinen Anteil zur dunklen Materiedichte bei. Aus Beobachtungen galaktischer Gamma-Emissionen läßt sich eine Schranke an die anfängliche Auslenkung des Radion-Feldwertes gegenüber seinem Vakuum-Erwartungswert herleiten.

Desweiteren untersuche ich die Auswirkungen eines typischen Moduli-Kähler-Potentials auf die kosmologische Dynamik eines solchen komplexen Skalarfeldes. Insbesondere diskutiere ich eine Klasse von Modellen mit steil abfallenden exponentiellen Potentialen und nicht-kanonischen kinetischen Termen, zu der auch das Radion-Beispiel zählt. Neben einer Präzisierung des bekannten 'overshooting'-Problems der kosmologischen Dynamik von Moduli-Feldern ergibt sich die interessante Möglichkeit von Lösungen, die ein Feld beschreiben, das langsam einen steilen Potentialhang 'hinabrollt'.

Abstract

We study the dynamical properties of geometric moduli in five- and six-dimensional supergravity compactified on flat orbifolds, focusing on the impact of the Kähler potential. In both cases, the Kähler potential exhibits no-scale structure at tree level. In five dimensions, the volume modulus (radion) can be stabilized by means of perturbative Kähler corrections. In six dimensions, the same holds for size and shape of the extra dimensions, only if the dilaton can be stabilized in a Minkowski vacuum by non-perturbative effects. We develop a systematic description of almost no-scale models and derive a model independent formula for the radion mass. The radion mass is suppressed compared to the gravitino mass. The suppression factor reflects the hierarchy between the Planck and the compactification scale. We analyze a specific example, where the compactification scale is determined by Fayet-Iliopoulos terms of a locally anomalous Abelian gauge group, which are $\mathcal{O}(M_{\mathrm{GUT}})$. In a scenario with gravitino dark matter, this leads to a radion mass of 1 - 10 MeV. In this mass range, the radion is cosmologically stable and contributes to the dark matter density. Based on galactic gamma ray data, we derive a tight bound on the initial displacement of the field value from its low energy vacuum. We also investigate implications of typical moduli Kähler potentials on the cosmological evolution of the scalar fields. In particular, we discuss a class of models with steep exponential potentials and non-canonical kinetic terms, motivated by our radion example. We consider the overshooting problem of cosmological moduli dynamics, and the possibility of slow-roll solutions despite the steepness of the scalar potential.

Contents

Chapter 1

Introduction

Superstring theory [1, 2] is widely believed to be the most promising candidate for a unified theory of interactions. Apart from being a UV completion of the field theoretical Standard Model of point particle physics, string excitations even allow for a renormalizable quantum description of spin two interactions, which is considered to provide a quantum theory of gravity. However, superstring theory is consistently formulated in ten space-time dimensions, and therefore dramatically contradicted by observations. Hence, one has to presume a mechanism that compactifies six spatial dimensions at a microscopically small length scale. At low energies, the excitations related to the extra dimensions decouple, and the resulting effective field theory can be formulated in four space-time dimensions. The low energy effective actions obtained from string theory are usually chosen to preserve at least N=1 supersymmetry [3]. Apart from the need to incorporate supersymmetry breaking at low energies, the issue of dynamical stability of the extra dimensions poses a considerable challenge for string model building. We shall address this problem in our work.

The fact that the Higgs boson mass receives divergent loop corrections, suggests that physics beyond the Standard Model becomes relevant at the TeV scale [4]. In particular, in supersymmetric theories the divergencies cancel. Therefore, one expects supersymmetry to be restored not far above the electroweak scale. Incorporated in supersymmetric grand unified theories (GUTs), the Higgs mechanism is challenged by the 'doublet-triplet splitting' problem: The unification of gauge interactions at a scale of $M_{\rm GUT} \simeq 10^{16}$ GeV is associated with an extension of the gauge symmetry, which implies larger representation spaces for the charged matter fields, including the Higgs boson. As a consequence, exotic particles can persist in the low energy limit of the theory. However, in the presence of extra dimensions, the exotic degrees of freedom can be identified with bulk fields that are not allowed to have light modes [5]. Together with a unified gauge sector in the higher dimensional theory, this leads to the picture of local grand unification [6, 7].

Recently, orbifold compactifications of the heterotic string [8] were shown to admit quasi-realistic vacua in four dimensions, yielding the particle spectrum of the supersymmetric Standard Model [9–11]. As discussed in [9], different anisotropic limits of the internal geometry lead to different gauge groups being realized in the corresponding bulk theory. In this way, heterotic orbifolds incorporate the originally field theoretical set-up of orbifold GUTs, and their appealing properties [6,12]. In [13], a six-dimensional local GUT was obtained as a specific anisotropic limit of a heterotic orbifold model with $E_8 \times E_8$ gauge symmetry. As a necessary prerequisite for the success of local grand unification within anisotropic orbifold compactifications, the size of the 'large' extra dimensions has to be stabilized on a scale close to the GUT scale [14].

The issue of the dynamical stability of anisotropic orbifolds has several different aspects. First, why are the extra dimensions microscopically small at all, and why is their size cosmologically stable, while the four-dimensional spacetime is known to expand? Second, how is it possible to achieve a hierarchy of scales within the compact space, rendering one or two of the extra dimensions significantly larger than the rest? Third, how can one incorporate a specific mass scale in the problem, in order to relate the compactification scale to the scale of grand unification? Finally, and presumably most importantly, does a stabilization mechanism, which provides at least some of these desirable features, lead to observable signatures, and does it possibly even allow for a falsification of the original set-up?

The first problem has been the subject of extensive research activities over the past two decades. In the following section, we shall briefly review the current status of moduli stabilization in string theory, including the possibility of stable compact extra dimensions. In this thesis, we will not address the second question. Concerning the third question, it was proposed [15] that localized Fayet-Iliopoulos terms of an anomalous U(1) gauge factor, which can be $\mathcal{O}(M_{\mathrm{GUT}})$ in heterotic orbifold compactifications [13], provide the relevant mass scale. In chapter 3, we shall present and analyze a specific realization of this proposal within a field theoretical, five-dimensional toy model. The main concern of this thesis, however, will be the last issue.

In order to confront genuinely stringy features with observational data, one should start with a fully-fledged string set-up in ten dimensions, and systematically deduce phenomenological consequences of the resulting low energy effective action, ranging from the realized particle spectrum to the issue of vacuum selection and the cosmological constant. However, this approach is far beyond the scope of this thesis. At present, it is not completely clear whether such a procedure could be carried out in principle, without resorting to field theoretical concepts at intermediate steps. Moreover, a lot of specifications are inevitable during the process, beginning with the choice of a specific string theory[1] and a specific compactification manifold, followed by model building issues related to the particle content and breaking of gauge symmetries, and the implementation of low energy supersymmetry breaking. Finally, the issue of vacuum selection inevitably requires to fix a large number of singlet expectation values by hand, since the complete scalar potential is not known explicitly [11]. As a consequence, observational tests can only be applied to very special points in the parameter space of stringy model building.

[1] At present, five different string theories are known, type I, type IIA and IIB, and heterotic with two different gauge groups, which are mutually interrelated by dualities. Moreover, M- and F-theory provide two different possible extensions, cf. [2].

In this thesis, we shall take a different point of view. Many generic features of string theories, including a supersymmetric particle spectrum and the existence of small extra dimensions, are easily incorporated in simple toy models. We will be interested in model independent consequences of a specific scenario of moduli stabilization, which we shall study in the framework of five- and six-dimensional supergravity theories. In particular, we intend to reconcile the idea of GUT scale extra dimensions with the prerequisite of TeV scale supersymmetry restoration and our current understanding of cosmology. In this way, we obtain a joint picture composed of several theoretically attractive model building blocks, which then, if taken altogether, allow for 'relational' predictions, such that falsification implies at least two of the ingredients to be mutually inconsistent.

Before we give a detailed outline of this thesis, we shall present some background material on the theoretical context of our work, including an overview of the challenges which are posed by modern observational cosmology.

1.1 Setting the stage

To begin this section, we give a brief introduction to the ΛCDM model of cosmology, and review its current observational status. Second, we recall the moduli problem of string theory and summarize the different strategies of moduli stabilization. Finally, we provide the technical basics of orbifold compactifications and their relation to supersymmetry breaking [16].

1.1.1 Dark energy and dark matter

Our current understanding of late time cosmology is based on observational evidence obtained from type Ia supernova lightcurves [17], the temperature anisotropies in the cosmic microwave background [18], and the large scale structure of matter distribution in the universe [19]. The 'standard model of cosmology', abbreviated ΛCDM, favors a flat universe dominated by a mixture of cold dark matter (CDM) and dark energy in the shape of a cosmological constant (Λ) with fractional density $\Omega_\Lambda \simeq 0.73$ [20]. The expansion of the universe is currently accelerating.

The universe is assumed to be isotropic and homogeneous on the largest scales, and therefore described by a Friedmann-Robertson-Walker (FRW) spacetime with metric

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1-kr^2} + r^2\big(d\theta^2 + \sin^2\theta d\varphi\big) \right] , \tag{1.1.1}$$

where $a(t)$ is the scale factor, and $k \in \{-1, 0, 1\}$ determines the geometry of spatial sections to be hyperbolic, flat and spherical, respectively. As a consequence, Einstein's field equations of gravity can be rewritten in terms of the Hubble rate $H = \dot{a}/a$,

$$M^2 \dot{H} = -\frac{1}{2} \sum_i (\rho_i + p_i) + \frac{k}{a^2} , \tag{1.1.2}$$

$$M^2 H^2 = \frac{1}{3}\sum_i \rho_i - \frac{k}{a^2}\,, \tag{1.1.3}$$

where M is the reduced Planck mass, $M \equiv (8\pi G_N)^{-1/2}$. The sum includes every contribution to the total energy density, each modeled as a perfect fluid with density ρ_i and pressure p_i. In terms of the *critical* density $\rho_{\rm crit} \equiv 3M^2H^2$, the second equation takes the form

$$1 = \sum_i \frac{\rho_i}{\rho_{\rm crit}} \equiv \sum_j \Omega_j + \Omega_\Lambda\,, \tag{1.1.4}$$

where we set $k = 0$ and split out the fractional density of a cosmological constant explicitly. The cosmological constant can be described as a perfect fluid with equation of state

$$w_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} = -1\,, \tag{1.1.5}$$

and is therefore often identified with the vacuum energy density of the universe.

Although the dark energy dominates the present universe, its density is extremely small in fundamental units,

$$\rho_\Lambda \simeq 10^{-120} M^4\,, \tag{1.1.6}$$

which corresponds to the smallness of the current Hubble parameter. In field theory, the vacuum energy is generated by zero point fluctuations of quantum fields, and can be estimated by the cut-off scale; it is conceivable that these contributions are exactly cancelled by a 'bare' cosmological constant, based on some (yet unknown) symmetry principle. However, the tiny *non-zero* value then poses a severe fine tuning problem. For this reason, dynamical models of dark energy were proposed, most prominent among them the 'cosmon' field [21], later on dubbed quintessence. In these models, the present day acceleration is due to a slowly rolling scalar field, with asymptotically vanishing potential energy. However, these models suffer from the *coincidence problem*: in order to explain why the quintessence energy density is of the same order as the dark matter density right now, one has to resort to an extreme fine-tuning of initial conditions, cf. [22, 23] and references therein.

In string inspired models, the dark energy density is also generated dynamically, namely by a non-zero vacuum value of the scalar potential. However, scalar fields of stringy origin usually couple to matter fields with gravitational strength, and inflict variations of fundamental constants, cf. the following subsection. Hence, at the position of the minimum the curvature of the potential has to be large enough for all the scalar fields to be stabilized during early stages of cosmic evolution; ideally, the lightest among these scalar fields would be responsible for the inflationary epoch. The residual potential energy after inflaton decay then behaves like a true cosmological constant throughout the succeeding cosmological eras. In this way, string theory favors dark energy that is indistinguishable from a cosmological constant. On the other hand, the cosmological constant will be clearly ruled out, if future observations confirm a deviation from the vacuum equation of state at present or during the recent past, cf. [24], in particular their Fig. 17. In this way, string theory may eventually turn out to be falsifiable by upcoming observational data.

While the accelerated expansion of the universe was discovered quite recently, observational evidence for the existence of dark matter was obtained by Fritz Zwicky already in 1933 [25], when he determined the mass-to-light ratio in the Coma cluster and estimated the mass fraction of luminous matter to be about ten percent. Later on, it was observed that the rotation curves of spiral galaxies [26] are nearly flat in the outer regions, indicating the existence of spherical halos of non-luminous, sc. 'dark' matter. On the other hand, the theoretical understanding of Big Bang nucleosynthesis (BBN) only allows for a fractional density $\Omega_{\rm mat} \simeq 0.04$ of baryons, in order to fit the observed abundances of primordial light elements, cf. [27]. Moreover, the temperature fluctuations in the CMB proved to be too small to account for the observed large scale structure displayed by luminous matter, since perturbations in a baryonic medium cannot start to grow before recombination [28]. Dark matter is most likely cold, featuring negligible velocity dispersion; numerical simulations (*e.g.* [29]) based on the cold dark matter paradigm successfully reproduce the statistical porperties of the observed large scale structure (cf. [30]). Taken together, there is overwhelming evidence for the existence of a non-baryonic species of matter, weakly or only gravitationally interacting, and more smoothly distributed than luminous matter.

In supersymmetric theories with R-parity [31], the lightest superpartner (LSP) of a Standard Model particle is stable, only weakly interacting, and therefore an attractive particle dark matter candidate. The cosmological relic abundance of weakly interacting massive particles (WIMPs) is mainly determined by their effective cross-section at decoupling from thermal equilibrium; if they interact with approximately electroweak strength, WIMP masses of some ten GeV up to a few TeV are consistent with the data on $\Omega_{\rm DM}$ [32].

In the case of small R-parity violating couplings, a gravitino LSP can still have a lifetime which is longer than the age of the universe. In [33] it was shown that (decaying) gravitino dark matter is consistent with BBN and thermal leptogenesis [34], if the gravitino has a mass $m_{3/2} \gtrsim 5$ GeV. BBN physics strongly constrains the lifetimes and / or the abundances of next-to-lightest supersymmetric particle (NLSP) candidates, since the yields of primordial elements are very sensitive to the presence of decaying particles during BBN, and to the entropy production which possibly results from these decays. In thermal leptogenesis the baryon asymmetry in the universe is generated at early times and high temperatures. Hence, it is impossible to dilute overabundancies of dark matter candidates by late time entropy production, because this would also lead to a wash-out of the primordial baryon asymmetry. Since oscillating scalar fields can contribute to the dark matter abundance as well, this restriction will turn out to be relevant when we face the cosmological moduli problem in chapter 4.

1.1.2 Moduli stabilization in string theory

By definition, a modulus field ϕ has the property that $\partial_\phi V = 0$ for any field value ϕ_0, where V denotes the tree level scalar potential.[2] Alternatively, we shall use the term 'flat direction of the scalar potential', which underlines the vacuum degeneracy that is associated with the existence of moduli. In string theory, phenomenologically relevant quantities of the low energy effective action depend on the vacuum expectation values (VEVs) of moduli fields, including mass terms, gauge and Yukawa couplings. In order to render the theory predictive, it is therefore essential to lift the flat directions in the scalar potential, which is equivalent to generating moduli masses. Since light scalar bosons would also give rise to fifth forces, these masses have to be large enough to pass the corresponding observational tests (cf. [35] and references therein). Hence, being a prerequisite of meaningful phenomenology, the stabilization of all moduli is a crucial task in string model building.

Moduli arise generically from metric degrees of freedom of the internal manifold, corresponding to force-free deformations of the geometry. These moduli come in two classes, dubbed Kähler and complex structure moduli. In terms of complex geometry,[3] Kähler deformations are of the type $\delta g_{i\bar{j}}$, i.e. associated with metric components that feature a holomorphic and an anti-holomorphic index, while complex structure deformations can be written as δg_{ij} or $\delta g_{\bar{i}\bar{j}}$, respectively. Also the dilaton, which controls the string coupling constant, is associated with a modulus at tree level. Both the dilaton and the Kähler moduli are combined with pseudoscalar partners that emerge from form fields of the ten-dimensional theory. Generically, there is also a large number of additional, non-geometric moduli, whose features depend on the details of the set-up. For instance, in the heterotic case vector bundle moduli [37] appear as gauge singlet scalar fields in the low energy effective action.

In the context of the supergravity limit of type IIB string theory, it was observed that non-trivial scalar potentials can be generated at tree level by 'turning on fluxes' [38–42]. These fluxes, which correspond to non-zero field strength background values of some of the various form fields in the action, can arise due to non-trivial topological properties of the compact space, and are quantized. The existence of fluxes gives rise to a discretuum of low energy vacua, dubbed 'landscape' [43]. As an apparently attractive side-effect, the present value of the cosmological constant may be 'explained' by the vast number of existing flux vacua [44]. In this perception, the problem of vacuum selection is circumvented by resorting to an anthropic principle (cf. [45]).

Compared to type IIB supergravity, the heterotic theory contains a lesser number of form fields in ten dimensions, cf. [2]. As a consequence, neither a comparative number of flux vacua is available, nor can a potential be generated, which removes as many flat directions from the low energy effective theory. However, it is possible to reduce the

[2] Whenever we use the term 'modulus' in the following, it is meant to be shorthand for 'modulus field'.

[3] The standard example of a smooth six-dimensional internal space is a Calabi-Yau threefold [36], which is, by definition, a solution of the vacuum Einstein equations.

number of moduli[4] by choosing compactification manifolds with generalized geometries, allowing for torsion (cf. [46] and references therein).

In general, promising strategies of moduli stabilization rely on non-perturbative dynamics. This fact is related to the Dine-Seiberg problem [47]: Consider the dilaton in heterotic string theory, which controls the gauge coupling at low energies as well as the string coupling itself. The runaway solution ($\phi \to \infty$) leads to a weakly coupled theory ($g_{\text{string}} \sim e^{-\phi}$), while the phenomenologically desired VEV corresponds to strong coupling. Hence, one cannot stabilize the dilaton in a perturbatively controlled way. In the strong coupling regime, it is impossible to verify that a vacuum solution obtained at a given loop order is stable against higher order corrections, which can be equally important.

In heterotic models, dilaton stabilization is usually achieved by means of hidden sector gaugino condensation (cf. [48] for a review and further references). So far, only field theoretical descriptions are available. In the formulation of [49], the hidden gaugino bilinear is identified with the lowest component of a chiral superfield,

$$Y \equiv \mathrm{Tr}\, W^\alpha W_\alpha \,, \tag{1.1.7}$$

where W^α is the (hidden sector) gauge field strength superfield, and we set $M \equiv 1$. Suppose the hidden sector gauge theory is asymptotically free. Gaugino condensation occurs at an energy scale, where the gauge theory becomes strongly coupled; the effect can be described in terms of a non-perturbative correction to the superpotential [50],

$$W^{\text{np}} = \frac{Y^3}{4}\left[S + b_0 \ln Y\right] \,, \tag{1.1.8}$$

where we took the gauge kinetic function to coincide with the dilaton superfield S, and b_0 is a constant associated with the one loop coefficient of the β-function. As a consequence, the field Y acquires a VEV; integrating out Y then leads to a dilaton-dependent contribution to the superpotential,

$$W^{\text{np}} \sim e^{-S/b_0} \,. \tag{1.1.9}$$

In order to stabilize the dilaton at the phenomenologically desired value, $\mathrm{Re}\, S \simeq 2$,[5] the hidden sector is subject to some fine-tuning; in racetrack models, one needs two factors of the gauge group with different but adjacent ranks, and additionally a collection of hidden matter fields, which are charged under at least one of the gauge group factors [51]. Alternatively, one may introduce a constant part W_0 in the superpotential, associated with approximate R symmetries, cf. [52].

Apart from the dilaton, heterotic models generically give rise to Kähler moduli in the low energy effective action, with (tree level) Kähler potential

$$K = -\ln(S + \bar{S}) - \ln\left[\frac{d_{ijk}}{6}(T^i + \bar{T}^i)(T^j + \bar{T}^j)(T^k + \bar{T}^k)\right] \,, \tag{1.1.10}$$

[4] Since fluxes and torsion coefficients generate tree level contributions to the scalar potential, we refrain from calling any field, which is stabilized in this way, a 'modulus' in the first place.

[5] With $\langle \mathrm{Re}\, S\rangle = g_4^{-2}$, this value corresponds to $\alpha_{\text{GUT}} \simeq 1/25$.

where d_{ijk} are the Calabi-Yau intersection numbers [36]. In the simplest possible case, where the overall volume of the internal space is controlled by a single scale factor associated with $\mathrm{Re}\, T$, the Kähler potential reduces to

$$K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T})\,. \tag{1.1.11}$$

We shall call this field the universal Kähler modulus throughout the following chapters.

Near the compactification scale the effective description in terms of four-dimensional field theory breaks down. Due to the scale dependence of the gauge coupling, the gauge kinetic function receives threshold corrections, corresponding to the energy scale where the gauge interactions become sensitive to the complete higher-dimensional spectrum. It is intuitive that these threshold corrections depend on the volume of the internal space. Hence, one obtains a T-dependent correction to the non-perturbative dilaton superpotential, and the Kähler modulus can be stabilized along with the dilaton. In this case, the volume generically turns out to be close to one in units of the string length [53].

Alternatively, Kähler moduli may be stabilized at larger values by means of perturbative corrections, including string-loop- as well as α'-corrections, the latter being associated with the finite string length [54–57]. Moreover, after supersymmetry is broken, further corrections arise which can be treated perturbatively, since they are sensitive to the size of the internal space; the so-called Casimir energy is a field theoretical effect that shall be discussed in chapter 3. We note that the universal Kähler modulus does not necessarily suffer from the Dine-Seiberg problem, if the compactification scale is small compared to the string scale. In this case, the ratio of string length and compactification radius defines a suitable expansion parameter, thus rendering the perturbative treatment controllable. Due to the variety of different contributions, stable vacua can already arise at leading order in the corrections, as we shall demonstrate in chapter 3.

Concluding this subsection, let us briefly comment on the possibility of anisotropic orbifold compactifications. Suppose we can realize a set-up with two Kähler moduli T_1 and T_2, associated to a four-dimensional and a two-dimensional submanifold of the compact space, respectively. Furthermore, assume that the gauge kinetic function receives threshold correction depending only on T_1 [58], such that two complex dimensions are compactified at the fundamental scale. In that case, perturbative effects can lead to a stabilization of T_2 at a value which corresponds to a significantly larger length scale. The hierarchy between the compactification scales of the two submanifolds is then related to the disparate regimes of non-perturbative and perturbative stabilization. Keeping in mind that the perturbative treatment presupposes larger volumes, we shall later on refer to perturbative methods in order to realize a concrete stabilization mechanism leading to GUT scale extra dimensions, cf. chapter 3.

1.1.3 Orbifolds and supersymmetry breaking

Consider a theory in $D = 4 + d$ space-time dimensions, compactified on $\mathcal{M}_4 \times C$, where $\mathcal{M}_4$ is four-dimensional Minkowski spacetime and C is compact. If C can be written as

$$C = M/G\,, \tag{1.1.12}$$

where G is a discrete group acting *freely* on the (non-compact) manifold M, the *covering space* of C, then C itself is a manifold. G acts on M by means of operators $O_g : M \to M$ for $g \in G$, which form a representation of G. The group acts freely on M if only O_{id} has fixed points in M, such that $O_{id}(y) = y$ for $y \in M$. Since id is the identity in G, O_{id} operates trivially on M. The space C is then obtained by identifying points $y \in M$ that belong to the same *orbit*,

$$y \equiv O_g(y)\,, \quad \forall\, g \in G\,. \tag{1.1.13}$$

Now let H be a group acting *non freely* on C. We mod out C by H by identifying points $y \in M$ with their images $P_h(y)$ for some $h \in H$. The fact that H acts non freely on C means that some transformations P_h have fixed points in C. As a consequence, the space C/H is no longer smooth but has singularities at the fixed point positions; it is therefore called an *orbifold*.

We shall discuss two simple examples, which will be relevant in the following. First, consider a five-dimensional set-up with $C = \mathcal{S}^1 = \mathbb{R}/\mathbb{Z}$, and take $H = \mathbb{Z}_2$. The only non-trivial element of $\mathbb{Z}_2$ is the inversion, acting on $\mathcal{S}^1$ as follows,

$$P_-(y) = -y\,, \quad P_-^2(y) = y\,, \quad \forall\, y \in (-L, L]\,. \tag{1.1.14}$$

For any field ϕ defined on $\mathcal{M}_4 \times \mathcal{S}^1$, this implies

$$\phi(x,-y) = Z\phi(x,y)\,, \quad \phi(x, P_-^2(y)) = Z^2\phi(x,y) = \phi(x,y)\,. \tag{1.1.15}$$

This means that Z can be represented as a matrix acting on field space, with eigenvalues ± 1. We shall call fields ($\mathbb{Z}_2$) *even*, if they correspond to eigenstates of Z with eigenvalue $+1$, and *odd* else. We note that, in the special case of five dimensions, there are no singularities associated with the fixed points; the orbifold $\mathcal{M}_4 \times \mathcal{S}^1/\mathbb{Z}_2$ is a manifold with two smooth co-dimension one boundaries, which we shall call *branes*.

As a second example, consider the compact space to be a torus, $C = \mathcal{T}^2 = \mathbb{C}/_{\mathbb{Z}\times\mathbb{Z}}$, with fundamental domain $(y^1, y^2) \in (-L_1, L_1] \times (-L_2, L_2]$. Modding out by $\mathbb{Z}_2$ again, we now find four different fixed points,

$$P_1 = (0,0)\,, \quad P_2 = (0, L_2)\,, \quad P_3 = (L_1, 0)\,, \quad P_4 = (L_1, L_2)\,, \tag{1.1.16}$$

and the resulting space resembles a pillow, with a conical singularity attached to each of its four 'corners'. Let us now consider a scalar field ϕ defined on $\mathcal{M}_4 \times \mathcal{T}^2$, with Fourier expansion

$$\phi(x,y) = \frac{1}{\sqrt{L_1 L_2}} \sum_{l,n} \phi_{ln}(x) \left[\cos\left(\frac{l\pi y^1}{L_1} + \frac{n\pi y^2}{L_2}\right) + i \sin\left(\frac{l\pi y^1}{L_1} + \frac{n\pi y^2}{L_2}\right)\right]\,. \tag{1.1.17}$$

After integrating over the extra dimensions, each mode ϕ_{ln} acquires a Kaluza-Klein mass

$$m_{ln}^2 = \frac{\pi^2 l^2}{L_1^2} + \frac{\pi^2 n^2}{L_2^2}\,. \tag{1.1.18}$$

Imposing the $\mathbb{Z}_2$ action, we immediately see that only even fields retain a (massless) *zero mode*. The lightest odd state gets a mass of the order of the compactification scale $M_{\text{comp}} = L^{-1}$. This is the crucial feature of orbifolding: The spectrum of the low energy effective action, containing only the zero modes, is reduced *wrt* the spectrum of the higher-dimensional theory, by choosing appropriate boundary conditions. In this way, one can eliminate unwanted states from a model, *e.g.* in order to break gauge symmetries at fixed points.

If we apply the orbifold projection to the parameter of supersymmetry transformations,

$$\epsilon = \begin{pmatrix} \epsilon_L^+ \\ \epsilon_R^- \end{pmatrix}, \tag{1.1.19}$$

where the superscript '+' denotes the even components, we can also break supersymmetry (SUSY) by imposing boundary conditions on a fixed brane. Let us consider an N=2 vector multiplet (A_M, Σ, λ) in five dimensions.[6] We decompose the gaugino analogously to the Killing spinor,

$$\lambda = \begin{pmatrix} \lambda_L^+ \\ \lambda_R^- \end{pmatrix}, \tag{1.1.20}$$

and assign even orbifold parity to A_μ on the one hand, odd parity to A_5 and the real scalar Σ on the other hand. The states (A_μ, λ_L) then form an N=1 vector multiplet, their SUSY transformations being generated by ϵ_L. After integrating over the fifth dimension, only the zero modes are present in the low energy effective action, which is invariant under the N=1 subset of supersymmetries generated by ϵ_L.

In chapter 2, we will apply this procedure to 6D N=2 supergravity, in order to obtain N=1 supergravity in four dimensions. Concluding this section, we shall briefly recall the basic features of N=1 supergravity, which is defined to be a theory invariant under *local* supersymmetry transformations. The supergravity multiplet (e_μ^α, Ψ_μ), consisting of the vielbein e_μ^α and the gravitino Ψ_μ, can be coupled to vector multiplets, as well as chiral multiplets (Φ^i, ξ^i, F^i), which we write down in off-shell notation: ξ^i are chiral fermions, Φ^i complex scalars, and F^i auxiliary scalar fields which are needed for the SUSY algebra to close. The theory is completely determined by two functions; the gauge kinetic function $f(\Phi^i)$ encodes how the gauge coupling depends on the scalar fields in the theory; the Kähler function

$$G = K(\Phi^i, \bar{\Phi}^i) + \ln |W(\Phi^i)|^2 \tag{1.1.21}$$

determines the scalar kinetic terms and the scalar potential. Here we decomposed G in terms of the real Kähler potential K and the holomorphic superpotential W, which are both defined up to Kähler transformations,

$$K \to K + f + \bar{f}, \quad W \to We^{-f}, \tag{1.1.22}$$

[6] We note that we call a theory with eight supercharges to be N=2 supersymmetric, irrespective of the number of space-time dimensions of the background the theory is defined on.

where f is an arbitrary holomorphic function of the chiral superfields.[7] The auxiliary fields of the chiral multiplets are fixed by algebraic equations of motion,

$$F_i = e^{G/2}\partial_i G\,. \tag{1.1.23}$$

N=1 supersymmetry is spontaneously broken if $\langle F_i\rangle \neq 0$ for at least one of the F_i. The linear combination of fermions $G_i\xi^i$ is called *goldstino* and 'eaten up' by the gravitino. As a consequence, the gravitino acquires a mass; this is called the super-Higgs effect [59].

1.2 Outline of the thesis

In this thesis, we study the impact of the Kähler potential on the dynamics and stabilization of moduli fields. In heterotic models, tree level Kähler potentials for geometric moduli are typically of the form [36, 60]

$$K = -\sum_i n_i \ln(\Phi_i + \bar{\Phi}_i)\,. \tag{1.2.1}$$

We shall focus on the specific case

$$\sum_i n_i = 3\,, \quad \Rightarrow \quad K^i K_i = 3\,, \tag{1.2.2}$$

which corresponds to no-scale supergravity models [61]. The possibility to obtain Minkowski or de Sitter vacua in no-scale models was discussed in [62, 63]. In order to stabilize the moduli using only the tree level Kähler potential, one heavily relies on non-perturbative effects and fine-tuning of superpotential parameters [64]. In this thesis, we shall employ a different strategy. If the scalar potential exactly vanishes at tree level, a non-trivial vacuum configuration can be generated by loop corrections to the Kähler potential. The effective scalar potential is then said to be of 'almost no-scale' type, and can give rise to moduli masses which are small compared to the gravitino mass.

Almost no-scale models are most easily realized within five- or six-dimensional supergravity compactified on flat orbifolds. The six-dimensional (6D) set-up gives rise to three complex moduli fields with $n_i = 1$, while in 5D there is just one single geometric modulus, corresponding to the radion superfield [65]. Moreover, these models can be viewed as intermediate steps in anisotropic orbifold compactifications of the heterotic string [66], being a top-down equivalent of local orbifold GUTs in five [12] and six dimensions [6]. For our purpose, it will be sufficient to consider simple higher-dimensional toy models, focusing on the gravity sector of the theory. In the presence of brane localized fields, the moduli Kähler potential is modified due to the bulk-brane coupling, however, the almost no-scale structure remains intact.

The non-standard form of the Kähler potential forces the real part of a Kähler modulus to couple to space-time derivatives of both its imaginary part and additional

[7]We use the term superfield interchangeably with (super-)multiplet, without making explicit reference to the superspace formulation.

brane scalar fields. The impact of these non-canonical kinetic terms on the dynamics of moduli stabilization has not yet raised much interest. It is quite common to focus on the properties of the scalar potential alone, when one discusses, *e.g.*, the cosmological moduli problems. However, having studied the cosmological evolution of complex scalar fields with non-standard kinetic terms, we find distinctive features in the phase-portrait of the corresponding dynamical models. Among the different evolutionary scenarios which can be realized subject to the moduli couplings, we highlight the existence of slow-roll solutions despite of a steep potential, and the possibility of cosmological scaling solutions, preceding the eventual stabilization of a modulus field.

In this introductory chapter we have motivated our work and laid the foundations of our analyses by providing some background material. The remainder of the thesis is organized as follows.

In chapter 2, we study 6D N=2 supergravity on a $\mathbb{Z}_2$ orbifold background. The bulk theory can be decomposed in terms of N=1 supermultiplets at an orbifold fixed point. Using the most general ansatz for the internal metric, we infer the corresponding supersymmetry transformation laws and identify the 4D supergravity multiplet. The internal degrees of freedom of the 6D supergravity sector give rise to three chiral superfields on the brane. We determine the bosonic and fermionic component fields of these 'moduli multiplets' explicitly. Both the transformation laws and the induced Lagrangean on the brane contain additional degrees of freedom, which are residues of $\mathbb{Z}_2$ odd components of the bulk spectrum. We conjecture that these terms can be understood in the framework of an off-shell description of N=1 supergravity. If this conjecture were proven to be correct, one could then construct the couplings to brane localized fields by means of off-shell methods, thereby generalizing results previously obtained via the Noether method [67]. Finally, we specify the internal space to be a flat torus orbifold, and write down the low energy effective action. The moduli Kähler potential is of no-scale form.

Chapter 3 is devoted to a model independent strategy to stabilize a no-scale Kähler modulus by means of perturbative corrections to the Kähler potential. The simplest example is provided by the radion field arising from five-dimensional supergravity compactified on an orbifolded circle. At tree level, the radion is a modulus and the size of the extra dimension remains undetermined. To avoid runaway, the radion effective potential has to vanish exactly. Only then it is possible to generate a stable non-supersymmetric Minkoswki vacuum by sub-leading corrections. We introduce a general description of the 'almost no-scale' scenario, and provide a model independent mass formula for the radion. We discuss generic contributions to the effective radion potential and analyze a specific example, in order to illustrate our strategy: In the presence of Fayet-Iliopoulos terms associated to a locally anomalous U(1), the compactification scale can be as large as the GUT scale. The radion mass turns out to be both volume and loop suppressed *wrt* the gravitino mass. Finally, we consider the possibility to generalize our findings to the 6D case with three complex moduli fields.

Cosmological constraints on the existence of light moduli are discussed in chapter 4.

We briefly review the most prominent cosmological moduli problems from the perspective of initial conditions. We distinguish two conceivable evolutionary scenarios for a light modulus with a generic Kähler and scalar potential. If the field value is initially located in the steep part of the potential, it may enter a fast-roll regime and is likely to overshoot the shallow barrier which separates the minimum from a runaway solution. If its initial value is already close to the low energy vacuum, the field starts to oscillate coherently, when the Hubble rate has become sufficiently small. Using gamma ray observations of the galactic center, we derive a bound on the initial displacement of the radion. In order to study the fast-roll scenario systematically, we implement methods of dynamical systems analysis. We classify the different dynamical models in terms of three parameters, accounting for the slope of the scalar potential, the modulus coupling to dark matter, and the non-standard kinetic coupling which typically arises from moduli Kähler potentials. In particular, we study the evolution of the radion during the radiation epoch, and find a cosmological scaling solution. Finally, we apply our general results to the interesting possibility that our universe recurrently undergoes stages of accelerated expansion, which may be related to dynamical models of dark energy.

Additional material of more technical nature, including notations, conventions and terminology, as well as intermediate calculational steps, is collected within three appendices, which are each related to one of the three main chapters. We shall separately conclude each chapter by a brief summary. Our main results are then reviewed in chapter 5.

Chapter 2

N=1 supermultiplets on a brane from 6D orbifold supergravity

The objective of this chapter is to study the decomposition of 6D N=2 supergravity in terms of N=1 degrees of freedom on a co-dimension two brane located at the position of an orbifold fixed point. This programme is a six-dimensional equivalent of the formalism presented in [68], generalized to the case of local supersymmetry. The authors of [68] constructed the N=1 couplings of chiral fields localized on a four-dimensional brane to a five-dimensional bulk gauge theory, by decomposing the bulk fields into N=1 degrees of freedom. In global supersymmetry, this decomposition is unambigiously defined everywhere in the bulk. In this respect, one couples actual N=1 bulk multiplets to N=1 brane multiplets according to the rules of N=1 super Yang-Mills theory. Moreover, in the global case, an off-shell formulation of 5D N=2 super Yang-Mills theory is known, which gives rise to an N=1 off-shell theory after decomposition. As a consequence, the authors of [68] were able to obtain bulk-brane couplings of on-shell degrees of freedom simply by integrating out auxiliary fields.

In the case of local supersymmetry, we face two major obstacles; first, a complete off-shell description of 6D N=2 supergravity is not available.[1] Second, as we shall see, a consistent decomposition of 6D N=2 supergravity into N=1 degrees of freedom is only possible at the fixed point. Due to the orbifold projection, exactly half of the bulk degrees of freedom are required to vanish at the brane position. The remaining part of the spectrum re-organizes into N=1 multiplets in four dimensions. To identify these multiplets, we decompose the N=2 local supersymmetry (SUSY) transformations of fermions in terms of fields with even orbifold parity. The resulting transformation laws are those of the N=1 gravitino and three chiral fermions emerging from the internal components of the 6D gravitino and dilatino. We identify the corresponding bosonic component fields, and rewrite the 6D bosonic bulk theory in terms of those degrees of freedom, which are non-zero at the brane position. We call the resulting object $\mathcal{L}_{\text{bulk-ind}}$, the 'bulk-induced' contribution to the brane Lagrangean. The complete brane

[1] In [69] an off-shell action is given, but without specifying the corresponding transformation laws.

Lagrangean is then given by

$$\mathcal{L}_{\text{brane}} = \mathcal{L}_{\text{bulk-ind}} + \mathcal{L}_{\text{brane-loc}} + \mathcal{L}_{\text{coupl}} \,,$$

where $\mathcal{L}_{\text{brane-loc}}$ is entirely given in terms of the brane chiral fields, and $\mathcal{L}_{\text{coupl}}$ contains their couplings to bulk fields. A set of couplings has been constructed in [67] by means of the Noether procedure. In principle, it should be possible to generalize that result using (off-shell) methods as provided by superconformal tensor calculus [69,70]. The gravitino transformation law we obtain is reminiscent of the new minimal off-shell formulation of N=1 supergravity discovered by Sohnius and West [71].[2] Based on this observation, and the globally supersymmetric analogue of [68], we conjecture that it should be possible to rewrite $\mathcal{L}_{\text{brane}}$ in terms of an N=1 off-shell formulation. We shall carry out the first steps to check this conjecture, and discuss the obstructions which we have to face. The completion of this task, if possible, is left for future work.

The chapter is organized as follows. In section 2.1, we work out the decomposition of the SUSY transformations laws. To illustrate the procedure, we start with the simple example of an Abelian vector multiplet and then discuss the gravity sector. In section 2.2, we carry out the reduction of the bulk action in terms of the even N=1 degrees of freedom. We also briefly review the results of [67] on the bulk-brane couplings. In section 2.3 we specify our set-up to a flat torus background and present the corresponding low energy effective action. Section 2.4 is devoted to our conjecture on the existence of an N=1 off-shell formulation derived from 6D N=2 supergravity. The results of this chapter are briefly summarized in section 2.5.

2.1 Decomposition of the SUSY transformation laws

Our set-up is a toy model with the supergravity $(E_M^A, \Psi_M, B_{MN}^{(+)})$ and tensor multiplet $(B_{MN}^{(-)}, \chi, \phi)$ propagating in the 6D bulk. In addition, we assume the gauge sector to be part of the bulk theory; here we shall restrict ourselves to the simplest possible case of an Abelian vector multiplet (A_M, λ). The bosonic bulk action then reads

$$S_B = \int d^4x\, d^2y\, e_6 \left\{ \frac{M^2}{2} \left(-R + \partial_M\phi\partial^M\phi\right) - \frac{e^{\phi}}{4} F_{MN}F^{MN} + \frac{e^{2\phi}}{12} G_{MNP}G^{MNP} \right\} \,, \tag{2.1.1}$$

where ϕ denotes the dilaton, and the Kalb-Ramond field strength is given by

$$G_{MNP} = \partial_M B_{NP} + \frac{1}{\sqrt{2}M} F_{MN}A_P + cycl.\,, \quad B_{NP} = B_{NP}^{(+)} + B_{NP}^{(-)}\,, \tag{2.1.2}$$

including the Chern-Simons term. The (anti-)self dual part of B_{NP} belongs to the supergravity (tensor) multiplet, respectively. The gravitational coupling constant is

[2] Interestingly, a connection between new minimal supergravity and Calabi-Yau compactifications of the heterotic string was reported and analyzed by Ovrut [72].

reparametrized as $M \equiv M_6^2$, which will later on simplify the transition between the brane Lagrangean and the 4D low energy effective action. We set the 6D gauge coupling $g_6 \equiv 1$. Below, we shall also consider chiral superfields on a brane located at the fixed point of a $\mathbb{Z}_2$ orbifold parity. We assume the brane to be embedded trivially, such that the induced metric on the brane coincides with the external block of the 6D metric. For all practical purposes, we may think of the internal space as a flat torus orbifold, however, our results also apply to more general geometries.

Negative orbifold parity is assigned to the field components $A_n, B_{\mu n}$ and $g_{\mu n}$, where Greek (Latin) indices refer to external (internal) legs. We parametrize the even parity degrees of freedom of the metric as follows,

$$g_{MN} = \begin{pmatrix} r^{-2} g_{\mu\nu} & \\ & r^2 g_{mn} \end{pmatrix} , \quad \text{where} \quad g_{mn} = -\frac{1}{\tau_2} \begin{pmatrix} 1 & \tau_1 \\ \tau_1 & \tau_1^2 + \tau_2^2 \end{pmatrix} . \tag{2.1.3}$$

leading to an Einstein frame action in 4D.[3] For the corresponding vielbeins, $g_{MN} = \eta_{AB} E_M^A E_N^B$, one may choose

$$E_M^A = \begin{pmatrix} r^{-1} e_\mu^\alpha & e_m^\alpha \\ 0 & r e_m^a \end{pmatrix} , \quad E_m^a = -\frac{1}{\sqrt{\tau_2}} \begin{pmatrix} 1 & \tau_1 \\ 0 & -\tau_2 \end{pmatrix} . \tag{2.1.4}$$

The inverse vielbeins are then given by

$$E_A^M = \begin{pmatrix} r e_\alpha^\mu & e_a^\mu \\ 0 & r^{-1} e_a^m \end{pmatrix} , \quad e_a^m = -\frac{1}{\sqrt{\tau_2}} \begin{pmatrix} \tau_2 & \tau_1 \\ 0 & -1 \end{pmatrix} . \tag{2.1.5}$$

The decomposition of the spinors can be found in appendix A.1, together with the complete set of 6D N=2 on-shell SUSY transformations.

We note that, as it stands, the set-up is inconsistent because of gravitational anomalies. These could be canceled by including a large number of hypermultiplets [73]. However, we shall not be concerned with the issue of anomaly cancellation in the following. We begin with a discussion of the gauge sector, which suits to expose and illustrate our strategy, before we address supergravity.

2.1.1 The gauge vector multiplet

We want to identify the N=1 vector multiplet that propagates on the brane located at an orbifold fixed point ($y = 0$ for simplicity). $\mathbb{Z}_2$ odd components of the 6D fields will be set to zero below, however, we shall carefully keep track of terms which contain their internal derivatives.

Consider the 6D N=2 gaugino transformation law,

$$\delta\lambda = -\frac{e^{\phi/2}}{2\sqrt{2}} F_{MN} \Gamma^{MN} \epsilon \,, \tag{2.1.6}$$

which we shall rewrite to make N=1 supersymmetry manifest,

[3] We always take $g_{\mu\nu} = g_{\mu\nu}(x)$ to depend on the external coordinates only. However, we note that our ansatz features the most general parametrization of the internal metric.

$$\delta \begin{pmatrix} \lambda_L \\ 0 \end{pmatrix} = -\frac{e^{\phi/2}}{2\sqrt{2}} F_{MN} E^M_\alpha E^N_\beta \begin{pmatrix} \gamma^{\alpha\beta} & 0 \\ 0 & \gamma^{\alpha\beta} \end{pmatrix} \begin{pmatrix} \epsilon_L \\ 0 \end{pmatrix}$$
$$- \frac{e^{\phi/2}}{4\sqrt{2}} F_{MN} E^M_a E^N_b (x^a y^b - y^a x^b) \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} \begin{pmatrix} \epsilon_L \\ 0 \end{pmatrix}, \tag{2.1.7}$$

where $x^{\dot 5} = y^{\dot 5} = i, x^{\dot 6} = 1 = -y^{\dot 6}$, cf. appendix A.1. Observing the parity assignments of the gauge field strength and vielbein components, we obtain

$$\delta\lambda_L = -\frac{e^{\phi/2}}{2\sqrt{2}} \left[F_{\mu\nu} e^\mu_\alpha e^\nu_\beta r^2 \gamma^{\alpha\beta} + \frac{1}{2} F_{\dot 5 \dot 6} \epsilon_{ab} (x^a y^b - y^a x^b) \right] \epsilon_L \,, \tag{2.1.8}$$

with

$$F_{\dot 5 \dot 6} = F_{mn} E^m_{\dot 5} E^n_{\dot 6} \,. \tag{2.1.9}$$

We define the 2D Levi-Civita tensor density ϵ_{ab} such that $\epsilon_{\dot 5 \dot 6} = 1$. Hence, (2.1.8) reduces to

$$\delta\lambda_L = -\frac{e^{\phi/2}}{2\sqrt{2}} \left[F_{\mu\nu} r^2 \gamma^{\mu\nu} - 2i F_{\dot 5 \dot 6} \right] \epsilon_L \tag{2.1.10}$$

on the brane.

The following rescaling,

$$\lambda_L \to r^{3/2} e^{\phi/2} \lambda_L \,, \tag{2.1.11}$$

renders a standard kinetic term for the 4D gaugino,

$$E_6 \bar\lambda \Gamma^M D_M \lambda \;\to\; e_4 r^{-2} \bar\lambda_L r \gamma^\mu D_\mu \lambda_L r^3 e^\phi = e_4 s \bar\lambda_L \gamma^\mu D_\mu \lambda_L \,, \tag{2.1.12}$$

where $s \equiv r^2 e^\phi$ fixes the 4D gauge coupling. Together with a compensating rescaling of the Killing spinor,

$$\epsilon_L \to \sqrt{2} r^{-1/2} \epsilon_L \,, \tag{2.1.13}$$

we recover the standard 4D expression of the N=1 gaugino transformation law in Wess-Zumino (WZ) gauge [74],

$$\delta\lambda_L = -\frac{1}{2} F_{\mu\nu} \gamma^{\mu\nu} \epsilon_L - i\mathcal{D}\epsilon_L \,, \tag{2.1.14}$$

if we identify[4] $\mathcal{D} = -r^{-2} F_{\dot 5 \dot 6}$.

As a cross check, we conclude by examining the SUSY transformation of the corresponding vector boson,

$$\delta A_M = -\frac{i e^{-\phi/2}}{\sqrt{2}} \bar\epsilon \Gamma_M \lambda \,, \tag{2.1.15}$$

and obtain, in terms of the even parity degrees of freedom,

$$\delta A_\mu = -\frac{i e^{-\phi/2}}{\sqrt{2}} \big(\sqrt{2} r^{-1/2} \bar\epsilon_L\big) r^{-1} \gamma_\mu \big(r^{3/2} e^{\phi/2} \lambda_L\big) = -i \bar\epsilon_L \gamma_\mu \lambda_L \,, \tag{2.1.16}$$

which is indeed the N=1 transformation law of a gauge field [74].

[4] The implications of this relation shall be discussed in subsection 2.4.1.

2.1.2 The N=1 supergravity multiplet

We shall now apply this procedure to the SUSY transformation law of the 6D N=2 gravitino. We start by identifying the 4D gravitino. Diagonalizing the kinetic term, we find the linear combination

$$\psi_{L\mu} = r^{-1/2} \left[\Psi_\mu + \frac{1}{2} \Gamma_\mu \Gamma^m \Psi_m \right]^+ , \tag{2.1.17}$$

where the superscript '+' means that we have to take the even parity components of the expression given on the RHS. The remaining even degrees of freedom of the internal spinor components will give rise to chiral fermions on the brane, cf. the subsequent subsection.

We decompose the gravitino transformation law

$$\delta\Psi_M = \sqrt{2}M\, D_M\epsilon - \frac{e^\phi}{24} G_{NPR}\Gamma^{NPR}\Gamma_M\epsilon \tag{2.1.18}$$

in two steps. First, we focus on the covariant derivative acting on the Killing spinor. In a second step, we shall consider the contribution from the Kalb-Ramond field strength. In terms of the even parity components of the Killing spinor, we find

$$\begin{aligned}[][D_\mu\epsilon]^+ = \partial_\mu\epsilon_L &+ \frac{1}{4}\left[\omega_\mu^{\alpha\beta} - \left(e_\mu^\alpha e^{\nu\beta} - e^{\alpha\nu} e_\mu^\beta\right)\frac{\partial_\nu r^2}{2r^2}\right]\gamma_{\alpha\beta}\epsilon_L \\ &+ \frac{1}{2}\omega_\mu^{ab}(x_a y_b - y_a x_b)\epsilon_L \,, \end{aligned} \tag{2.1.19}$$

where $\omega_\mu^{\alpha\beta}$ is the 4D Einstein frame spin connection, cf. appendix A.2, which supplements $\partial_\mu\epsilon_L$ to the 4D covariant derivative $D_\mu^{(4)}\epsilon_L$. If we now plug in ω_μ^{ab} according to (A.2.6) and rescale ϵ_L, cf. (2.1.13), we obtain

$$\begin{aligned}\sqrt{2}r^{1/2}\,[D_\mu\epsilon]^+ = 2D_\mu^{(4)}\epsilon_L &- \frac{1}{2}\left(\frac{\partial_\mu r^2}{r^2} + \frac{\partial_\nu r^2}{r^2}\gamma_\mu^{\ \nu}\right)\epsilon_L \\ &+ i\left(\frac{\partial_\mu\tau_1}{2\tau_2} - r^{-3}\, e_{\mu\alpha}\,\partial_{[5}e_{6]}^\alpha\right)\epsilon_L \,. \end{aligned} \tag{2.1.20}$$

From the second contribution to (2.1.17),

$$\frac{1}{2}\Gamma_\mu\Gamma^m D_m\epsilon = \frac{1}{4}\,\omega_m^{a\beta}\, e_c^m \Gamma_\mu\Gamma^c\Gamma_{a\beta}\,\epsilon \,, \tag{2.1.21}$$

using (A.2.8), we obtain, after some algebra,

$$\left[\frac{1}{2}\Gamma_\mu\Gamma^m D_m\epsilon\right]^+ = \frac{1}{2}\left(\frac{\partial_\nu r^2}{2r^2} + \frac{i}{r^3}\,\partial_{[5}e_{6]}^\beta\, e_{\beta\nu}\right)\gamma_\mu\gamma^\nu\epsilon_L \,. \tag{2.1.22}$$

Combining this result with (2.1.20), we finally end up with

$$
\sqrt{2}r^{1/2}\left[\left(D_\mu+\frac{1}{2}\Gamma_\mu\Gamma^m D_m\right)\epsilon\right]^+
$$
$$
=2D^{(4)}_\mu\epsilon_L+\frac{i}{2}\frac{\partial_\mu\tau_1}{\tau_2}\epsilon_L+\frac{i}{r^3}\,\partial_{[5}e_{6]}^{\beta}e_{\beta\nu}\gamma_\mu{}^\nu\epsilon_L\,. \tag{2.1.23}
$$

Let us now consider the contributions from the Kalb-Ramond field strength,

$$
\begin{aligned}
G_{NPR}\left(\Gamma^{NPR}\Gamma_\mu+\frac{1}{2}\Gamma_\mu\Gamma^m\Gamma^{NPR}\Gamma_m\right)\epsilon &= G_{\nu\pi\rho}\left(\Gamma^{\nu\pi\rho}\Gamma_\mu+\frac{1}{2}\Gamma_\mu\Gamma^m\Gamma^{\nu\pi\rho}\Gamma_m\right)\epsilon\\
&+3\,G_{n\pi\rho}\left(\Gamma^n\Gamma^{\pi\rho}\Gamma_\mu+\frac{1}{2}\Gamma_\mu\Gamma^m\Gamma^n\Gamma^{\pi\rho}\Gamma_m\right)\epsilon\\
&+3\,G_{np\rho}\left(\Gamma^\rho\Gamma^{np}\Gamma_\mu+\frac{1}{2}\Gamma_\mu\Gamma^\rho\Gamma^m\Gamma^{np}\Gamma_m\right)\epsilon\,.
\end{aligned}\tag{2.1.24}
$$

Notice that only the first and the third term lead to a non-vanishing contribution to the gravitino transformation. From the first term we get

$$
-\frac{e^\phi}{12}\left[G_{\nu\pi\rho}\Gamma^{\nu\pi\rho}{}_\mu\epsilon\right]^+=\frac{iM}{2\sqrt{2}}\frac{\partial_\mu a}{s}\,\epsilon_L\,, \tag{2.1.25}
$$

where we dualized the field strength term,

$$
G_{\nu\pi\rho}\equiv\frac{M}{\sqrt{2}}\frac{e^{-2\phi}}{r^4}\epsilon_{\nu\pi\rho\lambda}\partial^\lambda a\,. \tag{2.1.26}
$$

Now consider the third term on the RHS of (2.1.24), which yields

$$
-\frac{e^\phi}{4}\left[G_{np\mu}\Gamma^{np}\epsilon\right]^+=-\frac{e^\phi}{8}G_{ab\mu}(x^ay^b-y^ax^b)\epsilon_L\,. \tag{2.1.27}
$$

Let us define

$$
G_{ab\mu}\equiv\frac{M}{\sqrt{2}r^2}\,\epsilon_{ab}D^*_\mu b\,,\qquad \partial_\mu B_{\dot{5}\dot{6}}\equiv\frac{M}{\sqrt{2}r^2}\,\partial_\mu b\,, \tag{2.1.28}
$$

where $D^*_\mu b\equiv\partial_\mu b+B_\mu$, such that the latter incorporates any term containing internal derivatives of odd fields. Putting everything together, we then obtain the gravitino transformation law,

$$
\begin{aligned}
\delta\psi_{L\mu}=2M\,D^{(4)}_\mu\epsilon_L&+\frac{iM}{2}\left(\frac{\partial_\mu\tau_1}{\tau_2}+\frac{\partial_\mu a}{s}+\frac{D^*_\mu b}{t}\right)\epsilon_L\\
&+\frac{iM}{r^3}\,\partial_{[5}e_{6]}^{\beta}e_{\beta\nu}\gamma_\mu{}^\nu\epsilon_L\,,
\end{aligned}\tag{2.1.29}
$$

where we defined $t\equiv e^{-\phi}r^2$.

The corresponding bosonic transformation law can be deduced from the 6D vielbein variation,

$$
\delta E^A_M=-\frac{i}{\sqrt{2}M}\,\bar\epsilon\,\Gamma^A\Psi_M\,, \tag{2.1.30}
$$

if we observe that the 4D Einstein frame vielbein can be written as

$$e_\mu^\alpha = \sqrt{-\det E_m^a}\, E_\mu^\alpha \,. \tag{2.1.31}$$

Using this, and employing the rescalings of $\bar{\epsilon}_L$ and $\psi_{L\mu}$, we obtain

$$\begin{aligned} \delta e_\mu^\alpha &= -\frac{ir}{\sqrt{2}M}\left[\bar{\epsilon}\left(\frac{1}{2}E_\mu^\alpha(\Gamma^\mu\Gamma_\mu)E_a^m\Gamma^a\Psi_m + \Gamma^\alpha\Psi_\mu\right)\right]^+ \\ &= -\frac{i}{M}\bar{\epsilon}_L\gamma^\alpha\psi_{L\mu} \,. \end{aligned} \tag{2.1.32}$$

Hence, we confirmed that $\psi_{L\mu}$ is indeed the fermionic partner of the 4D Einstein frame vielbein.

Due to the presence of internal derivative terms, (2.1.29) is not as easily identified with a template N=1 transformation law, as the gaugino variation was. We can either consider an on-shell template, cf. [59, 75],

$$\delta\psi_{L\mu} = 2M\, D_\mu^{(4)}\epsilon_L - \frac{1}{2M}\big(K_i\partial_\mu\Phi^i - K_{\bar{j}}\partial_\mu\Phi^{\bar{j}}\big)\epsilon_L \,, \tag{2.1.33}$$

where the Φ^i are scalar components of chiral superfields, or try to identify (2.1.29) with an off-shell transformation law, following the example of the gaugino. In the first case, we would obviously have to truncate the off-diagonal part of the vielbein, in order to get rid of the contribution $\sim \gamma_\mu{}^\nu\epsilon_L$. Such a truncation would then correspond to dimensional reduction on a flat torus background with a block-diagonal metric. We shall be following this path in section 2.3. On the other hand, the off-shell option will be further discussed in section 2.4. Beforehand, we return to the 6D N=2 gravitino transformation law.

2.1.3 Chiral multiplets from the 6D supergravity sector

The internal metric degrees of freedom give rise to scalar fields in 4D. We intend to identify the corresponding fermions, which we expect to emerge from the internal components of the 6D gravitino. As a guideline to find the correct linear combinations, we shall first vary the two scalars s and t, which appeared in the gravitino transformation law. Let us start with

$$\delta r^2 = \frac{ir^2}{\sqrt{2}M}\left[\bar{\epsilon}\,\Gamma^a\Psi_a\right]^+ = M^{-1}r^{3/2}\bar{\epsilon}_L(\Psi_{R\dot{5}} - i\Psi_{R\dot{6}}) \,. \tag{2.1.34}$$

Combining this result with the dilaton transformation rule,

$$\delta\phi = \frac{1}{\sqrt{2}M}\bar{\epsilon}\chi \,, \tag{2.1.35}$$

we find

$$\delta s = M^{-1}e^{\phi}r^{3/2}\bar{\epsilon}_L\big[(\Psi_{R\dot{5}} - i\Psi_{R\dot{6}}) + \chi_R\big] \,, \tag{2.1.36}$$

$$\delta t = M^{-1}e^{-\phi}r^{3/2}\bar{\epsilon}_L\big[(\Psi_{R\dot{5}} - i\Psi_{R\dot{6}}) - \chi_R\big] \,. \tag{2.1.37}$$

In addition, we may consider

$$\delta\tau_2 = -\delta\left[r^2(E_{\dot 5}^{5})^{-2}\right] = -\frac{\tau_2}{r^2}\delta r^2 + \frac{2}{M}\frac{\tau_2^{3/2}}{r}\frac{i}{\sqrt{2}}\left[\bar\epsilon\Gamma^{\dot 5}\Psi_a E_{5}^{a}\right]^{+}$$
$$= M^{-1}\tau_2 r^{-1/2}\bar\epsilon_L(\Psi_{R\dot 5} + i\Psi_{R\dot 6})\,. \tag{2.1.38}$$

We are thus led to define

$$\psi_U = \tau_2 r^{-1/2}(\Psi_{R\dot 5} + i\Psi_{R\dot 6})\,, \tag{2.1.39}$$
$$\psi_T = t r^{-1/2}\left[(\Psi_{R\dot 5} - i\Psi_{R\dot 6}) - \chi_R\right]\,, \tag{2.1.40}$$
$$\psi_S = s r^{-1/2}\left[(\Psi_{R\dot 5} - i\Psi_{R\dot 6}) + \chi_R\right]\,. \tag{2.1.41}$$

Hence, we have to compute

$$\delta(\Psi_{R\dot 5} \pm i\Psi_{R\dot 6}) = (E_{\dot 5}^{m} \pm iE_{\dot 6}^{m})\left[\frac{M}{\sqrt{2}}\,\omega_m^{a\beta}\Gamma_{a\beta}\epsilon - \frac{e^{\phi}}{24}(G_{\nu\pi\rho}\Gamma^{\nu\pi\rho} + 3G_{np\rho}\Gamma^{np}\Gamma^{\rho})\Gamma_m\epsilon\right]\,, \tag{2.1.42}$$

which, as usual, yields the transformation laws up to fermion bilinears.

Since $(e_{\dot 5}^{m} + ie_{\dot 6}^{m})e_m^a y_a = 0$, we observe that $\delta\psi_U$ does not receive any contribution from the Kalb-Ramond field strength. Furthermore, inserting the spin connection, we find that only $\tilde\omega_m^{a\beta}$ (cf. (A.2.8) and (A.2.10)–(A.2.13)) leads to a non-vanishing term in the total variation,

$$\delta\psi_U = \frac{\tau_2 M}{r^2}\big(e_{\dot 5}^{m} + ie_{\dot 6}^{m}\big)\tilde\omega_m^{a\beta}y_a\gamma_\beta\epsilon_L = -iM\partial_\nu\tau\,\gamma^\nu\epsilon_L\,. \tag{2.1.43}$$

On the other hand, after rescaling the Killing spinor, we obtain

$$\delta(\Psi_{R\dot 5} - i\Psi_{R\dot 6}) = -\frac{e^{\phi}}{12}\left[r^3 G_{\nu\pi\rho}\gamma^{\nu\pi\rho} - \frac{3}{2}G_{ab\rho}(x^a y^b - y^a x^b)\gamma^\rho\right]\frac{\sqrt{2}}{r^{1/2}}\epsilon_L - \frac{2M}{r^{1/2}}\left[i\partial_\mu r - \frac{1}{r^3}\partial_{[5}e_{6]}^{\beta}e_{\beta\mu}\right]\gamma^\mu\epsilon_L\,, \tag{2.1.44}$$

to be combined with the variation of χ_R. From the 6D dilatino transformation law,

$$\delta\chi = -\frac{iM}{\sqrt{2}}\,\partial_M\phi\Gamma^M\epsilon - \frac{ie^{\phi}}{12}G_{NPR}\Gamma^{NPR}\epsilon\,, \tag{2.1.45}$$

we get

$$\delta\chi_R = -\frac{e^{\phi}}{12}\left[r^3 G_{\nu\pi\rho}\gamma^{\nu\pi\rho} + \frac{3}{2}G_{ab\rho}(x^a y^b - y^a x^b)\gamma^\rho\right]\frac{\sqrt{2}}{r^{1/2}}\epsilon_L - iMr^{1/2}\partial_\mu\gamma^\mu\epsilon_L\,. \tag{2.1.46}$$

If we now put everything together, employing the dualization (2.1.26) and our definition (2.1.28), we finally arrive at

$$\delta\psi_T = -iM\left[\partial_\mu t + iD^*_\mu b + \frac{2it}{r^3}\partial_{[5}e^\beta_{6]}e_{\beta\mu}\right]\gamma^\mu\epsilon_L\,, \tag{2.1.47}$$

$$\delta\psi_S = -iM\left[\partial_\mu s + i\partial_\mu a + \frac{2is}{r^3}\partial_{[5}e^\beta_{6]}e_{\beta\mu}\right]\gamma^\mu\epsilon_L\,. \tag{2.1.48}$$

We note that the transformation laws of ψ_S and ψ_T do not exactly coincide with those of N=1 chiral fermions, which again is a consequence of the internal derivative terms. As in the gravitino case, we have the choice to either resort to additional truncations, or to appropriatly redefine the scalar kinetic terms, in the context of a would-be off-shell formulation. We shall reconsider this issue in section 2.4.

In summary, we identified three chiral multiplets, which add up with the 4D gravity multiplet to exactly half the degrees of freedom of the 6D N=2 supergravity and tensor multiplet. The remaining half is projected out by the orbifold condition.

2.2 The bosonic brane Lagrangean

So far we identified the N=1 supergravity multiplet, three chiral superfields and an Abelian vector multiplet as part of the 4D theory. Now we have to reduce the bosonic bulk action in terms of these degrees of freedom, in order to infer the bulk-induced part of the brane Lagrangean. The only non-trivial part is due to the Ricci scalar,

$$\begin{aligned} E_6R_6\Big|_{\text{brane}} = e_4\Big\{ & R_4 - \frac{1}{r^4}\partial^\rho r^2\partial_\rho r^2 - \frac{1}{2\tau_2^2}g^{\mu\nu}(\partial_\mu\tau_1\partial_\nu\tau_1 + \partial_\mu\tau_2\partial_\nu\tau_2) \\ & + \frac{1}{r^6}g^{ms}g^{nr}\partial_{[m}e_{n]\gamma}\partial_{[r}e^\gamma_{s]} - \frac{1}{r}e^\mu_\alpha\partial_\mu\left(r^{-2}g^{mn}\right)\partial_m e^\alpha_n \\ & + r^{-2}g^{mn}\left(3\frac{\partial_m\partial_n r^2}{r^2} + \frac{\partial_m\partial_n\tau_2}{\tau_2}\right) \\ & + \frac{2}{r^2\tau_2}\left(\tau_2\partial_5\partial_5\tau_2 + \tau_1\partial_5\partial_5\tau_1 - \partial_5\partial_6\tau_1\right)\Big\}\,. \end{aligned} \tag{2.2.1}$$

Details of the computation can be found in appendix A.2.

At the orbifold fixed point, the gauge kinetic term reduces as follows,

$$-\frac{E_6}{4}\,e^\phi F_{MN}F^{MN}\Big|_{\text{brane}} = -\frac{e_4}{4}\left[s\,F_{\mu\nu}F^{\mu\nu} - \frac{2s}{r^4}F^2_{\dot{5}\dot{6}}\right], \tag{2.2.2}$$

From the Kalb-Ramond term[5] we get

[5] Strictly speaking, we are not allowed to dualize the external field strength in this naive way, treating G_{MNP} as the proper physical degree of freedom. The correct procedure would be to impose its Bianchi identity by means of a Lagrange multiplier; integrating out the field strength term then gives rise to the typical coupling of the axion a to the gauge kinetic term, which is due to the presence of the Chern-Simons term. However, the brane Lagrangean we consider here is not the integrand of a complete action, hence we cannot apply the standard method. For the time being, we simply neglect the missing coupling term; in the following section, we shall then introduce it by hand.

$$
\frac{E_6}{12}\, e^{2\phi} G_{MNP} G^{MNP}\Big|_{\text{brane}} = \frac{e_4}{12}\left[r^4 e^{2\phi} G_{\mu\nu\rho} G^{\mu\nu\rho} + 3e^{2\phi} G_{\mu ab} G^{\mu ab}\right]
$$
$$
= e_4\,\frac{M^2}{4}\left[s^{-2}\partial_\mu a \partial^\mu a + t^{-2} D^*_\mu b\, D^{*\mu} b\right] . \tag{2.2.3}
$$

Finally, putting everything together, we obtain

$$
\begin{aligned}
\mathcal{L}_{\text{bulk-ind}} = e_4 \Big\{ & -\frac{M^2}{2}(R_4 + R_{\text{res}}) - \frac{s}{4} F_{\mu\nu} F^{\mu\nu} + \frac{s}{2r^4} F^2_{\dot{5}\dot{6}} \\
& + \frac{M^2}{4t^2}\partial_\mu t \partial^\mu t + \frac{M^2}{4s^2}\partial_\mu s \partial^\mu s + \frac{M^2}{4\tau_2^2}\partial_\mu \tau \partial^\mu \bar{\tau} \\
& + \frac{M^2}{4t^2} D^*_\mu b D^{*\mu} b + \frac{M^2}{4s^2}\partial_\mu a \partial^\mu a + \frac{M^2}{r^6}\partial_{[5} e^\alpha_{6]} \partial_{[5} e_{6]\alpha} \Big\} .
\end{aligned} \tag{2.2.4}
$$

Here we defined

$$
\begin{aligned}
R_{\text{res}} \equiv\, & r^{-2} g^{mn}\left[3\,\frac{\partial_m \partial_n r^2}{r^2} + \frac{\partial_m \partial_n \tau_2}{\tau_2} + e_4^{-1}\partial_\mu\left(\frac{e_4}{r} e^\mu_\alpha \partial_m e^\alpha_n\right)\right] \\
& + \frac{2}{r^2\tau_2}\left(\tau_2 \partial_5 \partial_5 \tau_2 + \tau_1 \partial_5 \partial_5 \tau_1 - \partial_5 \partial_6 \tau_1\right) ,
\end{aligned} \tag{2.2.5}
$$

collecting the terms which do not fall in any of the N=1 multiplets we obtained by decomposing the SUSY transformations. Apart from this apparent remnant of the higher dimensional theory, the Lagrangean supposedly contains auxiliary degrees of freedom, i.e. fields lacking a brane localized kinetic term. Recalling the gaugino transformation law (2.1.14), we might be tempted to identify $r^{-4} F^2_{\dot{5}\dot{6}} \equiv \mathcal{D}^2$ as the auxiliary sector of the gauge theory, however, the term does not transform properly. Similarly, introducing a short-hand notation,

$$
V_\mu \equiv \frac{M}{r^3}\partial_{[5} e^\alpha_{6]} e_{\alpha\mu} \tag{2.2.6}
$$

is part of the gravitino transformation law, without being a physical degree of freedom on the brane. We shall discuss the implications of these terms further below, in section 2.4.

Let us now introduce complex scalar fields, and recall the modulini transformations (2.1.47) and (2.1.48), which imply the following redefinitions

$$
D_\mu S \equiv M\left[\partial_\mu s + i\left(\partial_\mu a + 2sr^{-3}\partial_{[5} e^\alpha_{6]} e_{\alpha\mu}\right)\right] , \tag{2.2.7}
$$
$$
D_\mu T \equiv M\left[\partial_\mu t + i\left(D^*_\mu b + 2tr^{-3}\partial_{[5} e^\alpha_{6]} e_{\alpha\mu}\right)\right] , \tag{2.2.8}
$$
$$
\partial_\mu U \equiv M\,\partial_\mu \tau . \tag{2.2.9}
$$

If we now replace the scalar kinetic terms within (2.2.4) using these redefinitions, the scalar kinetic Lagrangean takes the form

$$
\begin{aligned}
e_4^{-1}\mathcal{L}^{\text{scalar}}_{\text{kin}} = & \frac{1}{4\tau_2^2}\partial_\mu U \partial^\mu \bar{U} + \frac{1}{4t^2} D_\mu T D^\mu \bar{T} + \frac{1}{4s^2} D_\mu S D^\mu \bar{S} \\
& - M V^\mu\left(\frac{B_\mu}{t} + \frac{\partial_\mu b}{t} + \frac{\partial_\mu a}{s}\right) - V^\mu V_\mu ,
\end{aligned} \tag{2.2.10}
$$

where we included the term $V^\mu V_\mu$, which was already present in (2.2.4).

2.2.1 Coupling to chiral brane fields

We will now proceed and introduce a set of chiral supermultiplets on the brane, with scalar components Q^i. For simplicity, we take them to be uncharged under the bulk U(1) gauge theory, and focus on the couplings to the bulk gravity sector.

We start with a general ansatz for the corresponding kinetic Lagrangean,

$$\mathcal{L}^Q_{\text{kin}} \supset E_4 G^{\mu\nu}\Omega_{i\bar{j}}\partial_\mu Q^i \partial_\nu \bar{Q}^{\bar{j}} = e_4 r^{-2} g^{\mu\nu}\Omega_{i\bar{j}}\partial_\mu Q^i \partial_\nu \bar{Q}^{\bar{j}}\,, \tag{2.2.11}$$

where $G^{\mu\nu}$ is the external block of the 6D metric, and the function Ω depends on the $Q^i, \bar{Q}^{\bar{j}}$ only. The apparent coupling to r^{-2} results from the fact that the induced metric on the brane is not $g^{\mu\nu}$. From the SUSY transformation laws we know that r^2 by itself is not the lowest component of an N=1 chiral multiplet, hence we can deduce a coupling between the chiral brane fields to either T or S (or both). Since we assumed the brane to be transversal *wrt* the internal space, the chiral brane fields cannot couple to τ, since this would require a contact term with the internal block of the 6D metric. The correct kinetic coupling was constructed in [67] using the Noether method,

$$\begin{aligned} e_4^{-1}\mathcal{L}^{T,Q}_{\text{kin}} = {}& \frac{M^2}{4t^2}\left[\partial_\mu t \partial^\mu t + \left(\partial_\mu b + B_\mu - \frac{i}{M}(\bar{Q}\partial_\mu Q - Q\partial_\mu \bar{Q})\right)^2\right] \\ & + \frac{1}{t}\partial_\mu Q \partial^\mu \bar{Q}\,, \end{aligned} \tag{2.2.12}$$

which is consistent with a redefinition of $G_{\mu ab}$ in the presence of brane localized fields. The generalization to several chiral brane fields is straightforward,

$$\begin{aligned} e_4^{-1}\mathcal{L}^{T,Q}_{\text{kin}} = {}& \frac{M^2}{4t^2}\left[\partial_\mu t \partial^\mu t + \left(\partial_\mu b + B_\mu + 2tV_\mu - \frac{i}{M}(\Omega_i\partial_\mu Q^i - \Omega_{\bar{j}}\partial_\mu \bar{Q}^{\bar{i}})\right)^2\right] \\ & + \frac{\Omega_{i\bar{j}}}{t}\partial_\mu Q^i \partial^\mu \bar{Q}^{\bar{j}}\,, \end{aligned} \tag{2.2.13}$$

where we now also took account of the modification we infered from the SUSY transformation law. We are now ready to put everything together, and find the complete brane Lagrangean to be

$$\begin{aligned} \mathcal{L}_{\text{brane}} &= \mathcal{L}_{\text{bulk-ind}} + \mathcal{L}_{\text{brane-loc}} + \mathcal{L}_{\text{coupl}} \\ &= e_4\left\{-\frac{M^2}{2}(R_4 + R_{\text{res}}) - V^\mu V_\mu + \frac{s}{2r^4}F^2_{\dot{5}\dot{6}} - \frac{s}{4}F_{\mu\nu}F^{\mu\nu}\right. \\ &\quad + \frac{M^2}{4t^2}\left[\partial_\mu t \partial^\mu t + \left(\partial_\mu b + B_\mu + 2tV_\mu - \frac{i}{M}(\Omega_i\partial_\mu Q^i - \Omega_{\bar{j}}\partial_\mu \bar{Q}^{\bar{i}})\right)^2\right] \\ &\quad \left. + \frac{\Omega_{i\bar{j}}}{t}\partial_\mu Q^i \partial^\mu \bar{Q}^{\bar{j}} + \frac{1}{4\tau_2^2}\partial_\mu U \partial^\mu \bar{U} + \frac{1}{4s^2}D_\mu S D^\mu \bar{S}\right\}\,. \end{aligned} \tag{2.2.14}$$

Notice that the N=1 local supersymmetry of this Lagrangean can be made manifest by means of further truncations,

$$R_{\text{res}} = 0\,, \qquad V_\mu = 0\,, \qquad F_{\dot{5}\dot{6}} = 0\,, \tag{2.2.15}$$

which would correspond to dimensional reduction on a specific background geometry. In the next section, we shall proceed exactly along these lines, specifying to a flat torus orbifold and then carrying out the compactification. Thereafter, we will investigate the alternative possibility, namely to complement the brane Lagrangean in order to make contact with an off-shell formulation.

2.3 Torus compactification and low energy effective action

We shall now specify the internal space to be an orbifolded torus, $\mathcal{T}^2/\mathbb{Z}_2$. In this case, it is straightforward to derive the low energy effective action. Only degrees of freedom with even orbifold parity are allowed to have zero modes on the torus, and their massive Kaluza-Klein modes can be integrated out trivially. The 6D metric is then block-diagonal and a function of the external coordinates only,

$$ds^2 = r^{-2}(x)g_{\mu\nu}(x)dx^\mu dx^\nu + r^2(x)g_{mn}(x)dx^m dx^n \,, \tag{2.3.1}$$

which is a considerable simplification compared to our former set-up, in particular leading to

$$R_{\text{res}} = 0 \,, \qquad V_\mu = 0 \,. \tag{2.3.2}$$

However, the odd components of the Kalb-Ramond two-form couple to the brane chiral fields, leading to non-trivial localized sources for the bulk equations of motion. These equations are solved by a background value [67], which is constant *wrt* the internal coordinates y,[6]

$$\hat{G}_{\mu 56} = \partial_\mu B_{56} + \frac{i}{\sqrt{2}L^2 M}(\Omega_i \partial_\mu Q^i - \Omega_{\bar{j}} \partial_\mu \bar{Q}^{\bar{j}}) \,. \tag{2.3.3}$$

In (2.3.3) L^2 is the area of the fundamental domain of the torus, and

$$\hat{G}_{\mu 56} \equiv G_{\mu 56} - \delta(y)\frac{i}{\sqrt{2}}(\Omega_i \partial_\mu Q^i - \Omega_{\bar{j}} \partial_\mu \bar{Q}^{\bar{j}}) \tag{2.3.4}$$

the redefined Kalb-Ramond field strength [67] up to fermion bilinears. Let us briefly comment on the relevant mass scales in the problem. After compactification, the 4D Einstein frame Planck mass is

$$M_4^2 = M_6^4 L^2 \,, \tag{2.3.5}$$

where we define L such that the vacuum value of the extra-dimensional scale factor is $r_0 = 1$, provided all moduli are stabilized. Here we benefit from our sloppy notation in

[6] Here we only consider the case without a non-zero flux background, $\langle F_{\dot{5}\dot{6}} \rangle = 0$. See, however, [76] concerning the general situation.

(2.1.1), because we can simply rescale $M \to L^2 M$ when we make the transition from the bulk-induced brane Lagrangean to the low energy effective action,[7]

$$S_B = \int d^4x\, e_4 \Big\{ -\frac{M^2}{2} R_4 - \frac{s}{4} F_{\mu\nu}F^{\mu\nu} + \frac{ia}{4} F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{1}{4s^2}\partial_\mu S \partial^\mu \bar{S}$$
$$+ \frac{1}{4\tau_2^2}\partial_\mu U \partial^\mu \bar{U} + \frac{M^2}{4t^2}\partial_\mu t \partial^\mu t + \frac{\Omega_{i\bar{j}}}{t}\partial_\mu Q^i \partial^\mu \bar{Q}^{\bar{j}}$$
$$+ \frac{1}{4t^2}\Big[M\,\partial_\mu b + iM^{-1}(\Omega_i \partial_\mu Q^i - \Omega_{\bar{j}}\partial_\mu \bar{Q}^{\bar{j}})\Big]^2 \Big\}\,, \tag{2.3.6}$$

where now $M = M_4$. According to [67], the scalar kinetic terms are reproduced by the following Kähler potential,

$$M^{-2}K = -\ln\left(\frac{U+\bar{U}}{2}\right) - \ln\left(\frac{S+\bar{S}}{2}\right) - \ln\left(\frac{T+\bar{T}}{2} - \frac{\Omega(Q^i,\bar{Q}^{\bar{j}})}{M^2}\right)\,, \tag{2.3.7}$$

provided a redefinition of T,

$$T \equiv t + \frac{\Omega(Q^i,\bar{Q}^{\bar{j}})}{M^2} + ib\,. \tag{2.3.8}$$

We already know that S and U are indeed the scalar components of chiral superfields, which we denote by the same capital letters in a harmless abuse of notation. It remains to be shown that the same holds for the T multiplet, taking account of the necessary redefinitions. We shall first consider the kinetic part of the action. The relevant entries of the Kähler metric are given by

$$K_{T\bar{T}} = \frac{1}{4t^2}\,, \tag{2.3.9}$$
$$K_{T\bar{j}} = -\frac{\Omega_{\bar{j}}}{2Mt^2}\,, \tag{2.3.10}$$
$$K_{i\bar{T}} = -\frac{\Omega_i}{2Mt^2}\,, \tag{2.3.11}$$
$$K_{i\bar{j}} = \frac{\Omega_{i\bar{j}}}{t} - \frac{\Omega_i\Omega_{\bar{j}}}{M^2t^2} \equiv K^I_{i\bar{j}} + K^{II}_{i\bar{j}}\,. \tag{2.3.12}$$

The corresponding part of the scalar kinetic Lagrangean then reads

$$e_4^{-1}\mathcal{L}^{T,Q}_{\rm kin} = M^2 K_{T\bar{T}}\,(\partial_\mu t\partial^\mu t + \partial_\mu b \partial^\mu b) + K^I_{i\bar{j}}\partial_\mu Q^i \partial^\mu \bar{Q}^{\bar{j}}$$
$$+ iM(K_{T\bar{j}}\partial_\mu \bar{Q}^{\bar{j}} - K_{i\bar{T}}\partial_\mu Q^i)\partial^\mu b$$
$$+ \frac{1}{M^2}\left[\left(K_{T\bar{T}} + \frac{M}{\Omega_{\bar{j}}}K_{T\bar{j}}\right)(\Omega_{\bar{j}}\partial_\mu \bar{Q}^{\bar{j}})^2 + \left(K_{T\bar{T}} + \frac{M}{\Omega_i}K_{i\bar{T}}\right)(\Omega_i \partial^\mu Q^i)^2\right]$$
$$+ \left[\frac{2\Omega_i\Omega_{\bar{j}}}{M^2}K_{T\bar{T}} + \frac{\Omega_{\bar{j}}}{M}K_{i\bar{T}} + \frac{\Omega_i}{M}K_{T\bar{j}} + K^{II}_{i\bar{j}}\right]\partial_\mu \bar{Q}^{\bar{j}}\partial^\mu Q\,, \tag{2.3.13}$$

which indeed coincides with the terms given in (2.3.6).

[7] Here we include the coupling term $\sim aF\tilde{F}$, which was neglected in the brane Lagrangean.

Now for the SUSY transformation laws. Let us, for simplicity, focus on the case with a single brane superfield, and begin with the scalar component,

$$\begin{aligned}\delta T &= \delta(t+ib) + M^{-2}[\bar{Q}\delta Q + Q\delta\bar{Q}] \\ &= \delta(t+ib) + M^{-2}\left[\bar{Q}\psi_Q\bar{\epsilon}_L + Q\bar{\psi}_{\bar{Q}}\epsilon_L\right] .\end{aligned} \tag{2.3.14}$$

We redefine ψ_T such that

$$\begin{aligned}\delta\psi_T^{\text{new}} &= \delta\left[\psi_T + \frac{2\bar{Q}}{M}\psi_Q\right] \\ &= \delta\psi_T^{\text{old}} + \frac{i}{M}(\bar{Q}\partial_\mu Q - Q\partial_\mu\bar{Q})\gamma^\mu\epsilon_L - \frac{2i\bar{Q}}{M}\,\partial_\mu Q\,\gamma^\mu\epsilon_L \\ &= \delta\psi_T^{\text{old}} - \frac{i}{M}\partial_\mu|Q|^2\gamma^\mu\epsilon_L .\end{aligned} \tag{2.3.15}$$

The second term in the second line emerges from the redefinition of $G_{\mu ab}$. These two transformation laws can be reconciled by a modification of

$$\delta b \to \delta b - iM^{-2}\left[\bar{Q}\psi_Q\bar{\epsilon}_L - Q\bar{\psi}_{\bar{Q}}\epsilon_L\right] , \tag{2.3.16}$$

as proposed by [67].

In the presence of a brane localized superpotential $W(Q^i)$, the gravitino transformation law picks up an additional contribution [59, 75],

$$\begin{aligned}\delta\psi_{L\mu} &= 2M\,D_\mu^{(4)}\epsilon_L - \frac{1}{2M}\big(K_i\partial_\mu\Phi^i - K_{\bar{j}}\partial_\mu\Phi^{\bar{j}}\big)\epsilon_L - e^{K/(2M)}W\gamma_\mu\epsilon_L \\ &= 2M\,D_\mu^{(4)}\epsilon_L + \frac{iM}{2}\left(\frac{\partial_\mu a}{s} + \frac{\partial_\mu b}{t} + \frac{\partial_\mu\tau_1}{\tau_2}\right) - \frac{1}{2Mt}\big(\Omega_i\partial_\mu Q^i - \Omega_{\bar{j}}\partial_\mu\bar{Q}^{\bar{j}}\big) \\ &\quad - e^{K/(2M)}\,\frac{W(Q^i)}{M}\,\gamma_\mu\epsilon_L .\end{aligned} \tag{2.3.17}$$

The scalar F term potential is then given by

$$V_F = \frac{1}{st\tau_2}K^{i\bar{j}}W_i\bar{W}_{\bar{j}} , \tag{2.3.18}$$

where

$$W_i = \frac{\partial W}{\partial Q^i} . \tag{2.3.19}$$

Notice that, in contrast to the general case of N=1 supergravity, the negative definite contribution $\sim -3e^{K/M}|W|^2$ is absent, due to the special structure of the Kähler potential (2.3.7). Namely, it obeys the relation

$$K^iK_i = 3M^2 , \tag{2.3.20}$$

which characterizes so-called *no-scale* supergravity models [61]. In the next chapter, we shall consider the implications of this property on the issue of moduli stabilization. However, beforehand we shall conclude this chapter with a discussion of the non-standard contributions to the brane Lagrangean and SUSY transformations we obtained by decomposing the bulk theory.

2.4 Outlook: the issue of auxiliary fields

The contents of this section are rather speculative, and beyond the scope of the approach we have been following during the preceding parts of this chapter. However, it is tempting to ask, whether the complete bulk-brane action of our set-up could be re-organized in terms of the N=1 degrees of freedom we identified by decomposing the transformation laws. N=1 supersymmetry is a subset of the full symmetries of the bulk theory after all. The residual degrees of freedom, which do not fall into N=1 multiplets by themselves, might then be interpreted as auxiliary fields. In the case of global SUSY, it is known that the field content of higher-dimensional theories can be decomposed in terms of N=1 multiplets [77], prior to dimensional reduction and without even introducing an orbifold projection. In the 5D case, it was also attempted to include a radion superfield within a globally supersymmetric bulk action [65].

From the perspective of the low energy effective action, we may rephrase the problem as follows. We have identified the part of the bulk spectrum, which gives rise to zero modes on a flat torus orbifold. The zero modes fall into N=1 multiplets, and the low energy effective action can be consistently formulated as N=1 supergravity coupled to chiral and gauge fields. This decomposition, however, does not hold at the level of massive Kaluza-Klein states, since the corresponding massive multiplets contain twice as much degrees of freedom, and remix even and odd components of the bulk fields. By truncating the bulk theory to exactly those degrees of freedom that do have zero modes, we might hope to obtain a 'lift' of the low energy effective action back to 6D, following the spirit of [65]. Alternatively, we may start from the brane Lagrangean, and try to find a continuation of our field decomposition into the bulk, in order to obtain a complete action in terms of the brane degrees of freedom.

It is, however, easy to see that the decomposition of the supergravity and tensor multiplets we worked out at the brane position is, in general, invalid away from the fixed point. The linear combinations ψ_S, ψ_T and ψ_U, which we may call modulini collectively, do no longer transform as chiral fermions, since the variation of Ψ_{Rm} picks up a contribution from $\partial_m \epsilon_L$. Hence, we have to impose

$$\epsilon = \begin{pmatrix} \epsilon_L(x) \\ 0 \end{pmatrix} \tag{2.4.1}$$

globally.[8] Moreover, there are constraints on the physical fields to be observed from the SUSY transformations, in order to keep the truncation consistent. Consider, *e.g.*, the transformation properties of $\Psi_{R\mu}$ and Ψ_{Lm} (cf. (A.1.5) and (A.1.12)), which belong to the truncated part of the spectrum. Since they vary, amongst other things, into internal derivatives of the internal metric degrees of freedom, we seem to be forced to impose $G_{mn} = G_{mn}(x)$, in order to keep the resulting action invariant. We also note that we could not apply to τ the mode expansion for a 6D scalar field. Let us take a block-diagonal metric ansatz, where τ_1 and τ_2 are allowed to depend on both external and

[8] This corresponds to the fact, that N=1 Poincaré supergravity can only be formulated in 4D.

internal coordinates. Inserting this ansatz, we obtain from the Ricci scalar,

$$\mathcal{L}(\tau_1,\tau_2) \supset \frac{1}{r^2\tau_2}\left[-2\left(\partial_5\tau_1 - \frac{\tau_1}{\tau_2}\,\partial_5\tau_2\right)^2 - \frac{3}{\tau_2}\partial_5\tau_1\partial_6\tau_2 - \frac{1}{\tau_2}\partial_6\tau_1\partial_5\tau_2 \right.$$
$$\left. + \frac{4\tau_1}{\tau_2^2}\partial_5\tau_2\partial_6\tau_2 - 2\left(\frac{\partial_6\tau_2}{\tau_2}\right)^2\right], \tag{2.4.2}$$

which is obviously not of the form $g^{mn}\partial_m\tau\partial_n\bar{\tau}$.

Hence, it seems pointless to write down a 6D supergravity action in terms of N=1 degrees of freedom. Such an action would be a completely trivial embedding of 4D N=1 supergravity, because the fields of the supergravity sector have to be constant *wrt* the internal coordinates y, in order to allow for a continuation of the N=1 SUSY transformation laws into the bulk. However, we may also take the opportunity to keep and re-interpret the internal derivative terms, which appeared in the SUSY transformations on the brane, in order to investigate the possibility of a consistent off-shell formulation. Suppose we were able to identify some part of the truncated sector with the auxiliary fields of N=1 supergravity. In this case, one could incorporate at least a bit of information about the bulk theory within the N=1 formulation in terms of even parity states; this might enable us to construct the bulk-brane couplings using off-shell methods instead of the Noether procedure. In order to motivate this approach, we shall reconsider our results on the vector multiplet.

2.4.1 Revisiting the vector multiplet

Recall the SUSY transformation law of the gaugino,

$$\begin{aligned}\delta\lambda_L &= -\frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon_L + ir^{-2}F_{\dot{5}\dot{6}}\epsilon_L \\ &\equiv -\frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon_L - i\mathcal{D}\epsilon_L\,,\end{aligned} \tag{2.4.3}$$

and the reduced Lagrangean,

$$\begin{aligned}\mathcal{L}_{\text{gauge}} &= -\frac{e_4}{4}\left[s\,F_{\mu\nu}F^{\mu\nu} - \frac{2s}{r^4}F_{\dot{5}\dot{6}}^2\right] \\ &\equiv e_4\left[-\frac{s}{4}\,F_{\mu\nu}F^{\mu\nu} + \frac{s}{2}\mathcal{D}^2\right],\end{aligned} \tag{2.4.4}$$

where we identified $-r^{-2}F_{\dot{5}\dot{6}}$ with the N=1 auxiliary field $\mathcal{D}$, although it does not transform correctly. $F_{\dot{5}\dot{6}}$ varies into internal derivatives of an odd fermion, λ_R, which we intended to truncate from the 6D action. However, in global supersymmetry, where an off-shell description of the higher-dimensional Yang-Mills theory is known, the N=1 auxiliary field can be defined unambigiously.

Let us consider the five-dimensional analogue, which is extensively discussed in the literature [68, 78]. Here the auxiliary field of the N=1 gauge vector multiplet (in WZ gauge) is given by

$$\mathcal{D} = X^3 - D_5\Sigma\,, \tag{2.4.5}$$

where X^3 originates from the triplet of N=2 auxiliary fields in 5D, and $D_5\Sigma$ is the internal derivative of the scalar component field of the 5D N=2 vector multiplet and, hence, the analogue of our $F_{\dot{5}\dot{6}}$. The expression on the RHS exactly reproduces the transformation law of an auxiliary $\mathcal{D}$ field, since the terms $\sim \lambda_R$ emerging from the variation of Σ are precisely canceled by (part of) the transformation of X^3.

Now let us assume that we did instead start from the N=2 on-shell description in 5D, leading to

$$\mathcal{D} = -D_5\Sigma\,. \tag{2.4.6}$$

This expression was interpreted as an algebraic equation of motion in [77]. However, the authors showed that integrating out $\mathcal{D}$ then returns the original 5D bosonic action. Instead one might want to keep the term $-D_5\Sigma$ in order to *reconstruct* the N=1 off-shell formulation. One could then impose the correct transformation law and introduce an additional field, say X^3, whose SUSY transformation were to be arranged by hand in order to contribute the missing terms. Due to the existence of a 5D N=2 off-shell formulation we know that this is possible.

Since we seek to construct an N=1 invariant supersymmetric action, we shall not take $\mathcal{D} = -r^{-2}F_{\dot{5}\dot{6}}$ as an equation of motion, but as an identity instead, which is, however, incomplete as long as we do not refer to the off-shell formulation of the N=2 theory in 6D. But if we chose to complete it by hand, imposing a new transformation law for $F_{\dot{5}\dot{6}}$, and carefully keeping track of the fermionic terms, we would obtain the same result as from the 6D N=2 off-shell formulation.

For our purpose, the important feature is that $-r^{-2}F_{\dot{5}\dot{6}}$ couples to on-shell degrees of freedom, as the complete auxiliary field $\mathcal{D}$ would.[9] Hence, it can be used to rewrite the 6D action in N=1 off-shell form, as implied by (2.4.4). The algebraic equation of motion we obtain by varying the resulting action *wrt* $-r^{-2}F_{\dot{5}\dot{6}}$ will then coincide with the one which is obtained by integrating out $\mathcal{D}$ from a true off-shell action. If we could apply this strategy successfully also to the supergravity sector, we might be able to construct bulk-brane couplings in a way analogous to an off-shell method.

For completeness, we note that it is actually possible to derive a global off-shell super-Yang-Mills action by dimensional reduction of a higher-dimensional on-shell action via Legendre transformation [79]. In that case, the resulting auxiliary field corresponds to $-F_{\dot{5}\dot{6}}$, *including* the correct transformation law.

2.4.2 Toward N=1 off-shell supergravity?

In [69], the authors derived an off-shell action for 6D N=2 Poincaré supergravity coupled to the tensor multiplet, using superconformal tensor calculus. Amongst the independent fields, they list an auxiliary triplet vector of SU(2) and an auxiliary antisymmetric tensor field. As they note, this is equivalent of the new-minimal auxiliary formulation of N=1

[9]Notice that the term $F_{\dot{5}\dot{6}}A_\mu$ we obtained from the reduction of the Chern-Simons term is, by definition, part of the kinetic sector of the N=1 theory, cf. [76].

supergravity. In both cases, the off-shell formulation of Poincaré supergravity is obtained by means of fixing the gauge freedom within a linear compensating multiplet [70]. However, [69] does not provide us with SUSY transformation laws. Hence, unfortunately, we were not able to start directly from this off-shell formulation, when we decomposed the SUSY transformations at the orbifold fixed point.

Let us now reconsider the gravitino transformation law (2.1.29) we obtained at the brane position, which is reminiscent of the new-minimal formulation of Sohnius and West [71] as well,

$$\delta\psi_{L\mu} = 2D_\mu\epsilon_L - 2i(\mathcal{A}_\mu + \mathcal{V}_\mu)\epsilon_L + i\gamma_\mu{}^\nu\mathcal{V}_\nu\epsilon_L \,. \tag{2.4.7}$$

Here, we set the reduced 4D Planck mass $M_4 = 1$ for convenience. We may now identify

$$\mathcal{V}_\mu = \frac{1}{r^3}\,\partial_{[5}e^\alpha_{6]}e_{\alpha\mu} = V_\mu \,, \tag{2.4.8}$$

$$\mathcal{A}_\mu = -\frac{1}{4}\left(\frac{\partial_\mu\tau_1}{\tau_2} + \frac{D_\mu a}{s} + \frac{D_\mu b}{t}\right) , \tag{2.4.9}$$

where we also set the 6D Planck scale $M_6 = 1$. We shall stick to this convention in the following. In the equation for $\mathcal{A}_\mu$, we adopted the notation D_μ from the redefinition of the kinetic terms, given in (2.2.7) and (2.2.8).

As we already noted, from the brane point of view V_μ is an auxiliary degree of freedom in the Lagrangean (2.2.4). Therefore, we conjecture that

$$\mathcal{V}_\mu = \mathcal{V}^*_\mu + V_\mu \,, \tag{2.4.10}$$

where - in analogy to the gauge vector case - $\mathcal{V}^*_\mu$ is the (unknown) contribution from the 6D N=2 auxiliary sector that would guarantee the sum on the RHS to transform exactly like the N=1 auxiliary field. On the other hand, we have to consider (2.4.9) as an algebraic equation of motion, since it is explicitly given in terms of even fields which belong to other multiplets of the N=1 on-shell theory, and it fits the corresponding expression in the on-shell transformation law (2.3.17). Hence, we shall proceed by 'integrating in' $\mathcal{A}_\mu$, i.e. supplementing our Lagrangean with an auxiliary sector that would return (2.4.9) via an equation of motion.

Now consider [71]

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{new-min}} = {} & -\frac{M^2}{2}R + K_{i\bar{j}}\left(\partial_\mu\Phi^i\partial^\mu\bar{\Phi}^{\bar{j}} + F^iF^{\bar{j}}\right) \\ & - \mathcal{V}^\mu\left[i\big(K_i\partial_\mu\Phi^i - K_{\bar{j}}\partial_\mu\bar{\Phi}^{\bar{j}}\big) + 4\mathcal{A}_\mu + 3\mathcal{V}_\mu\right] , \end{aligned} \tag{2.4.11}$$

where Φ^i runs over all the scalar components of chiral superfields. We shall use $\mathcal{L}_{\text{new-min}}$ as a template to be compared with (2.2.4). We rewrite the latter as follows, using (2.2.10) and omitting the gauge theory part,

$$
\begin{aligned}
e^{-1}\mathcal{L} \supset & -\frac{M^2}{2}R + \frac{1}{4\tau_2^2}\partial_\mu U \partial^\mu \bar{U} + \frac{1}{4s^2}D_\mu S D^\mu \bar{S} + \frac{1}{4t^2}D_\mu T D^\mu \bar{T} \\
& - V^\mu \left[\frac{D_\mu a}{s} + \frac{D_\mu b}{t} - 3V_\mu\right] .
\end{aligned} \tag{2.4.12}
$$

As part of the unavoidable truncations, we set $R_{\text{res}} = 0$. In order to recover $\mathcal{A}_\mu$ as given in (2.4.9), we have to 'integrate in' $\mathcal{A}_\mu$ precisely by adding

$$
e^{-1}\mathcal{L}_+^{\text{aux}} = -V^\mu \left[\frac{\partial_\mu \tau_1}{\tau_2} + 4\mathcal{A}_\mu + 3V_\mu\right] \tag{2.4.13}
$$

to (2.4.12). The resulting Lagrangean,

$$
\begin{aligned}
e^{-1}\mathcal{L} \supset & -\frac{M^2}{2}R + \frac{1}{4\tau_2^2}\partial_\mu U \partial^\mu \bar{U} + \frac{1}{4s^2}D_\mu S D^\mu \bar{S} + \frac{1}{4t^2}D_\mu T D^\mu \bar{T} \\
& - V^\mu \left[\left(\frac{\partial_\mu \tau_1}{\tau_2} + \frac{D_\mu a}{s} + \frac{D_\mu b}{t}\right) + 4\mathcal{A}_\mu\right] ,
\end{aligned} \tag{2.4.14}
$$

as it is obtained from the decomposition of the bulk theory (2.4.12) by 'integrating in' $\mathcal{A}_\mu$ according to the gravitino transformation law, matches the new-minimal off-shell template up to the missing term $-3V^\mu V_\mu$. Notice that (2.4.12), (2.4.14) and (2.4.11) are mutually consistent, if and only if we take $\mathcal{V}_\mu = V_\mu = 0$. However, in new-minimal supergravity $\mathcal{V}_\mu = 0$ is nothing but the equation of motion that follows from varying *wrt* $\mathcal{A}_\mu$.

We shall proceed by introducing the brane localized kinetic sector. According to (2.4.11), the auxiliary sector must now take the form

$$
\begin{aligned}
e^{-1}\mathcal{L}^{\text{aux}} = -\mathcal{V}^\mu \Bigg[&\left(\frac{\partial_\mu \tau_1}{\tau_2} + \frac{\partial_\mu a}{s} + \frac{\partial_\mu b}{t}\right) + i\big(K_i \partial_\mu Q^i - K_{\bar{j}} \partial_\mu \bar{Q}^{\bar{j}}\big) \\
& + 4\mathcal{A}_\mu + 3\mathcal{V}_\mu\Bigg] ,
\end{aligned} \tag{2.4.15}
$$

when expressed in terms of the zero modes. On the other hand, starting from (2.4.9) we can write

$$
e^{-1}\mathcal{L}^{\text{aux}} = -V^\mu \left[\left(\frac{\partial_\mu \tau_1}{\tau_2} + \frac{\partial_\mu a}{s} + \frac{\partial_\mu b}{t}\right) + \frac{B_\mu}{t} + 4V_\mu + 4\big(\mathcal{A}_\mu - A_\mu^{\text{coupl}}\big)\right] , \tag{2.4.16}
$$

where we introduced the notation A_μ^{coupl}, encoding the modification to the auxiliary sector of (2.4.14) which is required in order to couple brane chiral fields, if we want the result to agree with (2.4.11). Equating these two expressions we obtain

$$
4A_\mu^{\text{coupl}} = -i\big(K_i \partial_\mu Q^i - K_{\bar{j}} \partial_\mu \bar{Q}^{\bar{j}}\big) + \frac{B_\mu}{t} - 3V_\mu , \tag{2.4.17}
$$

provided $\mathcal{V}_\mu = V_\mu$. Observing that V_μ is still set to zero by the algebraic equation of motion for $\mathcal{A}_\mu$, our coupling prescription amounts to nothing else but the replacement

$$B_\mu \;\rightarrow\; it\big(K_i\partial_\mu Q^i - K_{\bar{j}}\partial_\mu \bar{Q}^{\bar{j}}\big) = i\big(\Omega_i\partial_\mu Q^i - \Omega_{\bar{j}}\partial_\mu \bar{Q}^{\bar{j}}\big)\,. \tag{2.4.18}$$

This is equivalent to the corresponding result of [67], compare (2.3.3) and (2.3.6). If we now proceed and replace B_μ in the kinetic part of the Lagrangean, we are led to a redefintion of the T modulus and its Kähler potential, thereby reproducing the coupling found in [67]. However, our approach lacks a rigorous justification, since we did not consider the fermionic action. In order to prove our procedure to be reliable, the following programme would have to be carried out:

1. Decompose the fermionic bulk action in terms of the N=1 multiplets on the brane, including internal derivatives of the odd components.

2. Identify those parts of the action, which correspond to variations of $F_{\dot{5}\dot{6}}$ and V_μ, and make sure they can consistently be set to zero. Impose transformation laws for $\mathcal{D} \sim F_{\dot{5}\dot{6}}$ and V_μ, such that the supersymmetry algebra closes off-shell. Alternatively, introduce additional auxiliary degrees of freedom X and $\mathcal{V}^*_\mu$, such that $X - r^{-2}F_{\dot{5}\dot{6}}$ transforms like $\mathcal{D}$, and $\mathcal{V}^*_\mu + V_\mu$ as $\mathcal{V}_\mu$.

3. Check that the resulting action is invariant and reproduces the on-shell action constructed from only the even fields, corresponding to zero modes of a low energy effective action.

As a reasonable starting point for further research in this direction, one may try to deduce the SUSY transformation laws for the 6D N=2 off-shell formulation of [69]. Then the N=2 auxiliary fields could be decomposed *wrt* their orbifold parities analogously to the on-shell degrees of freedom. This might lead to a positive identification of $\mathcal{V}^*_\mu$.

2.5 Summary

In this chapter, we determined the N=1 supermultiplets which emerge from 6D N=2 supergravity on a co-dimension two brane. We decomposed the N=2 transformation laws in terms of $\mathbb{Z}_2$ even components at an orbifold fixed point, and explicitly identified the fermionic partners of the geometric moduli. Including also brane localized chiral fields, we obtained the low energy effective action corresponding to a flat torus background. The result agrees with earlier calculations [67], except that we took account of a gauge sector in the bulk. In addition, we discussed the possibility of an N=1 off-shell formulation made manifest within the full 6D bulk-brane action, which would enable us to construct more general bulk-brane couplings. Our preliminary results are inconclusive, however, we were at least able to recover the kinetic coupling derived in [67], which provides a starting point for further investigation.

Chapter 3

Moduli stabilization in almost no-scale supergravity

In this chapter, we develop a systematic and model independent strategy to stabilize a volume modulus by sub-leading corrections to a no-scale potential. We also comment on the possibility of generalizing this scenario to the case of three moduli, as it results from compactifying 6D supergravity, and on the necessary modifications. We illustrate our strategy by working out a specific example, motivated by [15]. The authors of [15] suggested that quantum effects, in particular the interplay of Casimir energy [80, 81] and localized Fayet-Iliopoulos (FI) terms [82], can lead to the stabilization of two compact dimensions. For a FI mass scale $\mathcal{O}(M_{\mathrm{GUT}})$, as it occurs in some compactifications of the heterotic string [13], one could then obtain the scale of the compact dimensions L to be of the order M_{GUT}^{-1}. The barrier which separates four-dimensional from six-dimensional Minkowski space turns out to vanish for unbroken supersymmetry.

The set-up of [15, 83] is a globally supersymmetric model on $\mathcal{M}_4 \times \mathcal{T}^2/_{\mathbb{Z}_2\times\mathbb{Z}_2}$, i.e. with the extra-dimensional space being a flat torus orbifold. The two radii of the torus and the angle between the corresponding lattice vectors are treated as free parameters of the model. Stabilization is achieved by extremizing the vacuum energy density *wrt* these parameters. However, size and shape of the internal space correspond to metric degrees of freedom. Hence, it is obvious that the dynamics of moduli stabilization could be treated consistently only within a supergravity description of the model. Moreover, the stabilization of the extra dimensions is closely related to the mechanism of supersymmetry breaking and its mediation to the observable sector. In the model of [15, 83] SUSY breaking is localized on a hidden brane and mediated via bulk effects to the observable sector, which is again (at least partly) realized on branes. Therefore, it is necessary to take account of the couplings between moduli and brane localized fields, cf. the previous chapter. In the present chapter, we shall demonstrate that the supergravity realization of the idea of [15] does not simply provide sub-leading supergravity corrections, but leads to a significantly different picture.

In [83] the dominant tree level potential resulting from the F-term of the SUSY breaking field was canceled by a scale-dependent brane tension that emerges at one loop, in order to downlift the vacuum to Minkowski spacetime. Since the one loop result is divergent, a renormalization procedure is required, which in turn would depend on the UV completion of the theory. The Casimir energy is a quantum correction to the Kähler potential, and in N=1 supergravity the Kähler potential is renormalized at every loop level. Non-perturbative corrections are also possible. A perturbative treatment of the problem is therefore only viable, if higher loop contributions are negligible. Moreover, we re-encounter the old cosmological constant problem of field theory. We note that it is impossible to quantify the fine-tuning of the cosmological constant that is hidden in the downlift procedure of [83]. Instead, we shall make sure that our results do not rely on fixing a scale-dependent, potentially divergent term to one specific value. We will comment on this issue again during the following analysis.

Finally, the scalar sector of the low energy effective theory is more complicated than the parameter space of [15,83]. In addition to size and shape of the internal dimensions, we have to include the dilaton, which is part of the 6D supergravity sector. In the previous chapter we obtained the Kähler potential which results from compactifying 6D supergravity on a flat torus orbifold,

$$M^{-2}K = -\ln\left(\frac{S+\bar{S}}{2}\right) - \ln\left(\frac{T+\bar{T}}{2} - \frac{\Omega}{M^2}\right) - \ln\left(\frac{U+\bar{U}}{2}\right),$$

where Ω is a function of brane localized chiral fields. We have to seek out a vacuum, which generates a positive mass for each of the six real scalars contained in S, T and U.

As we already noted, the given Kähler potential is of no-scale structure. The simplest prototype in the corresponding class of models is the Kähler potential

$$M^{-2}K = -3\ln\left(\frac{T+\bar{T}}{2}\right),$$

featuring a single modulus field, which can be identified with the universal Kähler modulus controlling the volume of a six-dimensional Calabi-Yau manifold. However, as we shall see, the same Kähler potential emerges from dimensional reduction of 5D supergravity. We also noted that the Casimir energy of bulk fields constitutes a correction to the Kähler potential, which guides us to consider *almost* no-scale supergravity models [84]. The single field case allows us to study generic features of the almost no-scale scenario for moduli stabilization. The more complicated case with three complex fields, corresponding to the 6D set-up, will be left for future work. We shall see that the 5D results provide a sound basis to address the 6D problem, however, there are also important differences. We will comment on the implications below.

As an example of our general prescription, we study the interplay of supersymmetry breaking and gauge symmetry breaking - induced by localized FI terms - within low energy effective supergravity, including the dynamics of the radion superfield, cf. [85–87]. At tree level, the radion potential exactly vanishes. The flat direction needs to be lifted by quantum corrections to the Kähler potential, which always include the Casimir energy, if SUSY is broken. The chiral superfield, which generates the expectation value of the superpotential, couples to bulk fields. This coupling induces an additional contribution to the radion potential. The resulting potential allows for locally stable Minkowski or de Sitter vacua, without the need of an additional 'uplifting' mechanism.

Let us emphasize that it is impossible to discuss moduli stabilization independently of the cosmological constant problem. It is meaningless to compute the size of the extra-dimensional space associated with an anti-de Sitter vacuum, because any generic uplifting procedure will introduce another moduli dependent term into the scalar potential. This additional term will then induce a shift of the moduli vacuum expectation values (VEVs), thereby invalidating the results obtained prior to the uplift. A possible way out is to compute the size of the extra dimension in a specific conformal frame, where the uplifting term can be represented by a constant. But we prefer to work in the Einstein frame and seek a stable Minkowski vacuum by taking account of any relevant term from the beginning.

The chapter is organized as follows. Section 3.1 introduces the no-scale set-up with the radion superfield coupled to a brane localized chiral superfield. The general structure of almost no-scale models is analyzed in section 3.2, where also a model independent formula for the radion mass is derived. As a specific example for the general set-up, stabilization induced by localized FI terms is worked out in section 3.3. Section 3.4 contains some preliminary results on the 6D case with three complex scalar fields, which is followed by a brief summary in section 3.5.

The main results of this chapter were published in [88], however, all the considerations and computations presented here were originally worked out by the author of this thesis.

3.1 No-scale supergravity from a 5D orbifold model

The objective of this chapter is to infer the Kähler potential of the 4D N=1 low energy effective supergravity, which is obtained by dimensional reduction of the 5D N=2 theory on an orbifold. We shall first consider the bosonic component action.

3.1.1 Dimensional reduction of the bosonic component action

Consider the bosonic part of the 5D N=2 supergravity action on $\mathcal{M}_4 \times S_1/\mathbb{Z}_2$, with bulk and brane contributions

$$S_5 = \int d^4x\, dy \big\{ \mathcal{L}_{\text{bulk}} + \delta(y)\mathcal{L}_{\text{vis}} + \delta(y-L)\mathcal{L}_{\text{hid}} \big\}\,, \tag{3.1.1}$$

where we distinguished between a 'visible' brane, where observable fields are located, and the 'hidden' brane. The bulk part is

$$S_{\text{bulk}} = \frac{M_5^3}{2} \int d^4x \int_0^L dy \sqrt{-g_5} \left\{ -R_5 + \frac{1}{2} H^{MN} H_{MN} + \mathcal{L}_{\text{bulk}}^{\text{mat}} \right\} . \tag{3.1.2}$$

Here $H_{MN} = \partial_M B_N - \partial_N B_M$ is the field strength of the graviphoton, which is the spin-1 component of the supergravity multiplet in 5D. Dimensional reduction of this action on the background metric

$$ds_5^2 = g_{\mu\nu}(x)dx^\mu dx^\nu + r^2(x)dy^2 \tag{3.1.3}$$

leads to

$$S_4 = \frac{M^2}{2}\int d^4x\sqrt{-g}r\left\{-R + \frac{1}{r^2}\partial_\mu B_5\partial^\mu B_5 + \mathcal{L}^{(4)}_{\text{bulk}}\right\} + S_{\text{branes}}[g^{\mu\nu}]\,, \tag{3.1.4}$$

where we only kept $g_{\mu\nu}, g_{55}$ and B_5, which are the fields with even $\mathbb{Z}_2$ parity. The remaining fields, $g_{\mu 5}$ and B_μ, are $\mathbb{Z}_2$ odd and thus do not have light modes. $M = \sqrt{M_5^3 L}$ is the 4D Planck mass. Without loss of generality, we assume the radion to be stabilized with $r_0 = 1$ in the vacuum. We note that the radion field, being the scale factor of the fifth dimension, is dimensionless and has no kinetic term. Due to the bulk-brane structure of the model, r couples non-universally to the matter sector, hence it is not a Brans-Dicke scalar.[1]

Performing a conformal transformation of the metric,

$$g_{\mu\nu} \to g^*_{\mu\nu} = r g_{\mu\nu}\,, \quad \sqrt{-g}r^2 = \sqrt{-g^*}\,, \tag{3.1.5}$$

such that

$$R[g] = rR[g^*] - \frac{3}{2}g^{*\mu\nu}\frac{\partial_\mu r\partial_\nu r}{r}\,, \tag{3.1.6}$$

one finds for the action in the Einstein frame,[2]

$$\begin{aligned} S_4 = \frac{M^2}{2}\int d^4x\sqrt{-g}\bigg\{-R &+ \frac{3}{2r^2}g^{\mu\nu}\partial_\mu r\partial_\nu r + \frac{1}{r^2}\partial_\mu B_5\partial^\mu B_5 \\ &+ \frac{1}{r}\mathcal{L}^{(4)}_{\text{bulk}}[rg^{\mu\nu}]\bigg\} + S_{\text{branes}}[rg^{\mu\nu}]\,, \end{aligned} \tag{3.1.7}$$

where we suppressed the * in the notation of the Einstein frame metric. This action contains a quadratic kinetic term for the radion field. Notice the presence of the unusual factor 3, which will reappear in the Kähler potential below. This factor indicates that the kinetic term is solely generated by the conformal transformation, which can be better understood within a superconformal framework.

3.1.2 The Kähler potential

In a globally supersymmetric theory, the scalar kinetic terms are determined by a real function $\Omega(z^i, \bar{z}^{\bar{j}})$,

$$\int d^2\theta d^2\bar{\theta}\,\boldsymbol{\Omega}(z^i, \bar{z}^{\bar{j}}) \supset \Omega_{i\bar{j}}\partial_\mu z^i\partial^\mu \bar{z}^{\bar{j}}\,, \quad \Omega_{i\bar{j}} \equiv \partial_i\partial_{\bar{j}}\Omega\,. \tag{3.1.8}$$

[1] See [23] for a brief introduction to scalar-tensor theories and further references.

[2] This computation is completely equivalent to our approach in the 6D case, where we chose to incorporate the necessary Weyl rescaling *a priori* within our metric ansatz, cf. appendix A.2 on the reduction of the 6D Ricci scalar.

In order to obtain the Kähler potential of the corresponding supergravity theory, which encodes the Einstein frame scalar kinetic terms,

$$K = -3M^2 \ln\left(-\frac{\Omega}{3M^2}\right), \tag{3.1.9}$$

we employ the compensator formalism of superconformal tensor calculus [89], with the superspace action

$$S = \int d^4x \left\{ \frac{1}{2} \int d^2\theta d^2\bar{\theta}\, \Phi\bar{\Phi}\mathbf{\Omega} + \int \left[d^2\theta\, \Phi^3 \mathbf{W} + h.c. \right] \right\}. \tag{3.1.10}$$

Here Φ denotes the chiral compensating superfield, $\mathbf{\Omega}$ is called the superspace kinetic energy, and $\mathbf{W}$ is the holomorphic superpotential. Superconformal tensor calculus provides us with the following $\mathcal{D}$-type action formula [89],

$$\begin{aligned} \frac{e^{-1}}{2}[\Phi\bar{\Phi}\mathbf{\Omega}]_{\mathcal{D}} = {} & \frac{1}{6}\tilde{\Omega}R - \tilde{\Omega}_{I\bar{J}}\left[\partial_\mu z^I \partial^\mu \bar{z}^{\bar{J}} - F^I \bar{F}^{\bar{J}}\right] \\ & - \frac{3}{4}\mathcal{A}_\mu \mathcal{A}^\mu + \frac{i}{2M}\mathcal{A}^\mu \left(z^0 \tilde{\Omega}_{0\bar{J}} \partial_\mu \bar{z}^{\bar{J}} - \bar{z}^{\bar{0}} \tilde{\Omega}_{\bar{0}I} \partial_\mu z^I \right) + \text{fermions}, \end{aligned} \tag{3.1.11}$$

where $\mathcal{A}_\mu$ is the residual auxiliary field of the supergravity multiplet, and identical - up to a numerical factor - with its counterpart in the new-minimal formulation, cf. [70] and section 1.4. Furthermore,

$$\tilde{\Omega} \equiv M^{-2}\phi\bar{\phi}\Omega \tag{3.1.12}$$

and ϕ denote the lowest components of the corresponding vector multiplet, and the chiral compensator multiplet, respectively. The indices $I, J \in \mathbb{N}_0$ run over all scalar component fields, including $z^0 = \phi$.

Within this framework, conformal transformations between different frames are implemented by an appropriate fixing of the remaining conformal gauge freedom. The choice

$$\Phi = M + \theta^2 F_\phi \tag{3.1.13}$$

corresponds to the component action (3.1.4), if we identify

$$\Omega = -3M^2 r\,. \tag{3.1.14}$$

On the other hand, the Einstein-Hilbert term of (3.1.11) is turned into canonical form if we set

$$\phi\bar{\phi}\Omega = -3M^4\,, \tag{3.1.15}$$

corresponding to the 4D Einstein frame.

Following [89], we can now verify that $\tilde{\Omega}$ gives rise to the correct (Einstein frame) scalar kinetic terms. Using $\partial_\mu\tilde{\Omega} = 0$, we can write

$$\partial_\mu z^0 = -\frac{z^0}{\Omega}\Omega_i \partial_\mu z^i - \frac{z^0}{6M^2}\left(K_i \partial_\mu z^i - K_{\bar{j}} \partial_\mu \bar{z}^{\bar{j}}\right), \tag{3.1.16}$$

where $K_i \equiv -3M^2\Omega^{-1}\Omega_i$, and get

$$\tilde{\Omega}_{0\bar{J}}\partial_\mu z^0 \partial^\mu \bar{z}^{\bar{J}} = \frac{\partial_\mu z^0}{M^2}\left[-z^0\Omega_i\partial^\mu z^i - \frac{z^0\Omega}{6M^2}\left(K_i\partial^\mu z^i - K_{\bar{j}}\partial^\mu \bar{z}^{\bar{j}}\right) + z^0\Omega_{\bar{j}}\partial^\mu \bar{z}^{\bar{j}}\right]$$
$$= \frac{\partial_\mu z^0}{M^2}\left[\frac{z^0\Omega}{6M^2}\left(K_i\partial^\mu z^i - K_{\bar{j}}\partial^\mu \bar{z}^{\bar{j}}\right)\right] , \tag{3.1.17}$$

as well as

$$\tilde{\Omega}_{I\bar{0}}\partial_\mu z^I \partial^\mu \bar{z}^{\bar{0}} = -\frac{z^0\Omega}{6M^4}\left(K_i\partial_\mu z^i - K_{\bar{j}}\partial_\mu \bar{z}^{\bar{j}}\right)\partial^\mu \bar{z}^{\bar{0}} . \tag{3.1.18}$$

Hence, we are left with

$$\begin{aligned}
\tilde{\Omega}_{I\bar{J}}\partial_\mu z^I \partial^\mu \bar{z}^{\bar{J}} &= \tilde{\Omega}_{0\bar{J}}\partial_\mu z^0 \partial^\mu \bar{z}^{\bar{J}} + \tilde{\Omega}_{I\bar{0}}\partial_\mu z^I \partial^\mu \bar{z}^{\bar{0}} - \tilde{\Omega}_{0\bar{0}}\partial_\mu z^0 \partial^\mu \bar{z}^{\bar{0}} + \tilde{\Omega}_{i\bar{j}}\partial_\mu z^i \partial^\mu \bar{z}^{\bar{j}} \\
&= \frac{|z^0|^2}{M^2\Omega}\left[-\Omega_i\Omega_{\bar{j}} + \Omega_{i\bar{j}}\Omega\right]\partial_\mu z^i \partial^\mu \bar{z}^{\bar{j}} + \frac{|z^0|^2\Omega}{36M^6}\left(K_i\partial_\mu z^i - K_{\bar{j}}\partial_\mu \bar{z}^{\bar{j}}\right)^2 \\
&= 3M^2\left[\ln\left(-\frac{\Omega}{3M^2}\right)\right]_{i\bar{j}}\partial_\mu z^i \partial^\mu \bar{z}^{\bar{j}} \\
&\quad - \frac{1}{12M^2}\left(K_i\partial_\mu z^i - K_{\bar{j}}\partial_\mu \bar{z}^{\bar{j}}\right)^2 ,
\end{aligned} \tag{3.1.19}$$

where in the last step we inserted $|z_0|^2 = -3M^4\Omega^{-1}$. The auxiliary sector of (3.1.11) is now given by

$$\begin{aligned}
-e^{-1}\mathcal{L}_{\text{aux}} &= \frac{3}{4}\mathcal{A}_\mu\mathcal{A}^\mu - \frac{i}{2}\mathcal{A}_\mu\frac{|z^0|^2\Omega}{3M^5}\left(K_i\partial^\mu z^i - K_{\bar{j}}\partial^\mu \bar{z}^{\bar{j}}\right) \\
&\quad - \frac{1}{12M^2}\left(K_i\partial_\mu z^i - K_{\bar{j}}\partial_\mu \bar{z}^{\bar{j}}\right)^2 - \tilde{\Omega}_{I\bar{J}}F^I\bar{F}^{\bar{J}} \\
&= \frac{3}{4}\left[\mathcal{A}_\mu + \frac{i}{3M}\left(K_i\partial_\mu z^i - K_{\bar{j}}\partial_\mu \bar{z}^{\bar{j}}\right)\right]^2 \\
&\quad - 3\left[\ln\left(-\frac{\Omega}{3M^2}\right)\right]_{i\bar{j}}F^i\bar{F}^{\bar{j}} - M^{-2}\Omega|\tilde{F}^0|^2 ,
\end{aligned} \tag{3.1.20}$$

where we redefined the compensator F term,

$$F^0 = \tilde{F}^0 - z^0\Omega^{-1}\Omega_i F^i . \tag{3.1.21}$$

Notice that we obtain

$$\mathcal{A}_\mu = -\frac{i}{3M}\left(K_i\partial_\mu z^i - K_{\bar{j}}\partial_\mu \bar{z}^{\bar{j}}\right) , \tag{3.1.22}$$

which coincides with the new-minimal result [71] upon redefining $\mathcal{A}_\mu \to \frac{4}{3}\mathcal{A}_\mu$.

We can now introduce the Kähler potential of N=1 supergravity, and match the bosonic component action (3.1.4) (recall $\phi = M$ in that frame) with

$$r = -\frac{\Omega}{3M^2} = \exp\left[-\frac{K}{3M^2}\right] \quad \Rightarrow \quad K = -3M^2\ln r , \tag{3.1.23}$$

such that

$$e^{-1}\mathcal{L}^{\rm local}_{\rm kin} = -3M^2 \left[\ln\left(-\frac{\Omega}{3M^2}\right)\right]_{i\bar{j}} \partial_\mu z^i \partial^\mu \bar{z}^{\bar{j}} = K_{i\bar{j}}\, \partial_\mu z^i \partial^\mu \bar{z}^{\bar{j}}\,. \tag{3.1.24}$$

Using the $\mathcal{F}$-type action formula of [89], we obtain from (3.1.10) and (3.1.20), after integrating out the auxiliary fields, the scalar potential of N=1 supergravity,

$$\begin{aligned} -e^{-1}\mathcal{L}_F &= -M^{-2}\left(\Omega F^\phi \bar{F}^{\bar\phi} + \phi\Omega_i F^i \bar{F}^{\bar\phi} + \bar\phi\Omega_{\bar{j}} \bar{F}^{\bar{j}} F^\phi + \phi\bar\phi\Omega_{i\bar{j}} F^i \bar{F}^{\bar{j}}\right) \\ &\quad - M^{-3}(\phi^3 F^i W_i + \bar\phi^3 \bar{F}^{\bar{j}} \bar{W}_{\bar{j}}) - 3M^{-3}(\phi^2 F^\phi W + \bar\phi^2 \bar{F}^{\bar\phi} \bar{W}) \\ &= e^{K/M^2}\left[(W_i + M^{-2}K_i W)K^{i\bar{j}}(\bar{W}_{\bar{j}} + M^{-2}K_{\bar{j}}\bar{W}) - 3M^{-2}|W|^2\right]. \end{aligned} \tag{3.1.25}$$

Instead of repeating the tedious decomposition of the higher dimensional SUSY transformations, we focus here on the lowest components of the supermultiplets and the bosonic action. It is easily verified[3] that

$$\Omega = -\frac{3}{2}(T + \bar{T}) \tag{3.1.26}$$

with

$$T = r + \sqrt{\frac{2}{3}} i B_5 \tag{3.1.27}$$

gives rise to the kinetic terms of both (3.1.4) and (3.1.7), depending on the respective choice of the compensator.

Now let us include a brane chiral field X, whose scalar component is supposed to have a canonical kinetic term on the brane,

$$\Omega_{\rm brane} = X\bar{X}\,. \tag{3.1.28}$$

The following Kähler potential[4] reproduces the scalar kinetic terms in the Einstein frame component action,

$$K = -3M^2 \ln\left(\frac{T+\bar{T}}{2} - \frac{X\bar{X}}{3M^2}\right), \tag{3.1.29}$$

where the scalar component of the radion superfield is now given by

$$T = r + \frac{X\bar{X}}{3M^2} + i\sqrt{\frac{2}{3}} B_5\,, \tag{3.1.30}$$

compensating for the non-diagonal entries in the Kähler metric [90]. Notice that, due to the definition of T, we still have $K = -3M^2 \ln r$.

[3] see, *e.g.*, [85] for more detail.

[4] The generalization to several chiral brane fields is straightforward, cf. section 1.3.

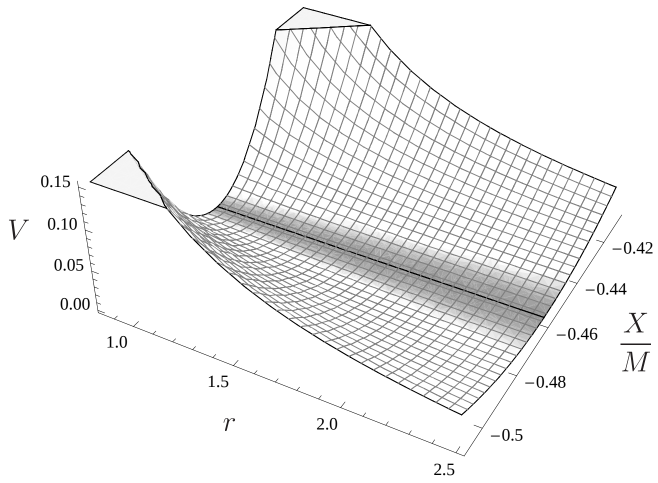

Figure 3.1: *Example of a no-scale potential, in units of* $m_{3/2}^2 M^2$. *It follows from the quadratic superpotential (3.3.29) for the choice* $\sqrt{\sigma_0} \simeq 0.46$.

The Kähler potential (3.1.29) has no-scale structure [91],

$$K^i K_i = 3M^2 \,. \tag{3.1.31}$$

Hence, the negative-definite contribution to the scalar potential vanishes, and one obtains

$$V_F = \frac{1}{r^2} W_X \bar{W}_{\bar{X}} \,. \tag{3.1.32}$$

The equations of motion

$$\partial_r V_F = 0 \,, \quad \partial_X V_F = 0 \,, \tag{3.1.33}$$

are simultaneously satisfied at stationary points of the superpotential,

$$\partial_X W|_{X_0} = 0 \,. \tag{3.1.34}$$

The potential then vanishes for all values of r, satisfying the Minkowski condition $V_F = 0$, and the size of the compact dimension remains undetermined (cf. Fig. 3.1).

Since the Kähler potential does not depend on B_5, the imaginary part of the complex scalar T is also a flat direction. The corresponding two scalar masses vanish,

$$M_1^2 = 0 \,, \quad M_2^2 = 0 \,, \tag{3.1.35}$$

whereas the masses of real and imaginary part of X are equal and positive,

$$M_3^2 = M_4^2 = \frac{1}{4} W_{XX} \bar{W}_{\bar{X}\bar{X}} \,. \tag{3.1.36}$$

For a non-vanishing superpotential, supersymmetry is spontaneously broken, indicated by a non-zero VEV of F_T. Hence, the 'radino', as we may call the fermionic component of the radion superfield, is to be identified with the goldstino. The gravitino mass, given by

$$m_{3/2}^2 = e^{K/M^2} \frac{|W|^2}{M^4} = r^{-3} \frac{|W|^2}{M^4} , \tag{3.1.37}$$

'slides' with the expectation value of the radion field, and remains undetermined at tree level.

The potential depicted in Fig. 3.1 illustrates the continuous vacuum degeneracy of the tree level potential. It is well known, that Kähler potentials of the type $K = -3M^2 \ln r$ do not admit non-supersymmetric Minkowski vacua with a positive definite mass matrix [62, 63, 92]. A necessary condition for the latter can be formulated as [62]

$$R_{i\bar{j}k\bar{l}} G^{\bar{l}} G^k G^{\bar{j}} G^i < 6M^2 , \tag{3.1.38}$$

where $R_{i\bar{j}k\bar{l}}$ denotes the Riemann curvature of the Kähler manifold, and

$$G = K + M^2 \ln \frac{|W|^2}{M^6} . \tag{3.1.39}$$

The scalar potential is then given by

$$V = m_{3/2}^2 \left(G^i G_i - 3M^2 \right) . \tag{3.1.40}$$

For the two-field no-scale Kähler potential (3.1.29), vanishing of the vacuum energy implies

$$R_{i\bar{j}k\bar{l}} G^{\bar{l}} G^k G^{\bar{j}} G^i = 6M^2 . \tag{3.1.41}$$

This result holds for any superpotential $W(X, T)$, even in the presence of non-perturbative corrections. Therefore at least one flat direction is unavoidable.[5] We conclude that sub-leading corrections are crucial in order to stabilize the radion in a Minkowski vacuum.

3.2 Almost no-scale models

In this section we introduce the κ formalism, which provides a general description of sub-leading corrections to the Kähler potential of no-scale supergravity, and allows us to derive a model independent mass formula.

3.2.1 κ formalism

In the presence of quantum loop corrections to the Kähler potential, the superspace kinetic energy Ω is modified,

$$\Omega = \Omega_0 + \Delta\Omega , \tag{3.2.1}$$

[5] We note that this argument also applies to the stabilization mechanism of [85], where the F-term uplift induces a flat direction in the hidden sector.

where

$$\Omega_0 = -3M^2 \left(\frac{T+\bar{T}}{2} - \frac{\sigma}{3} \right) , \quad \sigma \equiv \frac{X\bar{X}}{M^2} . \tag{3.2.2}$$

The corresponding Kähler potential is given by

$$\begin{aligned} K &= -3M^2 \ln \left[-\frac{\Omega_0}{3M^2} \left(1 + \frac{\Delta\Omega}{\Omega_0} \right) \right] \\ &= -3M^2 \left[\ln \left(\frac{T+\bar{T}}{2} - \frac{\sigma}{3} \right) + \ln(1-\kappa) \right] , \end{aligned} \tag{3.2.3}$$

where we defined κ via

$$\Delta\Omega \equiv 3M^2 r\kappa(r,\sigma) . \tag{3.2.4}$$

At this point, we only *assume* that the expansion procedure is well defined. As we shall see later, this condition can be met by compactifying at a volume that is significantly larger than the Planck length.

It is now straightforward to calculate the $\mathcal{O}(\kappa)$ correction to the scalar potential,

$$\begin{aligned} V_F =& \frac{1}{r^2} W_X \bar{W}_{\bar{X}} (1 + 2\kappa + \partial_r (r\kappa) - 3r\partial_\sigma (\sigma \partial_\sigma \kappa)) \\ &+ \frac{3 \left(X W_X \bar{W} + W \bar{X} \bar{W}_{\bar{X}} \right)}{M^2 r^2} \partial_r (r \partial_\sigma \kappa) - \frac{3W\bar{W}}{M^2 r^2} \left(2\partial_r \kappa + r\partial_r^2 \kappa \right) . \end{aligned} \tag{3.2.5}$$

The tree level minimum X_0 is shifted to $X_0 + \Delta X$. At linear order in ΔX, the extremum condition

$$\partial_X V_F|_{X_0+\Delta X} = 0 \tag{3.2.6}$$

implies

$$\Delta X = \frac{3W}{M^2 W_{XX}} \left(-\bar{X}\partial_r (r\partial_\sigma \kappa) + \frac{\bar{W}}{M^2 W_{XX}} X \left(2\partial_r \partial_\sigma \kappa + r\partial_r^2 \partial_\sigma \kappa \right) \right) \Big|_{X_0, r_0} . \tag{3.2.7}$$

Our systematic expansion in κ is consistent as long as $|\Delta X|/|X_0| \leq \mathcal{O}(\kappa)$. According to (3.2.7) this holds if $|W_{XX}| \geq \mathcal{O}(|W|/M^2)$, i.e.

$$M_{3,4} \geq \mathcal{O}(m_{3/2}) . \tag{3.2.8}$$

Notice that the corresponding fermion mass then satisfies the same bound, since there is no mass-splitting within the X multiplet at tree level [91].

The radion effective potential that results from (3.2.5) at leading order in κ,

$$V^{(1)}(r,\sigma) = -\frac{3|W|^2}{M^2} \left(\frac{2}{r^2} \partial_r \kappa(r,\sigma) + \frac{1}{r} \partial_r^2 \kappa(r,\sigma) \right) , \tag{3.2.9}$$

scales, as expected, with $m_{3/2}^2$. Let us now assume that the κ correction allows for the radion to be stabilized at $r_0 = 1$. Then we can compute the mass term for the corresponding scalar fluctuations,

$$r = r_0 + \delta r = 1 + \sqrt{\frac{2}{3}}\rho\,, \tag{3.2.10}$$

where the definition of ρ renders a canonical kinetic term. The mass matrix of the complex scalars T and X has eigenvalues

$$M_1^2 = 0\,, \quad M_2^2 = \frac{|W|^2}{M^4}\left(4\partial_r^3\kappa + \partial_r^4\kappa\right)\Big|_{X_0,r_0} + \mathcal{O}(\kappa^2 m_{3/2}^2)\,, \tag{3.2.11}$$

$$M_3^2 = M_4^2 = \frac{1}{4}W_{XX}\bar{W}_{\bar{X}\bar{X}}\Big|_{X_0} + \mathcal{O}(\kappa m_{3/2}^2)\,. \tag{3.2.12}$$

We observe that the imaginary part of the radion superfield, an axion, remains massless, whereas the real part acquires a small mass, which is $\mathcal{O}(\kappa)$ relative to the gravitino mass.[6] Notice that the origin of the suppression can be twofold: First, quantum corrections involve a loop factor, which needs to be small (< 1) if the perturbative treatment is to be well defined. Second, the order of magnitude of $\Delta\Omega$ is generically controlled by a specific mass scale, which should be lower than the 4D Planck scale. In particular, this additional mass scale will turn out to be crucial in order to stabilize the extra dimension at a volume significantly larger than the inverse Planck scale, as we shall demonstrate in the following section.

The κ formalism as presented here does not rely on any details of the model or the stabilization mechanism. Instead, if applicable, it covers a large class of models, which are characterized by the appearance of sub-leading corrections to the Kähler potential. Hence, the mass relation we obtained is a model independent prediction. It is inevitable whenever the vacuum is stabilized by means of quantum corrections. However, it is still necessary to ensure that a given model can be consistently treated within perturbation theory. Moreover, one has to carefully take account of any possibly relevant contribution to κ.

Before we continue with a specific example, we shall, for completeness, discuss whether an almost no-scale scenario can also arise in the presence of a radion-dependent superpotential [84, 85].

3.2.2 Gaugino condensation and κ correction

Let us consider a situation, where supersymmetry breaking is not (primarily) induced by a hidden brane field, but by non-perturbative effects in the bulk. The generic example is gaugino condensation in a hidden SU(N) part of the gauge sector [85], which induces a radion-dependent contribution to the superpotential,

$$W = W_0 + \Lambda^3 \exp\left[-\frac{16\pi^2 T}{3Ng_4^2}\right] \equiv W_0 + \Lambda^3 e^{-aT}\,. \tag{3.2.13}$$

[6] In the case of α'-corrections, a similar relation for the mass of the volume modulus has been obtained in [63].

Here g_4^2 is the unified 4D gauge coupling,

$$\Lambda = \frac{1}{Lg_4^2} \times \mathcal{O}(10^2)\,, \tag{3.2.14}$$

and W_0 is a constant superpotential that may arise from hidden brane dynamics, or along the lines of [52]. For simplicity, we will take W_0 to be real in the following.

In [84] the authors allow for both a Kähler correction and a sub-leading correction to the superpotential. They conclude that, if the latter effect dominates, there is no stable minimum at linear order in the correction. On the other hand, the stabilization mechanism of [85] requires

$$\Lambda^3 e^{-a} \sim \frac{W_0}{a} \gg W_0 \tag{3.2.15}$$

with $r_0 = 1$, and is therefore not of the almost no-scale type. We will now analyze the possibility that both the Kähler and the superpotential receive a radion-dependent, sub-leading correction. The underlying physics problem is, whether a κ correction could in principle provide an uplift for the stabilization mechanism of [85].

We rewrite

$$\kappa(r) \equiv \kappa_0 f_\kappa(r)\,, \qquad\qquad f_\kappa = \mathcal{O}(1)\,, \tag{3.2.16}$$

and obtain, to linear order in κ_0,

$$\begin{aligned} V(r) = \frac{4e^{-2ar}}{3M^2r^2} \Bigg\{ & a\Big((ar+3)\Lambda^3 - 3e^{ar}W_0\Big)\Lambda^3 \\ & + \kappa_0 \Bigg[12a\Big((ar+3)\Lambda^3 - 3e^{ar}W_0\Big)\Lambda^3 f_\kappa \\ & \quad - 6\Big((2ar+3)\Lambda^3 - 3e^{ar}W_0\Big)\Big(\Lambda^3 - e^{ar}W_0\Big)\frac{\partial f_\kappa}{\partial r} \\ & \quad - r\Big((2ar+3)\Lambda^3 - 3e^{ar}W_0\Big)^2 \frac{\partial^2 f_\kappa}{\partial r^2}\Bigg]\Bigg\}\,, \end{aligned} \tag{3.2.17}$$

where we already set the axion to its VEV, $\langle T_2 \rangle = \pi/a$. This potential will only be of almost no-scale type, if the terms given in the first line can be canceled up to a contribution $\mathcal{O}(\kappa_0)$.

We now seek for a stable Minkowski vacuum. After introducing an additional expansion parameter,

$$\omega \equiv \frac{\Lambda^3 e^{-a}}{W_0}\,, \tag{3.2.18}$$

and demanding $r_0 = 1$, the stationarity and Minkowski conditions imply

$$f_\kappa(1) = -\frac{1}{3\kappa_0} - \frac{1}{2a\omega}\left.\frac{\partial f_\kappa}{\partial r}\right|_{r=1} - \frac{1}{4a\omega}\left.\frac{\partial^2 f_\kappa}{\partial r^2}\right|_{r=1} + \mathcal{O}(1)\,. \tag{3.2.19}$$

We realize that f_κ is $\mathcal{O}(1)$, as required for the model to be well defined, if and only if

$$a\omega \approx \kappa_0 \tag{3.2.20}$$

and the three leading terms cancel out. This condition constitutes a significant amount of fine-tuning between parameters of the Kähler correction and the superpotential. Let us assume that $\kappa_0 = cLM$, where $c = \mathcal{O}(10^{-2})$ is a loop suppression factor. This holds *e.g.* if the κ correction is generated by Casimir energy, cf. the following section. Then we obtain

$$L^4 \approx \frac{\hat{c}^{-1}}{MW_0}\frac{16\pi^2}{3Ng_4^8}\exp\left[-\frac{16\pi^2}{3Ng_4^2}\right], \tag{3.2.21}$$

where $\hat{c} = \mathcal{O}(10^{-4})$. Notice that L depends on W_0 explicitely, which corresponds to the fact that both the Kähler and the superpotential corrections have to be of the same order. Hence, the fixing of L cannot be disentangled from the fine-tuning of the vacuum energy. Thus we cannot construct a viable stabilization mechanism, where the size of the extra dimension would be determined by an independent physical mass scale in the model.

3.3 Perturbative stabilization of the radion

In the previous section, we discussed how sub-leading Kähler corrections modify the no-scale scalar potential such that a stable, non-supersymmetric Minkowski vacuum becomes possible at leading order in the κ correction. We shall now present a specific example, where the Casimir energy of bulk fields and the effects of localized Fayet-Iliopoulos (FI) terms contribute to κ. We will show that the size L of the extra dimension can be explicitly calculated in terms of the model parameters, which control the order of magnitude of the κ correction. Beforehand we shall give an overview of those contributions to the radion effective potential, which generically appear as corrections to the Kähler potential, including the Casimir energy, a small warp factor, and also stringy corrections.

3.3.1 Generic contributions to κ

In general, κ receives a contribution from the Casimir energy of the gravitational multiplet [93] and other massless bulk fields,

$$\Delta\Omega_{\rm C}(r) = -\frac{1}{2L^2}\left(Ar^3 + 3Br^2 + \frac{C}{r^2}\right) \equiv 3M^2 r\kappa_{\rm C}(r), \tag{3.3.1}$$

which, according to (3.2.9), corresponds to the potential

$$V_{\rm C}^{(1)}(r) = \frac{3|W|^2}{M^4L^2r^2}\left(Ar + B + \frac{C}{r^4}\right). \tag{3.3.2}$$

The Casimir energy (3.3.2) vanishes for $W = 0$, i.e. for unbroken supersymmetry. The constant C is determined by the number of massless degrees of freedom in the bulk and will be specified below. More detail is given in appendix B. The constants A and B correspond to bulk and brane tensions, respectively. They are needed for the regularization of the divergent Casimir energy and depend on the renormalization scale, cf. [15, 94]. These constants have been used to stabilize the radion at a minimum with vanishing cosmological constant [81].[7] Strictly speaking, any stabilization mechanism which relies on the fixing of these parameters is only viable at one specific scale. If we consider the dynamics of radion stabilization on cosmological time scales, we may relate the renormalization scale to the Hubble parameter. Due to the scale dependence of A and B, the vacuum energy itself becomes a function of the Hubble rate. Hence, we obtain an effective cosmological constant, which depends on time.[8]

Our expansion around the no-scale potential is consistent if A and B are $\mathcal{O}(\kappa)$. For simplicity, we choose $A = B = 0$ in the following and neglect the tiny present time cosmological constant. This will allow us to fix the remaining free parameters of the model in an unambigious way, and to quantify the corresponding fine-tuning. Nevertheless, we shall emphasize that our stabilization mechanism can provide a Minkowski vacuum for any choice of the constants A and B, as long as they remain $\mathcal{O}(\kappa)$. We note that we are not able to contribute to the solution of the cosmological constant problem. However, it is worthwile to keep in mind that the vacuum energy in our set-up is a dynamical quantity, *not* a genuine cosmological constant.

In the case of massive bulk fields, the Casimir energy depends on their masses. The resulting term in the effective radion potential is known to take the form [81, 84]

$$\begin{aligned} V_{\mathrm{C}'}^{(1)}(r) = \frac{3|W|^2}{M^4 L^2 r^2} \frac{C'}{r^4} \bigg[& \frac{M_{\mathrm{bulk}}^2 L^2 r^2}{3} \mathrm{Li}_1\left(e^{-M_{\mathrm{bulk}} L\, r}\right) \\ & + M_{\mathrm{bulk}} L\, r \,\mathrm{Li}_2\left(e^{-M_{\mathrm{bulk}} L\, r}\right) + \mathrm{Li}_3\left(e^{-M_{\mathrm{bulk}} L\, r}\right) \bigg] , \end{aligned} \tag{3.3.3}$$

with the polylogarithmic functions

$$\mathrm{Li}_s\left(e^{-M_{\mathrm{bulk}} L\, r}\right) \equiv \sum_{k=1}^{\infty} \frac{e^{-k M_{\mathrm{bulk}} L\, r}}{k^s} . \tag{3.3.4}$$

The constant C' in (3.3.3) is related to the number of degrees of freedom with mass M_{bulk}, and will be specified below. Notice that $\kappa_{\mathrm{C}'}(r)$ can be obtained by integrating (3.2.9) for the potential (3.3.3), cf. [93], which is, however, not necessary for the following calculations.

The radion potential also receives corrections in the presence of brane-localized kinetic terms [97], which contribute to the Casimir energy as follows,

$$V_{\mathrm{D}}^{(1)} = \frac{3|W|^2}{M^4 L^2 r^2} \frac{D}{r^4} \frac{\ln(\alpha r)}{\alpha r} , \tag{3.3.5}$$

[7] Notice, however, that the choice of the constants has to be consistent with supersymmetry [95].

[8] In the context of semi-classical Einstein gravity, implications of a similar idea were analyzed in [96].

where $\alpha \sim g_4^{-2} \sim LM_5$ corresponds to the ratio of the 5D fundamental scale and the compactification scale. The constant D depends on the field content of the model. This contribution can be viewed as a two-loop effect and is sub-leading *wrt* the terms which arise at one loop, as long as $LM_5 > 1$. However, it has been shown that the interplay of one and two loop contributions can also lead to radion stabilization [98]. This mechanism is indeed a working example of the almost no-scale scenario, and can be reformulated in terms of our κ formalism. To be precise, we give an explicit expression for the κ correction corresponding to the model of [97],

$$\kappa_{\rm CD} = -\frac{C}{6r^3} - \frac{\hat{D}\ln(\hat{\alpha}r)}{12r^4} - kr\,, \tag{3.3.6}$$

where $\hat{D} = Dg_4^2$ and $\hat{\alpha} = e^{7/12}\alpha$. The term kr is due to a small warp-factor [84],

$$\Omega_{\rm warp} = \frac{3M^2}{k}\left(1 - e^{kr}\right) \simeq \Omega_0 - 3M^2 k\, r^2\,, \tag{3.3.7}$$

and provides a suitable uplifting mechanism.

The various effects we discussed so far already allow for a plethora of specific examples. As we shall illustrate in the next subsection, cf. Fig. 3.5, three different contributions to κ are needed to stabilize the radion in a Minkowski vacuum. First, a positive term that dominates at small distances, second, a positive term that dominates at large distances,[9] and third, a negative contribution which is relevant around $r = r_0$. Schematically, the potential can be written as

$$V(r) = X_1 r^{-x_1} - X_2 r^{-x_2} + X_3 r^{-x_3}\,, \qquad x_1 > x_2 > x_3 > 0\,, \tag{3.3.8}$$

and all X_i positive. Obviously, (3.3.6) gives rise to a potential of this form, if the brane-localized part is positive, and the bulk part dominated by gravity and gauge vector multiplets (leading to $C < 0$). In the general case, many different contributions to κ are possible. As long as the potential remains positive for $r \to 0$ and $r \to \infty$, and a negative term is present at leading order, any additional term will only modify, but not invalidate the position of a minimum. However, it is crucial to make sure that these additional terms are at most of the same order as the three relevant ones.

Stringy corrections to the Kähler potential include supersymmetric loop corrections and α'-corrections, which should be treated as additional contributions to the function κ. To give a specific example, we quote a result obtained for Calabi-Yau compactifications of the heterotic string [99]. There, α'-corrections give rise to a term

$$\kappa_{\alpha'} = \frac{\chi}{r}\,, \tag{3.3.9}$$

where χ is a real constant determined by the Euler characteristic of the Calabi-Yau manifold, and r is to be interpreted as the real part of a universal Kähler modulus, the scale factor of the internal metric. We note that χ is generically of $\mathcal{O}(10^2)$.

[9] Otherwise, an anti-de Sitter vacuum might appear at finite field value, thus rendering the Minkowski vacuum meta-stable.

We are not aware of any explicit results on stringy corrections in the framework of heterotic orbifold models with anisotropic compactification. Hence, we do not know how to represent the effect of stringy corrections in the field theoretic 5D toy model, which will be discussed in the following subsection. We have to keep in mind that our results might be altered if a consistent string embedding were available and realized. However, we are confident that α'-corrections would be treatable within our κ formalism, dominating the almost no-scale scalar potential at large volume.

3.3.2 Example: GUT scale extra dimension from brane localized FI terms

In orbifold compactifications of the heterotic string, FI terms of anomalous U(1) gauge symmetries generically arise at fixed points [13, 82]. They induce a non-trivial vacuum configuration of the scalar sector: Bulk fields that are charged under the U(1) symmetry develop vacuum expectation values and become massive. These VEVs ensure vanishing F- and D-terms in the bulk and at the fixed points. In the simplest case of one hypermultiplet, containing the N=1 chiral fields H and H^c, one has

$$\Delta\Omega_{\text{bulk}} = H\bar{H} + H^c\bar{H}^c\,, \tag{3.3.10}$$

$$\Delta\Omega_{\text{brane}} = \frac{\lambda'}{M_5^3}\left(H\bar{H} + H^c\bar{H}^c\right)X\bar{X}\,. \tag{3.3.11}$$

A detailed analysis [100] shows that, if the sum of the FI terms is non-zero, one of the two scalars, say H, develops an r-dependent VEV, while $\langle H^c\rangle = 0$. In the 4D theory[10] one then obtains

$$\begin{aligned}\Delta\Omega_{\text{FI}} &= \int_0^L dy\left[r\langle H\bar{H}\rangle + \delta(y-L)\,\frac{\lambda'}{M_5^3}\langle H\bar{H}\rangle\,X\bar{X}\right]\\ &= \xi\left(1 + \frac{\lambda X\bar{X}}{M^2 r}\right) \equiv 3M^2 r\kappa_{\text{FI}}(r,\sigma)\,,\end{aligned} \tag{3.3.12}$$

cf. (3.2.4). Here ξ is the sum of the two FI terms localized at the fixed points at $y=0$ and $y=L$, and $M_5^3 L = M^2$, provided $r_0 = 1$. The different couplings λ and λ' reflect the discrepancy between the condensate at $y=L$ and its average value. The function κ_{FI} corresponds to the effective radion potential

$$V_{\text{FI}}^{(1)}(r,\sigma) = -\frac{2\lambda\sigma}{r^3}\frac{\xi|W|^2}{M^4}\,, \tag{3.3.13}$$

cf. (3.2.9). Notice that the r-dependent background field value results in a deformation of the Kaluza-Klein spectrum. The special case $\xi = 0$, accompanied by strong localization of the bulk fields, was discussed in [102]. Here we consider the case of nearly constant VEVs, such that the backreaction on the internal geometry is negligible. Hence, the flat orbifold remains to be a viable approximation.

[10] We integrate out the heavy modes already at the level of the Kähler potential, and refer to the discussion in [101].

Furthermore, the VEV $\langle H\rangle$ breaks the anomalous U(1), and the corresponding gauge boson acquires a mass $M_V = \mathcal{O}(\sqrt{\xi})$, like the hyperscalars. For simplicity, we introduce a common mass parameter for the U(1) vector- and massive hypermultiplets.[11] $\mathcal{O}(1)$ mass differences would not change our results qualitatively. With $\xi = \mathcal{O}(M_{\rm GUT}^2)$, cf. [13], we set

$$M_H = M_V = M_{\rm bulk} = \mathcal{O}(M_{\rm GUT}) \,. \tag{3.3.14}$$

In terms of the dimensionless parameter ℓ, defined by

$$L = \frac{\ell}{M_{\rm bulk}} \,, \tag{3.3.15}$$

the resulting radion effective potential reads, to leading order in κ,

$$\begin{aligned} V^{(1)}(r,\sigma) &= V^{(1)}_{\rm FI}(r,\sigma) + V^{(1)}_{\rm C}(r) + V^{(1)}_{\rm C'}(r) \\ &= \frac{3|W|^2}{M^2 r^2}\frac{M_{\rm bulk}^2}{M^2}\left[-\frac{2\lambda\sigma}{3r^3}\frac{\xi}{M_{\rm bulk}^2} + \frac{C}{\ell^2 r^4}\right. \\ &\qquad \left. + \frac{C'}{\ell^2 r^4}\left(\frac{\ell^2 r^2}{3}{\rm Li}_1\left(e^{-\ell r}\right) + \ell r {\rm Li}_2\left(e^{-\ell r}\right) + {\rm Li}_3\left(e^{-\ell r}\right)\right)\right] \,. \end{aligned} \tag{3.3.16}$$

where we collected the contributions from (3.3.2), (3.3.3) and (3.3.13). The constant C (C') is determined by the number of massless (massive) vector- and hypermultiplets n_V, n_H (n'_V, n'_H), respectively,

$$C = \frac{\zeta(3)}{32\pi^2}\left(n_H - n_V - 2\right) \,, \qquad C' = \frac{1}{32\pi^2}\left(n'_H - n'_V\right) \,. \tag{3.3.17}$$

In the minimal case, $n_H = n_V = 0$, only the supergravity multiplet contributes to the massless sector. Notice that only hypermultiplets can give rise to positive contributions, leading to repulsive behaviour at small distances. Hence, a local minimum can be obtained for $C < 0$, $C' > 0$ (recall the discussion in the previous subsection), and therefore

$$n_H < n_V + 2 \,, \qquad n'_H > n'_V \,. \tag{3.3.18}$$

With $n'_H - n'_V = \mathcal{O}(10^2)$, as it is common in heterotic orbifolds [11], we obtain the parameter range

$$10^{-2} \lesssim C' \lesssim 1 \,. \tag{3.3.19}$$

We shall now demonstrate that the potential (3.3.16) admits a stable minimum with vanishing vacuum energy and determine the corresponding compactification scale. Therefore we have to solve the equations

$$\partial_r V^{(1)}\Big|_{r_0,\sigma_0} = 0 \,, \qquad V^{(1)}\Big|_{r_0,\sigma_0} = 0 \,. \tag{3.3.20}$$

[11] This contribution to the Casimir energy was disregarded in [15].

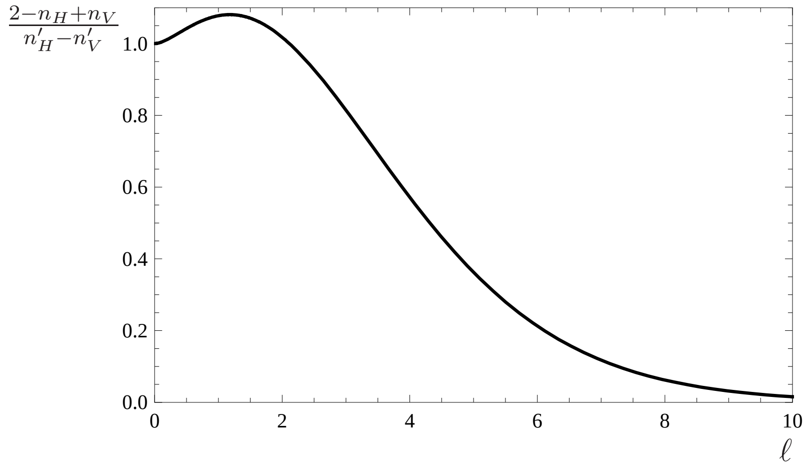

Figure 3.2: *The condition (3.3.21) corresponds to a relation between the bulk field content and the size of the compact dimension $L = \ell/M_{bulk}$. The ratio of multiplicities we plotted here exhibits a maximum at $\hat{\ell} \simeq 1.2$.*

Imposing $r_0 = 1$, we obtain two conditions on the quantities ℓ and σ_0,

$$\frac{C}{C'} = \frac{\ell^2}{3}\left[\frac{\ell}{1-e^{\ell}} - 2\,\mathrm{Li}_1(e^{-\ell})\right] - \ell\,\mathrm{Li}_2(e^{-\ell}) - \mathrm{Li}_3(e^{-\ell})\,, \tag{3.3.21}$$

$$\frac{\lambda\sigma_0}{C'} = \frac{M_{\text{bulk}}^2}{2\xi}\left[\frac{\ell}{1-e^{\ell}} - \mathrm{Li}_1(e^{-\ell})\right]. \tag{3.3.22}$$

The RHS of (3.3.21) is negative and bounded from below, which translates into a consistency condition on the field content (cf. Fig. 3.2),

$$0 < \frac{2-n_H+n_V}{n'_H-n'_V} \lesssim 1.1\,. \tag{3.3.23}$$

If this bound is satisfied in a given model with specified field content, (3.3.21) can be solved for ℓ. For local minima of the radion potential, this corresponds to the size L of the extra dimension, recall (3.3.15). Notice that the LHS of (3.3.23) is always rational, hence the allowed values of L are actually discrete.

Expanding the potential (3.3.16) around the local Minkowski vacuum and using (3.3.22), we obtain for the radion mass

$$\frac{m_\rho^2}{m_{3/2}^2} = C'\left(\frac{M_{\text{bulk}}}{M}\right)^2 f(\ell)\,, \tag{3.3.24}$$

where

$$f(\ell) = \frac{2}{3}\left[\frac{\ell\left(1+(\ell-1)e^{\ell}\right)}{(e^{\ell}-1)^2} - \mathrm{Li}_1(e^{-\ell})\right]. \tag{3.3.25}$$

The radion mass vanishes for $\ell = \hat{\ell} \simeq 1.2$, where the ratio C/C' is extremized (cf. Figs. 3.2, 3.3). For $\ell > \hat{\ell}$, m_ρ^2 is positive and we find a stable Minkowski vacuum

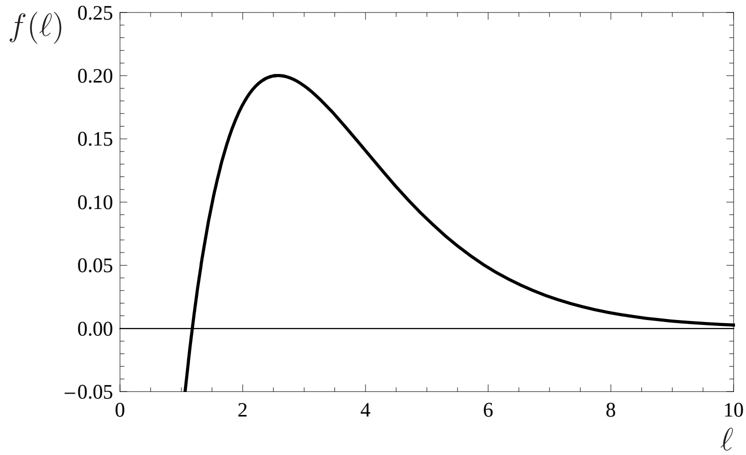

Figure 3.3: *The function $f(\ell)$, which determines the radion mass m_ρ, cf. (3.3.24).*

with

$$L \gtrsim 1/M_{\rm GUT}\,, \tag{3.3.26}$$

recall (3.3.14). From Fig. 3.3 we also get an upper bound on the radion mass. For $C' \lesssim 1$, one obtains

$$\frac{m_\rho^2}{m_{3/2}^2} \lesssim 0.2 \left(\frac{M_{\rm bulk}}{M}\right)^2 . \tag{3.3.27}$$

Notice that C and C', by definition, already contain a loop suppression factor which, however, can be compensated by including more and more fields in the model set-up. On the other hand, the volume suppression $M_{\rm bulk}M^{-1} \sim (LM)^{-1}$ turns out to be generic.

Having determined the size L of the compact dimension from (3.3.21), we still have to satisfy (3.3.22). This is a condition on $\lambda\sigma_0$. Since the RHS of (3.3.22) is negative, the coupling λ has to be negative as well. Given λ as a model parameter, this yields a condition on the expectation value $\sigma_0 = X_0\bar{X}_0/M^2$, and therefore on the parameters of the brane superpotential determining the VEV of the hidden brane singlet. This condition represents the unavoidable fine-tuning of the vacuum energy. With $M_{\rm bulk}^2/\xi \simeq 1$, one obtains the upper bound

$$|\lambda|\sigma_0 \lesssim 0.4\, C'\,. \tag{3.3.28}$$

Hence, for $C' < 1$ and $|\lambda| = \mathcal{O}(1)$, the expectation value of X is still smaller than the Planck mass.

As an example, consider the superpotential

$$W(X) = m_{3/2}M^2 \left[2\frac{X}{\sqrt{\sigma_0}M} + \left(\frac{X}{\sqrt{\sigma_0}M}\right)^2 \right] , \tag{3.3.29}$$

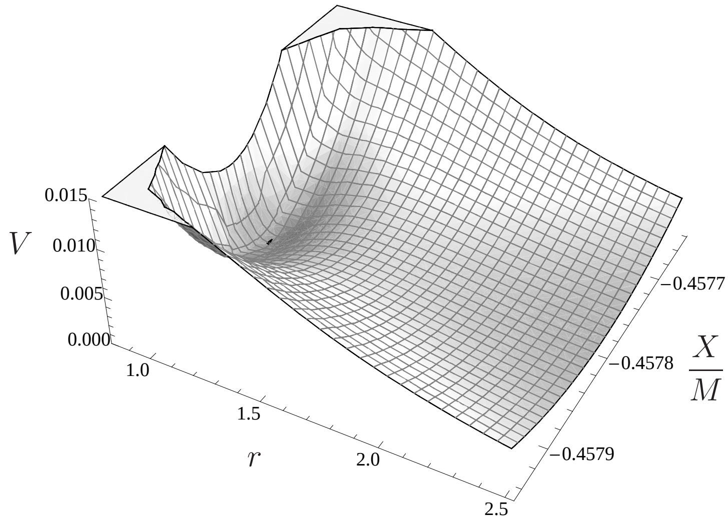

Figure 3.4: *The two-field potential $V_F(X, r)$ given by (3.3.30) is plotted in units of $m_{3/2}^2 M_{bulk}^2$, for the choice $\xi/M_{bulk}^2 = 1$, $\lambda = -1$, $C' \simeq 1.0$ and $\ell \simeq 2.1$.*

which gives $X = -\sqrt{\sigma_0} M$ up to terms $\mathcal{O}(\Delta X/\sqrt{\sigma_0})$, cf. (3.2.7). We note that (3.3.29) may represent the expansion of a non-perturbative brane superpotential up to second order in the field X. The corresponding two-field potential (3.2.5) can be simplified to

$$\begin{aligned} V_F(X, r) = &\frac{1}{r^2} W_X \bar{W}_{\bar{X}} - \lambda\xi \frac{\left(X W_X \bar{W} + W \bar{X} \bar{W}_{\bar{X}}\right)}{M^4 r^4} \\ &+ V^{(1)}(r, \sigma) + \mathcal{O}(|W_X|^2 \kappa) \,. \end{aligned} \tag{3.3.30}$$

We present a plot of the vicinity of X_0 and r_0 in Fig. 3.4, which illustrates the almost no-scale structure of the potential, when compared to the no-scale case shown in Fig. 1.

Figure 3.5 shows the effective radion potential $V_F(X_0, r)$ that results for $\ell \simeq 2.1$, corresponding to $n'_H - n'_V = 2 - n_H + n_V$. The three different contributions, according to our general discussion in the previous subsection, are also plotted separately. The stable Minkowski vacuum is separated from the runaway solution by a barrier of height

$$V_{\text{barrier}} \ll m_{3/2}^2 M_{\text{GUT}}^2 \,. \tag{3.3.31}$$

Vanishing of the vacuum energy in the local minimum requires a precise cancellation between three different contributions to the potential, all $\mathcal{O}(m_{3/2}^2 M_{\text{GUT}}^2)$. One may also introduce a small positive vacuum energy, $\Lambda \sim (10^{-3}\,\text{eV})^4$, which would then correspond to a fine-tuning

$$\frac{\Lambda}{m_{3/2}^2 M_{\text{GUT}}^2} \sim 10^{-80} \,, \tag{3.3.32}$$

provided a gravitino mass $m_{3/2} = \mathcal{O}(100\,\text{GeV})$.

It is instructive to compare the described mechanism of radion stabilization with the approach of [84]. In both cases, supersymmetry is broken by a superpotential localized

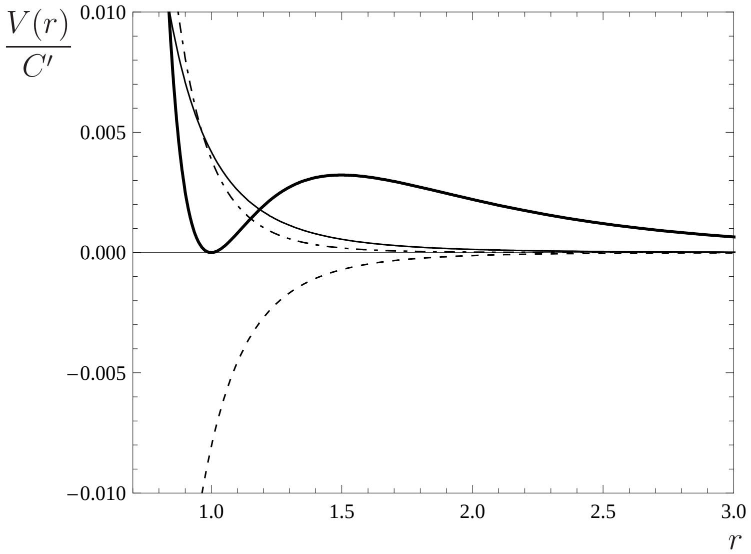

Figure 3.5: *The radion effective potential $V(r)/C'$ (bold curve), in units of $m_{3/2}^2 M_{bulk}^2$ for $\ell \simeq 2.1$. The dashed (dot-dashed) and the thin curve are the contributions to the Casimir energy of massless (massive) bulk degrees of freedom and the FI-term induced contribution, respectively, scaled by a factor $1/(100C')$.*

on a brane, and stabilization is achieved by the Casimir energy of massive and massless bulk fields. In [84], the bulk mass is a free parameter. In our case, it is induced by localized FI terms via the Higgs mechanism, leading to $M_{\text{bulk}} \simeq M_{\text{GUT}} \gg m_{3/2}$. On the other hand, the brane field that provides the non-zero superpotential couples to massive bulk fields. This yields an additional contribution to the potential, which is naturally of the same order of magnitude as the Casimir energy, since $LM_{\text{bulk}} = \mathcal{O}(1)$. As a consequence, we can realize a locally stable Minkowski or de Sitter vacuum without introducing an additional uplifting mechanism. To our knowledge, this is a genuine feature of our model.

3.4 Outlook: the 6D case

The starting point of this chapter was the suggestion of [15] to stabilize size and shape of two compact extra dimensions by the interplay of Casimir energy and brane localized FI terms. In the simpler case of a 5D set-up, we demonstrated how this idea can be realized consistently in the framework of almost no-scale supergravity. Having understood the single field class of models, we shall now turn back to our original intention. The (tree level) Kähler potential that results from a torus compactification of 6D supergravity is also of the no-scale type. Hence, we may hope that the 5D results can be generalized to the more complicated case with three complex scalars. If the superpotential only depends on the brane singlet, the resulting scalar potential is indeed very similar to the former case,[12] cf. (2.3.18),

$$V_F = \frac{1}{s\tau_2} W_X \bar{W}_{\bar{X}} \,. \tag{3.4.1}$$

However, we already know from [83] that both the Casimir energy and the contribution which is induced by the FI terms depend on the volume of the internal space. Hence, in the corresponding κ correction the dilaton S and the Kähler modulus T appear only in the combination $r^2 = \sqrt{st}$. Due to this degeneracy we cannot hope to achieve stabilization of all three moduli via perturbative Kähler corrections. On the other hand, stabilizing the dilaton via non-perturbative effects, such as gaugino condensation, will unavoidably spoil the flatness of the leading order potential, which turned out to be crucial for the success of the almost no-scale scenario in the single field case.

In this section, we will briefly discuss three different scenarios of dilaton stabilization and their limitations. With

$$\Omega_0 = \left[\left(\frac{T+\bar{T}}{2} - \frac{X\bar{X}}{M^2}\right)\left(\frac{S+\bar{S}}{2}\right)\left(\frac{U+\bar{U}}{2}\right)\right]^{1/3} = r^{4/3}\tau_2^{1/3} \,, \tag{3.4.2}$$

we can write

$$\begin{aligned} \kappa &= \kappa_{\mathrm{C}} + \kappa_{\mathrm{FI}} \\ &= -\frac{1}{st\tau_2}\Big[C_1 + C_2\tau_2^3 + C_3 f_\kappa(\tau_1,\tau_2)\Big] + \frac{\hat{\xi}}{(st)^{1/2}}\left(1 + \frac{\lambda X\bar{X}}{M^2(st)^{1/2}}\right) \,, \end{aligned} \tag{3.4.3}$$

where the C_i are constants depending on the field content of the model, and $\hat{\xi}$ is a non-zero linear combination of different brane localized FI terms. We took the result for the one loop Kähler correction from [67], which coincides with the findings of [83] for the Casimir energy of massless bulk fields. The contribution from massive fields is not yet included. Finally, $f_\kappa(\tau_1,\tau_2)$ is a complicated function of the shape parameters, cf. [81], for which we quote the following expansion [83],

$$f_\kappa(\tau_1,\tau_2) \simeq \frac{1}{\tau_2} - \frac{2(\tau_1 - 1/2)^2}{\tau_2^3} \,, \tag{3.4.4}$$

around one of the possible minima of τ_1. It is well known [103] that the Casimir energy of massless bulk fields can lead to the stabilization of τ_1 and τ_2, after supersymmetry is broken. Furthermore, we may hope to stabilize t exactly along the lines of the 5D scenario. However, as we have demonstrated for the single field case, our stabilization mechanism is spoiled if a moduli-dependent tree level term dominates over the quantum contribution. Hence, it is crucial not to generate such a leading order term in the process of dilaton fixing.

[12] Notice, however, that this term differs from the brane tension given in [83], and is therefore not so easily compensated within the scale dependent contribution arising at one loop.

3.4.1 A dilaton dominated scenario

We first discuss the simplest case without any dynamical brane field being present in the low energy effective action. In this case, the Kähler potential is simply

$$M^{-2}K = -\ln\left(\frac{S+\bar{S}}{2}\right) - \ln\left(\frac{T+\bar{T}}{2}\right) - \ln\left(\frac{U+\bar{U}}{2}\right) . \tag{3.4.5}$$

We take the superpotential to be a function of the 4D dilaton alone,

$$W = W_0 + W_1(S) , \tag{3.4.6}$$

where W_0 is a constant (brane) superpotential and W_1 is a non-perturbative contribution, *e.g.* due to gaugino condensation in the bulk. The resulting scalar potential is minimized if either

$$W_{SS} = 0 , \tag{3.4.7}$$

in which case the dilaton remains massless, or[13]

$$W - M(S+\bar{S})W_S = 0 , \tag{3.4.8}$$

which is equivalent to $F_S = 0$, i.e. SUSY is not broken by the dilaton F term. If, in addition, we impose the Minkowski condition, we find that necessarily $W = 0$. Hence, supersymmetry is preserved, and it is impossible to generate an effective moduli potential by means of Casimir type Kähler corrections.

3.4.2 Brane induced SUSY breaking

We will now analyze the situation that corresponds to the set-up of [15, 83] and our 5D toy model,

$$M^{-2}K = -\ln\left(\frac{S+\bar{S}}{2}\right) - \ln\left(\frac{T+\bar{T}}{2} - \frac{X\bar{X}}{M^2}\right) - \ln\left(\frac{U+\bar{U}}{2}\right) , \tag{3.4.9}$$

and the superpotential given by

$$W = W_0(X) + W_1(S) . \tag{3.4.10}$$

The resulting scalar potential is

$$V_F = \frac{1}{s\tau_2} W_X \bar{W}_{\bar{X}} - \frac{2}{t\tau_2}\left[\frac{1}{M}\big(W\bar{W}_{\bar{S}} + W_S\bar{W}\big) - (S+\bar{S})W_S\bar{W}_{\bar{S}}\right] + \mathcal{O}(\kappa) . \tag{3.4.11}$$

As expected, extremizing the potential *wrt* X implies $W_X = 0$. Using this, it follows again that $F_S = 0$ guarantees stationarity of the potential *wrt* S. However, applying these conditions we obtain

$$V_F = -\frac{2(S+\bar{S})}{t\tau_2}\big(1 + 3\kappa + ...\big) W_S \bar{W}_{\bar{S}} , \tag{3.4.12}$$

[13] As always, we take the modulus field to be dimensionless. For consistency we define $W_S = \partial_{(MS)} W$.

where the parentheses refer to additional terms including first and second deratives of κ. Hence, we realize that the first term always dominates the potential and therefore spoils the possibility of moduli stabilization via the κ correction: After the dilaton is stabilized, we are left with a runaway potential for t and τ_2 to leading order.

3.4.3 Bulk induced SUSY breaking

We conclude that the dilaton problem forces us to go beyond the almost no-scale scenario. Let us consider the Kähler potential

$$M^{-2}K = -\ln\left(\frac{T+\bar{T}}{2}\right) - \ln\left(\frac{S+\bar{S}}{2}\right) - \ln\left(\frac{U+\bar{U}}{2}\right) + M^{-2}K_{\text{bulk}}(X,\bar{X})\,, \tag{3.4.13}$$

where we now assume the singlet X to be part of the bulk spectrum, *e.g.* resulting from the decomposition of a hypermultiplet. This Kähler potential is no longer of the no-scale type, hence we can hope to circumvent the problems associated to the no-go theorems of [62, 63]. We restrict ourselves to the simplest case,

$$K_{\text{bulk}} = X\bar{X}\,, \tag{3.4.14}$$

and some qualitative remarks. We note that the two-field case,

$$K = -n_Y\, M^2 \ln(Y+\bar{Y}) + X\bar{X}\,, \tag{3.4.15}$$

was extensively discussed in [104] with $Y = T, n_Y = 3$, and in [105] with $Y = S$ and $n_Y = 1$.

As in the previous scenario, we take the superpotential to be a function of both S and X, cf. (3.4.10).[14] Again, we find that the scalar potential is extremized *wrt* S, if we take $F_S = 0$ and $W_X = 0$. Using these identities, we get

$$V_F = \frac{2(S+\bar{S})e^{X\bar{X}/M^2}}{t\tau_2}\left(\frac{X\bar{X}}{M^2} - 1\right) W_S \bar{W}_{\bar{S}} + \mathcal{O}(\kappa)\,. \tag{3.4.16}$$

Minimizing *wrt* X, we obtain the condition[15]

$$\frac{M^4 W_{XX}}{\bar{X}^2} = -M(S+\bar{S})W_S = W\,, \tag{3.4.17}$$

where the second equality reflects $F_S = 0$. We assume that it is possible to adjust the parameters of the superpotential such that $X = \bar{X} = M$, and that (3.4.17) can be

[14] For consistency, brane fields must also be present in the set-up; the superpotential can only depend on X by means of bulk-brane coupling terms. A non-vanishing brane singlet VEV can then induce a non-vanishing VEV of W_X.

[15] Here we face a serious tuning problem: The contributions $W_0(X)$ and $W_1(S)$ are supposed to be of completely different origin, however, the involved parameters have to be carefully adjusted to admit a viable solution.

solved for a phenomenologically viable dilaton VEV. Then the scalar potential vanishes to leading order, and the matrix $V_{i\bar{j}}$ has eigenvalues (up to $\mathcal{O}(\kappa)$)

$$V_{T\bar{T}} = V_{U\bar{U}} = 0\,, \tag{3.4.18}$$

$$V_{S\bar{S}} = \frac{2\,e}{t_0\tau_{2,0}}\left[(S_0+\bar{S}_0)W_{SS}\bar{W}_{\bar{S}\bar{S}} + \frac{W_{XX}\bar{W}_{\bar{X}\bar{X}}}{(S_0+\bar{S}_0)^3}\right]\,, \tag{3.4.19}$$

$$V_{X\bar{X}} = \frac{12\,W_{XX}\bar{W}_{\bar{X}\bar{X}}}{t_0\tau_{2,0}(S_0+\bar{S}_0)}\,, \tag{3.4.20}$$

where all scalars are taken to be replaced by their VEVs. After integrating out X we obtain, to leading order in κ,

$$\begin{aligned} V_F = -\frac{12e\,s^2\,W_S\bar{W}_{\bar{S}}}{A\tau_2}\Bigg[&\tau_2\left(\frac{\partial\kappa}{\partial\tau}+\frac{\partial\kappa}{\partial\bar{\tau}}+2\tau_2\frac{\partial^2\kappa}{\partial\tau\partial\bar{\tau}}\right) \\ &+A\,\frac{\partial\kappa}{\partial A}+\frac{A^2}{2}\frac{\partial^2\kappa}{\partial A^2}+A\tau_2\left(\frac{\partial^2\kappa}{\partial A\partial\tau}+\frac{\partial^2\kappa}{\partial A\partial\bar{\tau}}\right)\Bigg]\,, \end{aligned} \tag{3.4.21}$$

where now $s = s_0 + \Delta s$, and we defined $A \equiv s_0 t$ to account for the degeneracy in κ. Note that the shift in s may also induce a non-zero expectation value of $F_S(\Delta s)$. The systematic expansion in κ remains valid as long as $F_S(\Delta s) = \mathcal{O}(\kappa)$.

We presented a simple example, how to obtain a Minkowski vacuum at leading order of the scalar potential, with both the dilaton and the SUSY breaking singlet fixed and massive. Thereby, we realized the necessary prerequisite for the stabilization of τ_1, τ_2 and A via quantum corrections to the Kähler potential, along the lines of [83]. We did not work out the unavoidable fine-tuning of the superpotential parameters (cf. [64]), nor did we investigate how to obtain the phenomenologically desired value of $\mathrm{Re}\,S \simeq 2$. In a similar approach, this was shown to be possible in [106]. We leave the details for future work in a more realistic set-up.

3.5 Summary

In this chapter, we studied moduli stabilization within the low energy effective field theories obtained by compactifying five- and six-dimensional supergravity on flat orbifolds. In both cases, the moduli Kähler potential is of no-scale form at tree level. The 5D model gives rise to a single modulus field in the low energy effective action, whose real part is the radion. We showed that the radion can be stabilized in a non-supersymmetric Minkowski or de Sitter vacuum, in the framework of almost no-scale supergravity. In our set-up, supersymmetry breaking is induced by a chiral superfield localized at one of the fixed points, resulting in a non-vanishing superpotential. This generates a non-zero expectation value of the F-term of the radion multiplet, leading to radion mediated SUSY breaking.

Our κ formalism is a general description of almost no-scale models with a single modulus: The radion corresponds to an exactly flat direction at tree level, which is then lifted by perturbative corrections to the Kähler potential. We computed the effective radion potential to leading order in κ, and derived a model independent formula for the radion mass, which turns out to be suppressed compared to the gravitino mass. The axionic partner of the radion remains massless. The presence of light moduli is an unavoidable consequence of the proposed stabilization mechanism. This property is in contrast to models where the non-perturbative dependence of the superpotential on moduli fields plays a crucial role. In such models the moduli fields can be heavier, cf. [107, 108].

In the 6D case, we found that it is necessary to go beyond the almost no-scale scenario. Due to a degeneracy between $\mathrm{Re}\,S$ and $\mathrm{Re}\,T$ in the κ correction, the dilaton cannot be stabilized by quantum effects along with the Kähler and shape moduli. If the dilaton is stabilized by non-perturbative effects, the almost no-scale structure of the effective potential is spoiled. However, if supersymmetry is broken by means of a bulk matter field, it turns out to be possible to stabilize the dilaton, while ensuring the tree level flatness of the effective potential for the remaining moduli. Hence, we re-encounter a crucial property of our 5D almost no-scale scenario: The relevant scalar fields fall in two disparate sets, and a complete stabilization requires a two-step procedure. First, some fields (X and S) are already fixed at tree level or by non-perturbative effects incorporated in the superpotential. As a prerequisite for the next step, we have to impose stationarity also *wrt* to the remaining fields. This corresponds to vanishing vacuum energy at leading order. In a second step, the residual flat directions are lifted by perturbative Kähler corrections, leading to the stabilization of τ and t. As a consequence, the latter fields are parametrically lighter.

In the 5D case, we worked out a specific example, where the κ correction is generated by Casimir energy and brane localized Fayet-Iliopoulos terms, which give rise to supersymmetric mass terms of bulk fields. As a consequence, the size of the compact dimension is fixed at $L \gtrsim M_{\mathrm{bulk}}^{-1} \simeq M_{\mathrm{GUT}}^{-1} > M^{-1}$. The corresponding hierarchy of scales is reflected by the radion mass. Moreover, the FI terms induce vacuum expectation values of bulk hyperscalars. Their coupling to the hidden brane scalar, which is responsible for SUSY breaking, results in a direct contribution to the Kähler correction, being of the same order as the Casimir energy. Hence, the radion can be stabilized in a Minkowski or de Sitter vacuum without the need for an additional uplifting mechanism.

In addition, a tiny mass for the pseudoscalar partner of the radion, an axion, can be generated by non-perturbative effects of non-Abelian gauge theories. Depending on the cosmological evolution and initial conditions, coherent oscillations of such ultralight axions (and also the light radion) may yield an unacceptably large contribution to dark matter. Cosmological implications of the existence of light moduli will be discussed in the next chapter.

Chapter 4

Cosmology with a light modulus field

The prediction of moduli fields as light as the gravitino, or even lighter, is seriously challenged by observational cosmology, in particular if connected with a low scale of SUSY breaking, cf. [109]. If the scalar field is light enough to be stable on cosmological timescales, the various cosmological moduli problems are all related to the issue of initial conditions, where 'initial' refers to the field value after inflation or reheating.

Given a modulus potential of the type (3.3.16), cf. Fig. 3.5, we can distinguish three different regions of possible initial field values ϕ_{init}. First, for $\phi_{\text{init}} < \phi_0$, to the left of the minimum where the potential is steep, one faces the so-called overshooting problem [110,111]. Rolling down the steep potential, the field may gain enough kinetic energy to overcome the shallow barrier separating the (meta-)stable[1] vacuum from the runaway solution.

Second, in the proximity of ϕ_0, where a quadratic approximation is valid, the modulus field remains *frozen* to its initial value until the Hubble friction drops below the mass associated with the potential curvature. At this point, the modulus field starts to oscillate around the minimum, and the corresponding excitations may decay within the lifetime of the universe. The related moduli problem, also called Polonyi problem [113], can be circumvented if the modulus is heavier than ~ 40 TeV and decays early enough to avoid conflicts with the standard model of Big Bang nucleosynthesis (BBN) [114]. If, on the other hand, the modulus lifetime is large compared to the age of the universe, the energy density stored in the oscillations could overclose the universe. Depending on the modulus mass, this leads to a tight bound on the allowed range of initial conditions, i.e. the misalignment $\phi_{\text{init}} - \phi_0$ [115].

Finally, if $\phi_{\text{init}} > \phi_{\text{barrier}}$, decompactification is inevitable.[2] In this context, also thermal effects are important, as they can destabilize even heavy moduli [117]. At fi-

[1] If an uplift to the tiny present day cosmological constant is included, the resulting de Sitter vacuum is indeed metastable (cf. [112]), with a non-zero tunneling probability toward decompactification, but sufficiently long-lived compared to the age of the universe.

[2] At first sight, one may be tempted to distinguish also the case $\phi_{\text{init}} \simeq \phi_{\text{barrier}}$, where the modulus *sits and waits* in the vicinity of the barrier. However, even close to the maximum the potential is too strongly curved to admit a slow-roll solution, corresponding to an η parameter of $\mathcal{O}(1)$. This observation reflects the well-known η problem of inflationary model building in supergravity, cf. [116].

nite temperature, the gauge kinetic term acquires an expectation value, which induces a negative linear term in the effective potential of the modulus determining the gauge coupling [118]. Even a modulus which was already stabilized during inflation can be driven to runaway, if the reheating temperature is high enough to extinguish the minimum at finite field value. In the case of light moduli this effect is less relevant, since the end of inflation generically leaves the modulus field displaced from its low energy vacuum value; however, it strongly favors field values $\phi_{\text{init}} > \phi_{\text{barrier}}$, if the temperature dependent contribution to the potential is strong enough to overcome the Hubble friction.[3]

In this chapter, we analyze the cosmological implications of a modulus mass in the range 1-10 MeV. Within the almost no-scale scenario, such a small mass is consistent with a gravitino mass of 0.01-1 TeV and the fifth dimension being compactified at $M_{\text{comp}} \lesssim \mathcal{O}(M_{\text{GUT}})$, cf. Fig. 3.3. If the modulus were heavier, it would not be stable on timescales comparable to the age of the universe; if it were lighter, it would inflict dangerous variations of fundamental constants during BBN [120]. In particular, we are interested in a situation where the modulus field is stabilized close to the onset of BBN, but behaves like radiation during the preceding stage of radiation era, hence does not alter the standard picture of cosmic evolution. We shall demonstrate that steep exponential potentials admit a cosmological scaling solution of this type, which is also an attractor in a significant region of parameter space. Unfortunately, as we shall see, this evolutionary scenario generically implies overshooting. Alternatively, we consider the oscillatory scenario and derive a bound on the initial displacement of the field value using galactic gamma ray data.

The plan of the chapter is as follows. In section 4.1, we discuss some relevant couplings of moduli to observable fields, which are generic in our model set-up. Section 4.2 is devoted to the bounds on coherently oscillating scalar fields and their implications for our model of radion stabilization. In section 4.3, we present a general and systematic treatment of scalar dynamics in cosmology, and apply the results to the dynamics of the complex radion field during the radiation era. In section 4.4, as a second application, we investigate the viability of certain, recurrently accelerating solutions in cosmology. A brief summary is provided in section 4.5.

Part of the results of this chapter were published in [121], however, every calculation which is presented here was done by the author of this thesis.

4.1 Couplings of moduli to the observable sector

In the previous chapters, we discussed simple higher-dimensional toy models as a framework for moduli stabilization. We did not specify the complete particle spectrum. However, certain moduli couplings do not depend on the details of model building. In this section, we will not consider the couplings of moduli to Standard Model fermions, since they can either be part of the bulk spectrum, or belong to brane-localized chiral mul-

[3] See also [119] concerning similar considerations in connection with the scale of inflation.

tiplets. In particular, we do not specify the origin of the Higgs field, which in turn may either live on the brane [122], emerge from a bulk hyperscalar [123] or correspond to the internal components of bulk gauge fields [124, 125]. However, we note that the non-universal coupling of the radion to the matter sector may give rise to distinctive observable signatures in a model with specified field content.

On the other hand, in both the five- and six-dimensional case we took the gauge sector to be part of the bulk theory. Hence, the 4D gauge coupling is a function of a modulus,

$$e_4^{-1}\mathcal{L}_{\text{gauge}} = -\frac{e_4^{-1}}{4g^2}\left[\Phi W^\alpha W_\alpha\right]_{\mathcal{F}} \supset -\frac{\phi_1}{4g^2}F^a_{\mu\nu}F^{\mu\nu}_a + \frac{i\phi_2}{4g^2}F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a\,, \tag{4.1.1}$$

where W^α denotes the the field strength superfield, and we used the notation of superconformal tensor calculus [89]. In the 5D case, we have $\phi = T$, and in the 6D case $\phi = S$. While the real part of the modulus field fixes the 4D gauge coupling, the imaginary part, an axion, couples to $F\tilde{F}$.

To determine the coupling strength, we have to canonically normalize the kinetic terms. Recall, in the 5D case,

$$\frac{3M^2}{4r^2}\,\partial_\mu r\partial^\mu r + \frac{M^2}{2r^2}\,\partial_\mu B_5\partial^\mu B_5 \longrightarrow \frac{1}{2}g^{\mu\nu}\left(\partial_\mu\rho_1\partial_\nu\rho_1 + \partial_\mu\rho_2\partial_\nu\rho_2\right)\,, \tag{4.1.2}$$

where ρ was defined by

$$r = \exp\left(\sqrt{\frac{2}{3}}\frac{\rho_1}{M}\right)\,, \tag{4.1.3}$$

and

$$\rho_2 \equiv \frac{M}{r_0}B_5 = M\sqrt{\frac{3}{2}}\,T_2\,. \tag{4.1.4}$$

We ignore the mixing with the hidden scalar X, which is of $\mathcal{O}(\kappa)$. Notice that, by expanding around the VEV of ρ_1, we also get a derivative coupling to ρ_2,

$$e_4^{-1}\mathcal{L}_{\rho_1\rho_2} = \frac{\rho_1}{\sqrt{6}M}\,\partial_\mu\rho_2\partial^\mu\rho_2\,. \tag{4.1.5}$$

Analogously, we can define

$$s \equiv \exp\frac{\sqrt{2}\sigma_1}{M}\,, \qquad \sigma_2 \equiv \frac{M}{\sqrt{2}s_0}S_2\,, \tag{4.1.6}$$

for the complex dilaton emerging from the 6D model.

From up here, we shall focus on the single modulus case. From (4.1.1) we get

$$e_4^{-1}\mathcal{L}_{\text{gauge}} = -\frac{1}{4g^2}\left(1 + \sqrt{\frac{2}{3}}\frac{\rho_1}{M}\right)F^a_{\mu\nu}F^{\mu\nu}_a + \frac{i\rho_2}{2\sqrt{6}Mg^2}F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a\,, \tag{4.1.7}$$

where we can read off the radion-photon coupling,

$$g_{\rho_1\gamma\gamma} = \frac{\sqrt{2}}{\sqrt{3}Mg^2}\,, \tag{4.1.8}$$

and the axion decay constant

$$f^a_{\rho_2} = \frac{\sqrt{6}Mg^4}{16\pi^2} \simeq 10^{16}\mathrm{GeV}\,. \tag{4.1.9}$$

The additional numerical factors emerge from the definition of f^a, see *e.g.* [126], and we took $g^2/(4\pi) = \alpha_{\mathrm{GUT}} \approx 1/25$. The resulting value corresponds to the known result for the model-independent axion in string theory [127]. The coupling of the axion to non-Abelian gauge fields can give rise to a small axion mass via non-perturbative effects [126]. The contribution from QCD corrections can be expressed in terms of the pion mass and decay constant [128],

$$m_a \sim \frac{m_\pi f_\pi}{M} \sim 10^{-10}\,\mathrm{eV}\left(\frac{10^{18}\,\mathrm{GeV}}{M}\right)\,. \tag{4.1.10}$$

Without specifying further details of the spectrum, it is obvious that the radion couples to matter fields with gravitational strength, since the fermion masses depend on the radion VEV,

$$-e_4^{-1}\mathcal{L}_{\mathrm{mass}} \supset e^{K/(2M)}\lambda H Q\bar{Q} \longrightarrow \frac{\rho_1}{M}\,\lambda' m_Q Q\bar{Q}\,, \tag{4.1.11}$$

where H denotes the Higgs and Q an arbitrary matter field. The effective coupling λ' is of order one, although it will depend on the details of the model, compare *e.g.* [129]. Finally, the radion coupling to the gravitino is again model independent, since

$$\frac{1}{3}K_{i\bar{j}}F^i\bar{F}^{\bar{j}} = e^{K/M}\frac{|W|^2}{M^4} = \left(1 - \frac{\sqrt{6}\rho_1}{M}\right)\frac{|W|^2}{M^4}\,, \tag{4.1.12}$$

where the first equality only holds in a Minkowski vacuum.

4.2 Cosmological bounds

The objective of this section is to obtain constraints on the radion mass and axion decay constant by estimating the corresponding energy densities. We will see that the resulting bounds can be re-interpreted in terms of initial conditions for the cosmological evolution of the fields. From up here we assume both the hidden scalar X and its fermionic partner to be heavy enough not to cause cosmological problems by themselves.

4.2.1 Axion dark matter

Coherent oscillations of an ultra-cold condensate of ultra-light axions redshift like non-relativistic matter, and thereby contribute to the dark matter density [130]. This situation is effectively described by the equation of motion of the background field value,

$$\ddot{\theta} + 3H\dot{\theta} + m^2(T)\theta = 0\,, \tag{4.2.1}$$

where $\theta \equiv (\rho_2 - \langle\rho_2\rangle)/f^a_{\rho_2}$, and the mass term is generated when the universe has cooled down to $T \sim \Lambda_{\mathrm{QCD}}$. The field remains frozen to its initial value, θ_0, until $H \sim m(T_1)$. At this point, characterized by the temperature T_1, the axion starts to oscillate around its minimum value. The corresponding energy density is given by [126, 131]

$$\Omega_{\rho_2} h^2 \simeq 0.1\, \theta_0^2 \left(\frac{f^a_{\rho_2}}{10^{12}\,\mathrm{GeV}}\right)^{1.19}\,, \tag{4.2.2}$$

where h is the present day Hubble expansion rate in units of 100 km per second per megaparsec. Notice the unusual exponent, which is due to the scaling of T_1. Assuming the axion condensate to be solely responsible for today's dark matter density, we obtain from (4.1.9) a constraint on the initial misalignment value,

$$\theta_0 \lesssim 4 \times 10^{-3}\,, \tag{4.2.3}$$

while one would naturally assume $\theta_0 \simeq 1$, or even $\rho_2^{\mathrm{init}} \simeq M$. In the following, we shall assume that coherent oscillations of the axion only contribute a small fraction to the total dark energy density, in which case the given bound tightens significantly.

4.2.2 Radion decays and dark matter

Here we closely follow the analogous discussion in [129], but see also [132]. The coupling (4.1.8) induces radion decays into two photons, the decay width being

$$\Gamma^{\mathrm{tree}}_{\rho_1 \to \gamma\gamma} = \frac{g^2_{\rho_1\gamma\gamma}}{64\pi}\, m^3_{\rho_1}\,, \tag{4.2.4}$$

and the corresponding lifetime

$$\tau_{\rho_1 \to \gamma\gamma} = \frac{7.5 \times 10^{23}\,\mathrm{s}}{M^2 g^2_{\rho_1\gamma\gamma}} \left(\frac{m_{\rho_1}}{\mathrm{MeV}}\right)^{-3} \simeq 2.8 \times 10^{23}\,\mathrm{s} \left(\frac{m_{\rho_1}}{\mathrm{MeV}}\right)^{-3}. \tag{4.2.5}$$

Here we set

$$M g_{\rho_1\gamma\gamma} = \frac{\alpha^{-1}_{\mathrm{GUT}}}{2\sqrt{6}\pi} \simeq 1.6\,. \tag{4.2.6}$$

This is of course only a crude estimate, since the electromagnetic coupling changes during the lifetime of the universe from $\alpha_{\mathrm{GUT}} \simeq 1/25$ to $\alpha_{\mathrm{em}} \simeq 1/137$. However, it is clear that, over a wide range of masses, the radion is stable compared to the age of the universe,[4] $\tau_{\rm cosm} \sim 10^{17}$ s, and therefore contributes a fraction $\Omega_{\rho_1}/\Omega_{\rm DM}$ to the dark matter density. This contribution can be constrained by astrophysical observations.

In [133] the authors reported the absence of gamma ray emission lines from the galactic center, with line strength above 5×10^{-5} photons per cm^2 per second, in the range of $0.02 - 8$ MeV.[5] In [129], the total photon flux due to intragalactic moduli decay was computed, integrating the number density of the decaying particles along the line of sight. The authors used a Navarro-Frenk-White (NFW) profile for the galactic halo, assuming that the radion density distribution coincides with the profile of the dominant cold dark matter contribution. Here we only quote the result, referring to [129] for computational details,

$$\mathcal{N}_\gamma = \frac{\Omega_{\rho_1}}{\Omega_{\rm DM}} \left(\frac{m_{\rho_1}}{\rm MeV}\right)^{-1} \left(\frac{\tau_{\rho_1\to\gamma\gamma}}{10^{25}\,\rm s}\right)^{-1} \times \left[0.377\,\frac{\rm photons}{\rm s\,cm^2}\right] , \tag{4.2.7}$$

which leads to the bound

$$\frac{\Omega_{\rho_1}}{\Omega_{\rm DM}} \lesssim 3.7 \times 10^{-6} \left(\frac{m_{\rho_1}}{\rm MeV}\right)^{-2} . \tag{4.2.8}$$

We can combine this constraint with an estimate of the energy density stored in coherent oscillations of the radion field. If the modulus starts to oscillate during radiation epoch, the following approximation is valid [137],

$$m_{\rho_1} Y_{\rho_1} \simeq 6 \times 10^8 \left(\frac{\delta\rho_1}{M}\right)^2 \left(\frac{m_{\rho_1}}{\rm GeV}\right)^{1/2} {\rm GeV} , \tag{4.2.9}$$

where Y_{ρ_1} is the mass density normalized to the entropy density, and $\delta\rho_1$ denotes the initial displacement from the vacuum value.[6] A dark matter candidate can dominate the energy density of the universe unless, at matter-radiation equality,

$$m_{\rm DM} Y_{\rm DM} \lesssim 3\,{\rm eV} . \tag{4.2.10}$$

Obviously, combining this with (4.2.8) we obtain a much more stringent constraint,

$$m_{\rho_1} Y_{\rho_1} \lesssim 10^{-5}\,{\rm eV} , \tag{4.2.11}$$

[4] There are also other decay channels, *e.g.* into e^+e^-, neutrinos, or axions, but since all interactions are Planck suppressed, the total decay width will at most be $\mathcal{O}(10)$ times the partial decay width into photons. Hence, (4.2.5) should be a reasonable estimate of the true lifetime of the radion.

[5] The search was conducted by the space-borne INTEGRAL observatory, using the spectrometer SPI. In [134], the H.E.S.S. collaboration reported on observations in the energy range of $0.3 - 15$ TeV. The resulting bounds on lifetimes can be found in [135]. Fermi LAT data on the extragalactic gamma ray background in the range of $30 - 200$ MeV was published in [136], including a discussion of implications on dark matter physics.

[6] This expression differs from the result in the axion case (4.2.2), where one has specific knowledge of both the axion potential and its temperature dependence.

which translates into the conservative bound,

$$\frac{\delta\rho_1}{M} \lesssim 2.3 \times 10^{-11} \left(\frac{m_{\rho_1}}{\mathrm{MeV}}\right)^{-1/4} . \tag{4.2.12}$$

Notice that this estimate relies on the assumption that $\delta\rho_1$ is the initial value of the field at the beginning of the oscillatory regime, and that the radion background value was frozen to this point since the end of inflation. In a quadratic potential, the freezing regime ends when the Hubble rate drops below the curvature of the potential, $H \lesssim m$. The radion potential we derived in the previous chapter is very steep in the range $r < r_0$, and the quadratic approximation is only valid in the vicinity of the minimum, but certainly applies to a misalignment value obeying (4.2.12). However, the assumption of an enduring freezing regime is probably not justified further away from the minimum. Hence, $\delta\rho_1$ does not necessarily coincide with the field background value at the end of inflation, thereby alleviating the problem of initial conditions. We shall address this possibility in the next section, using methods of dynamical systems analysis.

4.3 Cosmological dynamics of a modulus field

We consider the cosmological evolution of a complex scalar field,

$$Z = \exp\left(\frac{\gamma\Phi}{2M}\right) + i\frac{\gamma\sigma}{2M} , \tag{4.3.1}$$

with its kinetic term being determined by the Kähler potential

$$K(Z,\bar{Z}) = -n_Z M^2 \ln(Z+\bar{Z}) , \tag{4.3.2}$$

such that $\gamma = \sqrt{8/n_Z}$ leads to a standard kinetic term for Φ. As long as we have to consider the fields as dynamical quantities, the kinetic term of σ is non-canonical, because it is impossible to rescale with a fixed vacuum value of $\mathrm{Re}Z$. This is a characteristic feature of non-standard Kähler potentials of the type (4.3.2). The resulting interaction term can have significant consequences on the cosmological evolution of the complex scalar field. In the following, we shall classify the various scenarios in terms of three parameters, which characterize the kinetic coupling, the modulus potential and its coupling to the dark matter density. To begin with, we present a general treatment of dynamical systems associated with (4.3.2); later on, we shall specialize to the radion.

Motivated by our analysis in the previous chapter, we assume the scalar potential to depend on $\mathrm{Re}Z$ only. Furthermore, we will set $M = 1$ in the following.

4.3.1 The general case

In a flat Friedmann-Robertson-Walker (FRW) universe containing cosmological background fluids, the equations[7] which govern the cosmological evolution of a complex

[7]Notice that we assumed the scalar to be stable on the timescale of the age of the universe. We therefore neglect the modification of the classical equations of motion due to the decay width. Moreover,

scalar field of the type (4.3.1) are given by

$$\ddot{\Phi} = -3H\dot{\Phi} - \frac{\gamma}{2}e^{-\gamma\Phi}\dot{\sigma}^2 + \lambda(\Phi)V(\Phi) + Q(\Phi)\rho_{\rm DM}\,(1-3w_{\rm DM})\,, \tag{4.3.3}$$

$$\ddot{\sigma} = -(3H - \gamma\dot{\Phi})\,\dot{\sigma}\,, \tag{4.3.4}$$

$$\dot{H} = -\frac{1}{2}\left[(1+w_{\rm fluid})\rho_{\rm fluid} + \dot{\Phi}^2 + e^{-\gamma\Phi}\dot{\sigma}^2\right]\,, \tag{4.3.5}$$

$$\dot{\rho}_{\rm fluid} = \left[-3(1+w_{\rm fluid})\,H - (1-3w_{\rm fluid})\,Q(\Phi)\,\dot{\Phi}\right]\rho_{\rm fluid}\,. \tag{4.3.6}$$

Here λ and Q are defined by

$$\lambda \equiv -\frac{1}{V}\frac{\partial V}{\partial\Phi}\,, \qquad Q \equiv -\frac{1}{m_{\rm DM}}\frac{\partial m_{\rm DM}}{\partial\Phi}\,, \tag{4.3.7}$$

and depend on Φ generically. The background fluid energy density $\rho_{\rm fluid}$, which collectively describes radiation (including relativistic matter) and dark matter, is characterized by its equation of state parameter $w_{\rm fluid} \in \left[0, \frac{1}{3}\right]$, the limit values corresponding to pure dark matter and pure radiation, respectively.

The displayed system of differential equations is supplemented by the second Friedmann equation,

$$H^2 = \frac{1}{3}\left[\rho_{\rm fluid} + \frac{1}{2}(\dot{\Phi}^2 + e^{-\gamma\Phi}\dot{\sigma}^2) + V(\Phi)\right]\,, \tag{4.3.8}$$

which imposes a constraint on the dynamical system and can be interchanged for any of the evolution equations (4.3.3)–(4.3.6). Introducing the following dynamical variables,

$$x_1^2 \equiv \frac{\dot{\Phi}^2}{6H^2}\,, \quad x_2^2 \equiv \frac{e^{-\gamma\Phi}\dot{\sigma}^2}{6H^2}\,, \quad y^2 \equiv \frac{V(\Phi)}{3H^2}\,, \quad z^2 \equiv \frac{\rho_{\rm rad}}{3H^2}\,, \tag{4.3.9}$$

the system (4.3.3)–(4.3.6) can be rewritten in autonomous form, cf. appendix C.1 for a brief introduction to dynamical systems. The resulting equations are

$$\begin{aligned}\frac{dx_1}{dN} &= \frac{3}{2}\,x_1(x_1^2 + x_2^2 - y^2 + \frac{1}{3}z^2 - 1) \\ &\quad + \sqrt{\frac{3}{2}}\big[-\gamma x_2^2 + \lambda y^2 + Q\,(1 - x_1^2 - x_2^2 - y^2 - z^2)\big]\,,\end{aligned} \tag{4.3.10}$$

$$\frac{dx_2}{dN} = \frac{3}{2}\,x_2(x_1^2 + x_2^2 - y^2 + \frac{1}{3}z^2 - 1) + \sqrt{\frac{3}{2}}\gamma\,x_1x_2\,, \tag{4.3.11}$$

$$\frac{dy}{dN} = \frac{3}{2}\,y\,(x_1^2 + x_2^2 - y^2 + \frac{1}{3}z^2 + 1) - \sqrt{\frac{3}{2}}\lambda\,x_1 y\,, \tag{4.3.12}$$

$$\frac{dz}{dN} = \frac{3}{2}\,z\,\left[x_1^2 + x_2^2 - y^2 + \frac{1}{3}\,(z^2 - 1)\right]\,, \tag{4.3.13}$$

we neglect high temperature effects. Our approximative description is only valid in the range between the TeV scale, where we assume the dark matter to become non-relativistic, and the onset of the oscillatory regime prior to BBN.

where $N = \ln a/a_0 = -\ln(1+z)$.[8] Our approach is a straightforward generalization of the single-field set-up of [138], λ and Q taken to be constants. We shall comment on the implications of these restrictions in the following subsection.

The given system of evolution equations defines a three-parameter family of dynamical models with four-dimensional, compact phase-space,

$$x_1^2 + x_2^2 + y^2 + z^2 \leq 1\,, \tag{4.3.14}$$

and can be further restricted to $(x_2, y, z) \geq 0$, since the system is invariant under change of sign in any of these variables. The fractional energy density associated with the complex scalar,

$$\Omega_{\text{sc}} = x_1^2 + x_2^2 + y^2\,, \tag{4.3.15}$$

and the fractional dark matter density,

$$\Omega_{\text{DM}} = 1 - x_1^2 - x_2^2 - y^2 - z^2\,, \tag{4.3.16}$$

are conveniently expressed in terms of the new dynamical variables. Moreover, the effective equation of state parameter is given by

$$w_{\text{eff}} \equiv \frac{p_{\text{sc}} + p_{\text{fluid}}}{\rho_{\text{sc}} + \rho_{\text{fluid}}} = x_1^2 + x_2^2 - y^2 + \frac{1}{3}z^2\,. \tag{4.3.17}$$

The equation

$$w_{\text{eff}}(x_1, x_2, y, z) = -1/3 \tag{4.3.18}$$

defines the boundary of the domain of accelerated expansion in phase-space.

Each dynamical model (γ, λ, Q) can be characterized by the corresponding set of stationary solutions, which are fixed points in the four-dimensional compact phase-space, cf. appendix C.1. We assume both λ and Q to be positive. We list all possible fixed points below, and display their relevant properties in table 4.3.1. The eigenvalues of the Jacobi matrix evaluated at the respective fixed point, which determine its stability properties, are listed in appendix C.2. We write the fixed points as $P^s = (x_1^s, x_2^s, y^s, z^s)$,

$$\begin{aligned}
A:&\quad \left(\sqrt{\frac{2}{3}}Q, 0, 0, 0\right)\,,\\
B_1, B_2:&\quad \Big(\pm 1, 0, 0, 0\Big),\\
C:&\quad \Big(0, 0, 0, 1\Big),\\
D:&\quad \left(\frac{1}{\sqrt{6}Q}, 0, 0, \sqrt{1 - \frac{1}{2Q^2}}\right)\,,
\end{aligned}$$

[8] Here, and only here, z is the redshift parameter, not to be confused with the dynamical variable z.

$$E:\quad \left(\frac{2\sqrt{2}}{\sqrt{3}\lambda},0,\frac{2}{\sqrt{3}\lambda},\sqrt{1-\frac{4}{\lambda^2}}\right),$$

$$F:\quad \left(\frac{\sqrt{\frac{3}{2}}}{\lambda-Q},0,\sqrt{\frac{2Q(Q-\lambda)+3}{2(\lambda-Q)^2}},0\right),$$

$$G:\quad \left(\frac{\lambda}{\sqrt{6}},0,\sqrt{1-\frac{\lambda^2}{6}},0\right),$$

$$H:\quad \left(\frac{\sqrt{\frac{3}{2}}}{\gamma+Q},\sqrt{\frac{2Q(\gamma+Q)-3}{2(\gamma+Q)^2}},0,0\right),$$

$$J:\quad \left(\frac{\sqrt{6}}{\gamma+\lambda},\sqrt{\frac{\lambda(\gamma+\lambda)-6}{(\gamma+\lambda)^2}},\sqrt{\frac{\gamma}{\gamma+\lambda}},0\right).$$

f.p.	**existence**	**stability**	$\Omega_{\mathbf{sc}}$	$w_{\mathbf{eff}}$
A	$Q \leq \sqrt{\frac{3}{2}}$	stable: $Q^2 < min\{\frac{1}{2},\frac{3}{2}-\gamma Q,\lambda Q-\frac{3}{2}\}$	$\frac{2}{3}Q^2$	$\frac{2}{3}Q^2$
B_1 B_2	$\forall(\gamma,\lambda,Q)$	saddle point: $\lambda>\sqrt{6}\ \wedge\ Q>\sqrt{\frac{3}{2}}\ \wedge\ \gamma<0$ unstable	1	1
C	$\forall(\gamma,\lambda,Q)$	unstable	0	$\frac{1}{3}$
D	$Q \geq \frac{1}{\sqrt{2}}$	stable: $\lambda > 4Q > 2\gamma$	$\frac{1}{6Q^2}$	$\frac{1}{3}$
E	$\lambda \geq 2$	stable: $2\gamma < \lambda < 4Q$	$\frac{4}{\lambda^2}$	$\frac{1}{3}$
F	$\frac{3}{2Q}+Q \geq \lambda \geq \frac{Q+\sqrt{Q^2+12}}{2}$ $\wedge\, \lvert\lambda-Q\rvert \leq \sqrt{\frac{3}{2}}$	stable: $\lambda > max\{4Q, 2Q+\gamma\}$	$\frac{3+Q^2-Q\lambda}{(Q-\lambda)^2}$	$\frac{Q}{\lambda-Q}$
G	$\lambda \leq \sqrt{6}$	stable: $\lambda^2 < min\{4, 3+Q\lambda, 6-\gamma\lambda\}$	1	$-1+\frac{\lambda^2}{3}$
H	$\gamma \geq max\{0,\frac{3}{2Q}-Q\}$	stable: $\lambda > \gamma+2Q$ $\wedge\ \gamma > 2Q$	$\frac{Q}{\gamma+Q}$	$\frac{Q}{\gamma+Q}$
J	$\lambda(\lambda+\gamma) > 6$ $\wedge\ \gamma \geq 0$	stable: $\lambda < min\{2\gamma, \gamma+2Q\}$	1	$\frac{\lambda-\gamma}{\lambda+\gamma}$

Table 4.3.1: *Properties of the fixed points of the system (4.3.10–4.3.13).*

As it is obvious from table 4.3.1, a plethora of evolutionary scenarios are possible, depending on the different parameter values. However, notice that each dynamical model is characterized by exactly one stable solution, which implies independence of initial conditions. For the remainder of this section, we shall consider a certain subclass of

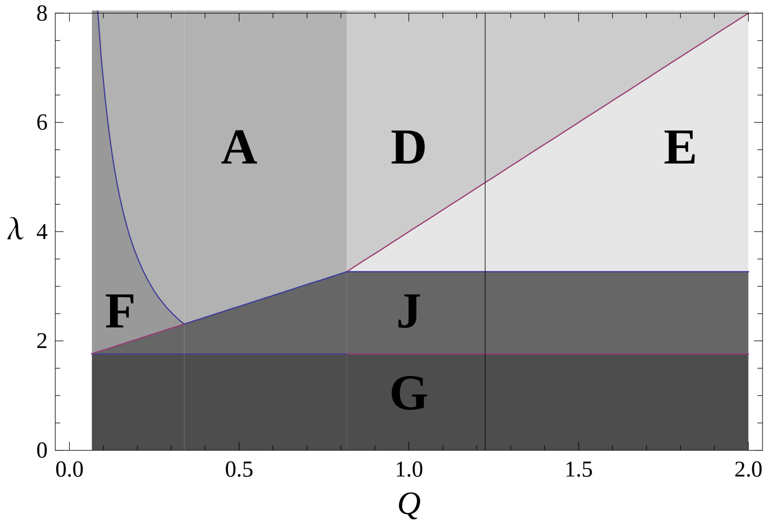

Figure 4.1: *The parameter space of the system (4.3.10)–(4.3.13) for fixed $\gamma = \sqrt{8/3}$, corresponding to the kinetic Lagrangean of the radion-axion model. Recall that Q parametrizes the radion-dependence of the dark matter mass, whereas λ is the slope of the potential. The different stable fixed points are indicated by different shadings and capital letters. The vertical line denotes the value of the parameter Q in the case of gravitino dark matter. More detail is given in the text.*

models corresponding to a modulus field with steep exponential potential and a specific non-canonical kinetic Lagrangean.

4.3.2 Radion dynamics in the radiation era

Given a Kähler potential of the type (4.3.2), the parameter γ is fixed by n_Z. We shall now focus on the radion model of the preceding chapter, and therefore set $\gamma = \sqrt{8/3}$.[9] In Fig. 4.1 we show the relevant part of the resulting parameter space, corresponding to the range $0 < Q \leq 2$ and $0 \leq \lambda \leq 8$, where six different stable solutions are possible. The parameter Q is determined by the radion dependence of the dark matter particle's mass. If the dark matter density is entirely due to the gravitino relic abundance, cf. [33], we have $Q = \sqrt{3/2}$ according to (4.1.12). Otherwise, the precise value of Q depends on the details of model building, such as the specific realization of the particle spectrum in the bulk and on the branes, the mediation scheme of SUSY breaking, and the resulting renormalization group running of couplings, masses and soft terms. However, for any fermionic LSP dark matter candidate,

$$m_{\rm DM} \sim e^{K/(2M)} = \exp\left(-\sqrt{\frac{3}{2}}\frac{\rho_1}{M}\right), \tag{4.3.19}$$

hence we can take $Q = \mathcal{O}(1)$ to be a reasonable assumption.[10]

[9] We neglect the $\mathcal{O}(\kappa)$ correction to the kinetic terms.

[10] Physicswise, this estimate corresponds to the usual gravitational strength coupling of geometric moduli to matter fields.

As we observe from Fig. 4.1, two different types of stable fixed points are possible, subject to the respective steepness of the potential. If the modulus potential is rather flat, $\lambda \leq 4\sqrt{2}/3$ for $Q \geq \sqrt{2/3}$, or $\lambda \leq 2(Q + \sqrt{2/3})$ for smaller values of Q, we find stationary solutions that are dominated by the energy density of the scalar fields: The associated fixed points, G and J, feature $\Omega_{\rm sc} = 1$, which is in obvious conflict with the current understanding of BBN. Physicswise, the limit $\lambda \to 0$ corresponds to the freezing regime of $H \gg m_\Phi$, where a light scalar field *sits and waits* until the Hubble rate drops below the curvature of the potential. The corresponding potential energy will inevitably dominate the energy density of the universe as $t \to \infty$. However, our set-up only applies to the regime, where the scalar potential $V(\Phi)$ is well approximated by an exponential. This approximation breaks down, when the modulus field enters the oscillatory regime of its cosmological evolution, cf. Fig. 4.3.

On the other hand, if the potential is sufficiently steep, we find one of the fixed points A, D, E or F to be stable. Associated with these fixed points are so-called *cosmological scaling solutions*, cf. [139], where the energy density of the scalar fields is fixed *wrt* the energy density of the dominating cosmological fluid. For $Q \geq \sqrt{2/3}$, the universe is radiation dominated, while the regime of A or F corresponds to $0 \leq w_{\rm eff} \leq 1/3$, and the universe is dominated by a mixture of dark matter and $\rho_{\rm sc}$. The contribution of the scalar fields to the total energy density is entirely kinetic at the attractor solutions A and D, while the fixed point E corresponds to a solution with $w_{\rm sc} = w_{\rm rad} = 1/3$, i.e. the scalar energy density redshifts exactly like radiation.

We note that the existence of the attractor D is related to the overshooting problem of moduli stabilization, cf. [111]. Although scalar kinetic energy redshifts with a^{-6}, i.e. faster than radiation, at the attractor $\Omega_{\rm sc}^{\rm kin}$ is fixed *wrt* $\Omega_{\rm rad}$ due to the specific couplings, and the scalar field inevitably enters the oscillatory regime with an excess of kinetic energy.[11]

We shall proceed by specializing to the radion potential of our example in the preceding chapter. In Fig. 4.2 we plotted the radion effective potential in terms of the canonically normalized field Φ, and in Fig. 4.3 we show the parameter function $\lambda(\Phi)$ for different values of $\ell = M_{\rm bulk}L$, in order to demonstrate that the variation with ℓ is insignificant. We observe that λ approaches constant values, $\lambda_- = 6\sqrt{2/3}$ for $\Phi \to -\infty$, and $\lambda_+ = 5\sqrt{2/3}$ for $\Phi \to \infty$, as expected from the asymptotic behaviour of the radion potential. However, approaching $\Phi = 0$ from below, the approximation by a constant λ becomes increasingly less viable, which corresponds to the oscillatory regime taking over. As long as $\lambda(\Phi) \leq 4\sqrt{3/2}$, we may consider the stationary solution E as quasi-stable in the sense of an 'instantaneous fixed point' [140], which moves as a function of $\lambda(N)$ in phase-space. However, at $\Phi_{\rm transit} \approx -M$, $E(N)$ ceases to be stable, and the radion enters

[11] The authors of [111] distinguish between a bound and an unbound case, corresponding to successful stabilization *vs.* overshooting, respectively. The scaling solution E is reminiscent of the latter case, cf. their Fig. 4. Whether the field settles to the potential minimum or overshoots, entirely depends on the initial conditions. Strictly speaking, if the field value overshoots, there is no oscillatory stage; nevertheless, in the following we shall always refer to the evolutionary stage, where the field value crosses the quadratic domain of its potential, by the term 'oscillatory regime'.

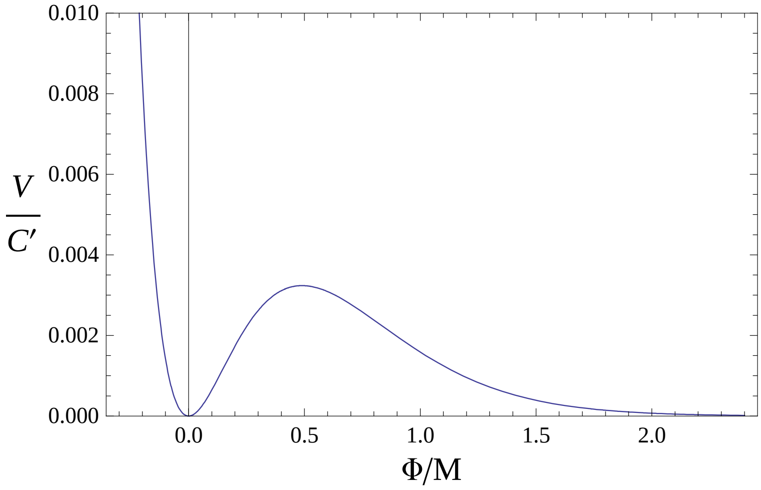

Figure 4.2: *The radion effective potential V/C' as a function of the canonically normalized field $\Phi = \sqrt{3/2}M \ln r$. We took $\ell = 2.2$.*

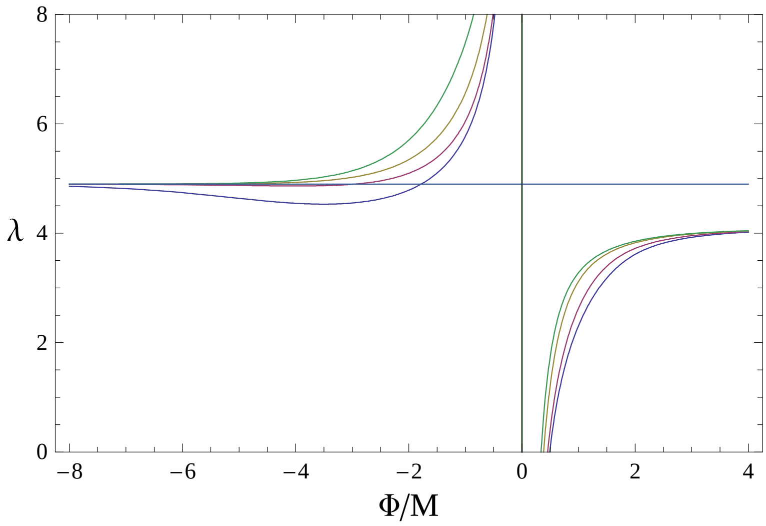

Figure 4.3: *The slope parameter λ of the radion potential (3.3.16) as a function of the field value Φ in units of M, plotted for $\ell = 2.2, 3, 6, 10$, from bottom to top. The horizontal line corresponds to the approximation $V(r) \sim r^{-6}$, and at the same time marks the boundary between the stability domains of the fixed points E (below) and D (above).*

the oscillatory regime with an excess of kinetic energy. Moreover, the corresponding field value $\Phi_{\rm transit}$ differs dramatically from the allowed displacement according to (4.2.12). Due to the steepness of the potential in this regime, we find

$$V(\Phi_{\rm transit}) = \mathcal{O}(10^3) \times V(\Phi_{\rm barrier})\,, \tag{4.3.20}$$

and the radion is bound to overshoot rather then to overclose the universe, cf. [110].

Our considerations do not only apply to the specific radion effective potential we studied so far. Expanding a generic almost no-scale modulus potential around $\Phi_0 = 0$,

$$V(\Phi) = V_2 \frac{2m_{3/2}^2}{3L^2}\left(\frac{\Phi}{M}\right)^2 + \ldots\,, \tag{4.3.21}$$

where the parentheses refer to terms of higher order in Φ/M and V_2 is an $\mathcal{O}(1)$ parameter, we realize that Fig. 4.3 covers the general situation. The quadratic approximation is valid in the range $-M < \Phi < M$, irrespective of any additional mass scale in the potential. Hence, there is no hope to circumvent the bound (4.2.12) by means of a fast-roll regime prior to the oscillatory stage.

Therefore, we have to impose that the radion field value is constrained to the close vicinity of its low energy vacuum value from the beginning of the radiation era. Furthermore, we have to assume this 'initial' condition to be generated by high energy dynamics during inflation and reheating. If we were to estimate the primordial 'initial' field value $\Phi_{\rm prim}$, the only relevant physical scale we could refer to is the 5D Planck scale,

$$Lr_{\rm prim} \sim M_5^{-1} \quad \Rightarrow \quad \Phi_{\rm prim} \simeq -5.64M\,, \tag{4.3.22}$$

hence it is natural to assume that the field value is thereafter driven toward zero by interactions with the inflaton or thermal effects. However, at high energies the potential barrier is likely to be extinguished by temperature corrections, and we cannot conceive a mechanism to pre-stabilize the field value within the allowed range, unless the Hubble friction keeps the field frozen during reheating.

In order to illustrate the impact of scalar interactions, let us consider the dynamical interplay between the stabilization of X and the evolution of the radion field, which can also be treated using our general prescription. We assume that X undergoes a stage of coherent oscillations before it settles at X_0, and that it is sufficiently long-lived for the energy density stored in the oscillations to redshift like dark matter. Using a quadratic expansion around X_0, we obtain the scalar Lagrangean,

$$\begin{aligned} e_4^{-1}\mathcal{L}_{\rm sc} = \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \frac{1}{2}\exp\left(-\sqrt{\frac{8}{3}}\,\Phi\right)\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\exp\left(-\sqrt{\frac{2}{3}}\,\Phi\right)\partial_\mu\chi\partial^\mu\chi \\ - \exp\left(-2\sqrt{\frac{2}{3}}\,\Phi\right)m_\chi^2\chi^2 - \exp\left(-6\sqrt{\frac{2}{3}}\,\Phi\right)\left|W(X_0)\right|^2, \end{aligned} \tag{4.3.23}$$

where we assume that the fluctuation $\chi = X - X_0$ is real, for simplicity. We only take into account the leading contribution to the radion effective potential in the range $\Phi < 0$, cf. (3.3.30). The radion equation of motion, cf. (4.3.3), now contains a force term due to the coupling to the energy and pressure density associated with χ,

$$\ddot{\Phi} = -3H\dot{\Phi} - \sqrt{\frac{2}{3}}\, e^{-\sqrt{8/3}\Phi}\dot{\sigma}^2 + \lambda\, V_{\rm eff}(\Phi) + Q\,\rho_\chi\,(1 - 3w_\chi)\,, \tag{4.3.24}$$

with

$$\lambda = 6\sqrt{\frac{2}{3}}\,, \qquad Q = \sqrt{\frac{2}{3}}\,, \tag{4.3.25}$$

and

$$\begin{aligned} \rho_\chi &= \frac{1}{2}e^{-\sqrt{2/3}\Phi}\dot{\chi}^2 + e^{-2\sqrt{2/3}\Phi}m_\chi^2\chi^2\,, \\ p_\chi &= \frac{1}{2}e^{-\sqrt{2/3}\Phi}\dot{\chi}^2 - e^{-2\sqrt{2/3}\Phi}m_\chi^2\chi^2\,. \end{aligned} \tag{4.3.26}$$

Taking a time average over a sufficient number of subsequent oscillations, we have $p_\chi = 0$, cf. [113], and thus can treat ρ_χ as dark matter. According to Fig. 4.1, the corresponding dynamical system resides exactly on the boundary between the regimes of fixed point A and D, respectively. The attractor solution characterizes a universe that is dominated by a mixture of kinetic energy associated with a fastly rolling radion, and the energy density stored in the coherent oscillations of χ. With

$$\Omega_{\rm kin}(\Phi) = \frac{2}{3}\,, \qquad \Omega_{\rm DM}(\chi) = \frac{1}{3}\,, \tag{4.3.27}$$

we find an effective equation of state $w_{\rm eff} = 1/3$, mimicking a radiation dominated universe. We observe that the oscillatory stage prior to the stabilization of X induces the radion to roll fastly in the steep region of its potential, thereby driving it to overshoot.

To conclude this section, we note that we found a unique possibility to circumvent the various cosmological moduli problems within our light radion model; the field value must settle within a close vicinity of its low energy vauum value, cf. (4.2.12), after reheating. Any stage of fast-roll prior to the oscillatory regime inevitably drives the field to overshoot the shallow minimum.

4.4 The issue of recurrent acceleration

It was pointed out recently [141] that theories of the type

$$S_{\rm AD} = \int d^4x\sqrt{-g}\left\{ -\frac{1}{2}R + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \frac{1}{2}e^{-\gamma\Phi}\partial_\mu\sigma\partial^\mu\sigma - e^{-\lambda\Phi}\right\} \tag{4.4.1}$$

admit cosmological solutions which give rise to the interesting phenomenon of recurrent acceleration. By a detailed phase-space analysis of the corresponding dynamical system, the authors verified the generic occurence of recurrent acceleration in the regime of a spiral focus associated to runaway behaviour of both scalar fields. In this picture, the present acceleration does not appear as a peculiar stage of cosmic history, being likely a transient or even recurring phenomenon. In particular, the authors conclude that the future evolution of the universe is by no means determined to be accelerating forever, in obvious contrast to standard ΛCDM cosmology. However, the dynamical models of [141] comprised only gravity and two real scalars, namely axion and dilaton; in order to relate these results to the observed accelerated expansion of the universe, it is crucial to take the non-gravitational sector of the theory into account.

For a general discussion of quintessence and related models, we refer to [22, 23] and references therein. Due to the bounds on a fifth force [142], it is impossible to explain the present dark energy density by means of a slowly rolling scalar field with gravitational strength coupling,[12] see also [143]. However, we may take the suggestion of [141] as a matter of principle. Since we have the required technical tools at hand, cf. table 4.3.1, we can proceed and check the viability of recurrent acceleration in dark energy model building. Therefore, we simply assume the complex scalar to be completely decoupled from the Standard Model. We are then able to study the evolution of an axion-dilaton (AD) dynamical system in the realistic background of a universe filled with (dark) matter and radiation, and compare the results to the pure AD and gravity case. We shall first translate the set-up of [141] into our dynamical variables, and then analyze the system in detail. In a second step, we consider the implications of our general results on the idea that the present time acceleration of the universe may be transient.

4.4.1 Recurrent acceleration in pure axion-dilaton cosmology

Provided a flat FRW universe and absence of cosmological perfect fluids, we obtain the autonomous system of evolution equations associated with (4.4.1) directly from (4.3.10)-(4.3.13) by setting $z = 0$, and imposing $x_1^2+x_2^2+y^2 = 1$. Hence, the corresponding phase-space is two-dimensional. We choose it to be spanned by $\{x_1, x_2\}$. After eliminating y from the system, we obtain

$$\frac{dx_1}{dN} = 3\,x_1(x_1^2 + x_2^2 - 1) + \sqrt{\frac{3}{2}}\big[- \gamma x_2^2 + \lambda(1 - x_1^2 - x_2^2)\big]\,, \tag{4.4.2}$$

$$\frac{dx_2}{dN} = 3\,x_2(x_1^2 + x_2^2 - 1) + \sqrt{\frac{3}{2}}\gamma\, x_1 x_2\,. \tag{4.4.3}$$

The equation of state is given by

$$w_{\text{eff}} = w_{\text{AD}} = 2(x_1^2 + x_2^2) - 1\,, \tag{4.4.4}$$

[12] This argument relies on a particle interpretation of both gravitational and quintessence-scalar interactions. Since there is no consistent theory of quantum gravity known so far, it may seem premature to exclude a whole class of models on these grounds. Instead, we may take the point of view that quintessence provides merely an *effective* description of unknown physics in terms of a scalar field with only gravitational and self-interactions.

and we find the following set of stationary points, given in terms of the model parameters,

$$\begin{aligned}
B_1, B_2 : &\quad \Big(\pm 1, 0\Big), \\
G : &\quad \left(\frac{\lambda}{\sqrt{6}}, 0\right), \\
J : &\quad \left(\frac{\sqrt{6}}{\gamma+\lambda}, \sqrt{\frac{\lambda(\gamma+\lambda)-6}{(\gamma+\lambda)^2}}\right).
\end{aligned}$$

fixed point	**existence**	**stability**	w
B_1	$\forall(\gamma,\lambda)$	stable: $\gamma < 0 \wedge \lambda > \sqrt{6}$	1
B_2	$\forall(\gamma,\lambda)$	saddle point: $\gamma > 0$	1
G	$\lambda < \sqrt{6}$	stable: $\lambda(\lambda+\gamma) < 6$	$-1+\frac{\lambda^2}{3}$
J	$\gamma \geq 0 \ \wedge \ \lambda(\lambda+\gamma) \geq 6$	stable: $\gamma > 0 \ \wedge \ \lambda(\lambda+\gamma) > 6$	$\frac{\lambda-\gamma}{\lambda+\gamma}$

Table 4.4.1: *Properties of the fixed points of the reduced dynamical system.*

Properties of the fixed points are displayed in table 4.4.1. The Jacobi matrix eigenvalues are listed in appendix C.2. In particular, we shall note that the fixed point J is a spiral focus if

$$3\gamma\,(9\gamma+8\lambda) - 4\,\gamma\lambda(\gamma+\lambda)^2 < 0\,. \tag{4.4.5}$$

According to table 4.4.1, and in agreement with [141], accelerated expansion is possible at the fixed point G, if $\lambda < \sqrt{2}$, or at the fixed point J, if $\gamma > 2\lambda$. Hence, we can distinguish three possibilities to realize a model which generically allows for periods of accelerated expansion. We give examples of phase-portraits of the different cases below. Recurrent acceleration is associated with the part of parameter space where the fixed point J is stable and a spiral focus.

G stable

If $\lambda < \sqrt{2}$, the attractor G is situated within the domain of accelerated expansion. In this case, once acceleration has set in, it will last forever. If $\sqrt{2} < \lambda < \sqrt{6}$, it can be a transient phenomenon along a subset of trajectories, hence the evolution depends on the choice of initial conditions. In Fig. 4.4, we plotted the phase-portrait of the system for the limiting case $\lambda = \sqrt{2}$.

J stable, G saddle point

If J is the attractor, the domain of acceleration in parameter space is bounded by $\gamma = 2\lambda$. The phase-portrait of the system depends crucially on the progress of the special trajectory connecting the saddle point G with the attractor. We will hereafter call it

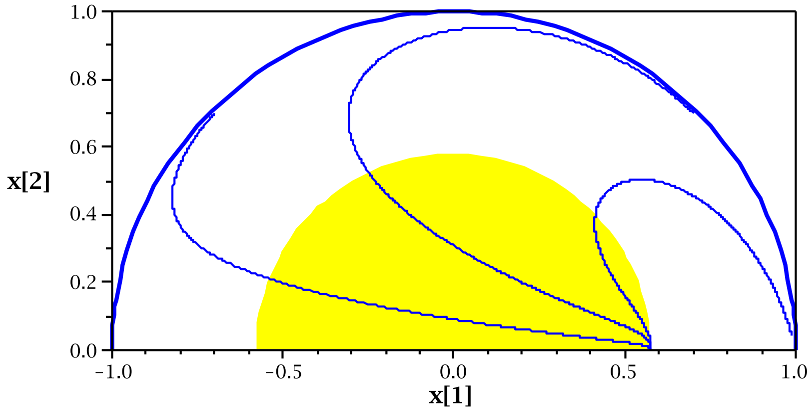

Figure 4.4: *Model with parameters* $(\lambda, \gamma) = (\sqrt{2}, 2)$. *Shaded (yellow) area corresponds to accelerated expansion.*

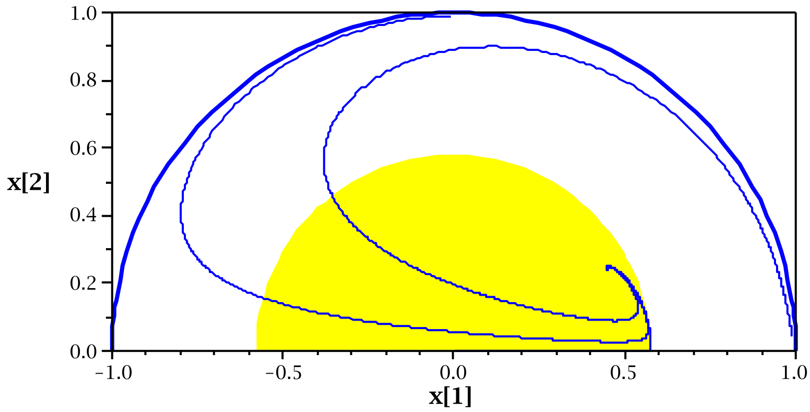

Figure 4.5: *Model with parameters* $(\lambda, \gamma) = (\sqrt{2}, 4)$.

the *connecting trajectory*. If $\lambda < \sqrt{2}$, both G and J are situated within the domain of acceleration in phase-space, and hence the connecting trajectory is completely contained within this domain as well. Since different trajectories cannot cross each other in phase-space, any trajectory approaching the fixed point remains within the accelerated regime once having entered it, cf. Fig. 4.5. If, on the other hand, G is situated outside, recurrent acceleration is generically realized, *if* the spiral focus J is located close enough to the acceleration boundary, such that any trajectory approaching the attractor crosses the boundary repeatedly, as does the connecting trajectory, cf. Fig. 4.6.

J spiral focus, G non-existing

In this case, recurrent acceleration is indeed a generic phenomenon. At $\lambda = \sqrt{6}$, the fixed point G merges with B_1. The dynamical evolution of the system is now totally

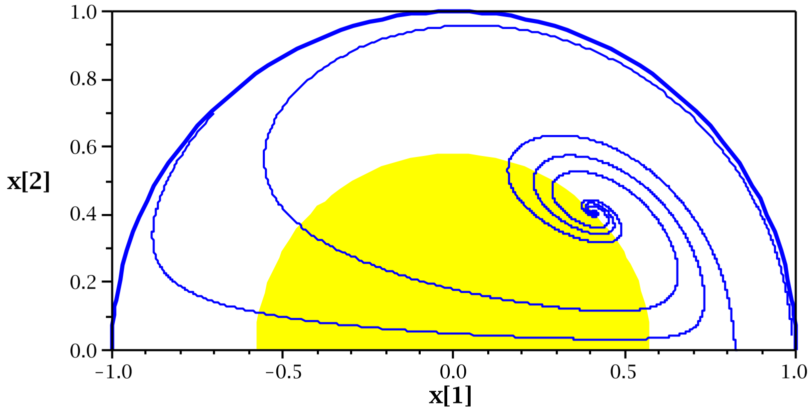

Figure 4.6: *Model with parameters* $(\lambda, \gamma) = (2, 4)$.

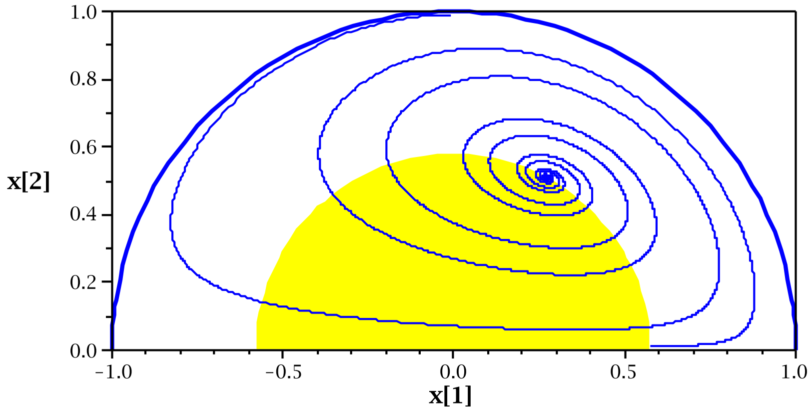

Figure 4.7: *Model with parameters* $(\lambda, \gamma) = (3, 6)$. *Only two different trajectories are shown.*

determined by the saddle points B_1 and B_2, situated at the phase-space boundary, and the spiral focus J. The condition (4.4.5) is trivially fulfilled in this part of parameter space. Each trajectory winds around the attractor several times, undergoing subsequent stages of accelerated and decelerated expansion, irrespective of the location of the fixed point *wrt* the acceleration boundary, cf. Fig. 4.7.

In summary, we observe that the existence of saddle points and their location in phase-space can have significant effects on the dynamical evolution of the system. Recurrent acceleration occurs generically, i.e. independently of initial conditions, if the saddle points are located on or close to the phase space boundary. Otherwise, the trajectories are fastly focused toward the connecting trajectory. If the attractor J is a spiral focus, the connecting trajectory itself may cross the boundary of acceleration more than once, depending on the location of J in phase-space. Models which generically feature recurrent acceleration are characterized by steep potentials, $\lambda \geq \sqrt{6}$. On the attractor solution, the energy density is a mixture between potential and kinetic energy of both the axion and the dilaton. This is a consequence of the axion's non-canonical kinetic term, which is dynamically relevant even though the axion potential is completely flat.

4.4.2 Axion-dilaton cosmology with perfect fluid background

We shall now consider the dynamical evolution of the axion-dilaton system in the presence of radiation and dark matter according to the standard model of cosmology. We therefore consider the results of table 4.3.1 with $Q = 0$, neglecting any matter coupling of the scalar fields, and seek out models (γ, λ) which admit a spiral focus J and may therefore give rise to recurrent acceleration. For convenience, we summarize the properties of the allowed fixed points in table 4.4.2.

fixed point	**existence**	**stability**	$\Omega_{\mathbf{AD}}$	$w_{\mathbf{eff}}$
A	$\forall(\gamma,\lambda)$	saddle point	0	0
B_1, B_2	$\forall(\gamma,\lambda)$	unstable	1	1
C	$\forall(\gamma,\lambda)$	unstable	0	$\frac{1}{3}$
E	$\lambda \geq 2$	saddle point: $\lambda > max\{2, 2\gamma\}$	$\frac{4}{\lambda^2}$	$\frac{1}{3}$
F	$\lambda \geq \sqrt{3}$	stable: $\lambda > max\{\sqrt{3}, \gamma\}$	$\frac{3}{\lambda^2}$	0
G	$\lambda \leq \sqrt{6}$	stable: $\lambda < \sqrt{3}$ $\wedge\ \lambda(\lambda+\gamma) < 6$	1	$-1 + \frac{\lambda^2}{3}$
J	$\lambda(\lambda+\gamma) > 6$ $\wedge\ \gamma \geq 0$	stable: $\gamma > \lambda$	1	$\frac{\lambda-\gamma}{\lambda+\gamma}$

Table 4.4.2: *Properties of the fixed points of system (4.3.10-4.3.13), with $Q = 0$.*

Recall that the existence of stationary solutions with $x_2 \neq 0$ is due to the modified friction term in the axion equation of motion (4.3.4), i.e. the quantity $3H - \gamma\dot{\Phi}$, which can have either sign. As long as $3H - \gamma\dot{\Phi} > 0$, the axion evolves towards a configuration where $\dot{\sigma} = 0$ and therefore $x_2 = 0$. On the other hand, stability of the fixed point J with $x_2 \neq 0$ implies $x_1|_J > \gamma^{-1}\sqrt{3/2}$, which is equivalent to $3H - \gamma\dot{\Phi} < 0$.

In Figs. 4.8 and 4.9 we show the different domains of stability in parameter space of both scenarios, with and without a cosmological fluid background. Most relevant is the appearance of the new fixed point F, which is either a stable focus or a saddle point in a significant range of parameter space. With $Q = 0$, the corresponding stationary solution is a matter dominated scaling solution, i.e. the energy density of the complex scalar redshifts exactly like dark matter. In particular, the domain of recurrent acceleration, as identified in subsection 4.4.1, is now completely superimposed by the existence of a

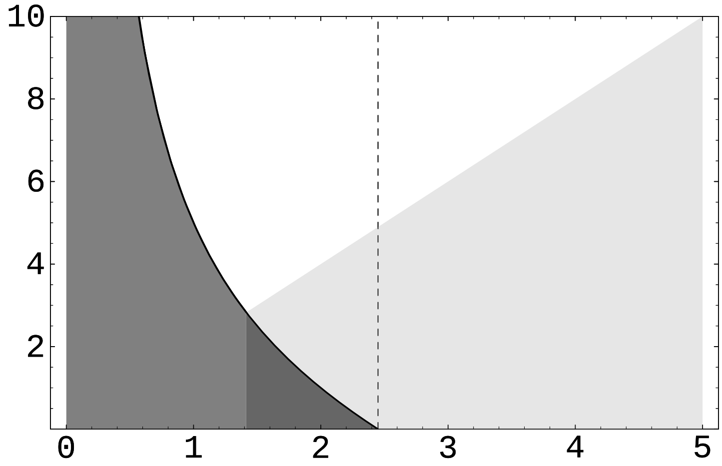

Figure 4.8: *The parameter space of the pure axion-dilaton system. Horizontal axis: λ, vertical axis: γ. In the gray region fixed point G is stable. The dashed line is the existence boundary of G. In the lightly shaded region acceleration is impossible at the attractor.*

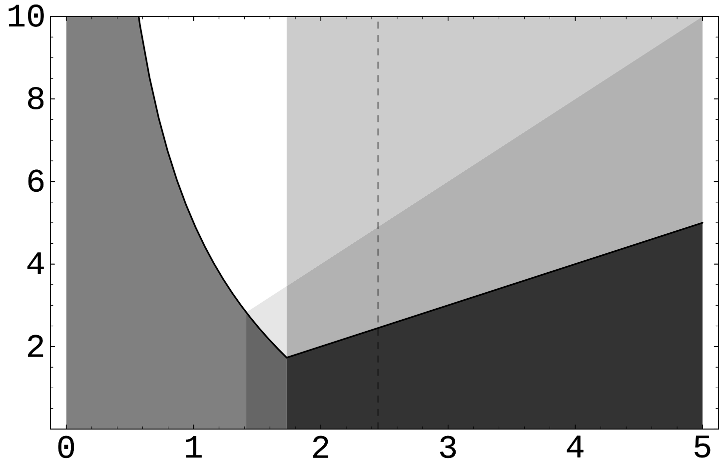

Figure 4.9: *As above, now the general case. The existence of the new fixed point F is indicated for $\lambda \geq 3$, in the dark gray region F is the attractor.*

saddle point scaling solution.

Following our discussion presented above, we assume J to be a spiral focus, located close to the acceleration boundary in phase-space. In other words, we restrict ourselves to a subclass of models, corresponding to the neighborhood of the line $\gamma = 2\lambda$ in parameter space. We have to discuss three different cases in turn.

- $\lambda < \sqrt{2}$: G saddle point, accelerated expansion at G;
- $\sqrt{2} < \lambda \leq \sqrt{3}$: G saddle point, deceleration at G;
- $\lambda > \sqrt{3}$: F saddle point.

We shall focus on the behaviour of the trajectory connecting the saddle point, be it F or G, with the attractor.

Starting with the first case, we note that always $w_{\text{eff}}(G) > w_{\text{eff}}(J)$. This follows from the existence condition of J, which can be rewritten as $\lambda^2/3 > 2\lambda(\lambda+\gamma)^{-1}$. Moreover, since $\Omega_{\text{AD}} = 1$ holds for both J and G, the connecting trajectory is constrained to the corresponding section of the phase-space boundary. Hence, we do not expect any qualitative difference compared to the situation described in section 4.4.1, cf. Fig. 4.5. Once the scalar sector starts to dominate, the fate of the universe is determined to be accelerating forever.

In the second case, the connecting trajectory itself crosses the acceleration boundary while spiraling onto the attractor, cf. Fig. 4.6. In particular, any trajectory which enters the acceleration domain *before* approaching the saddle point G will display at least two distinct stages of accelerated expansion. The first stage corresponds to the well-known *freezing* regime of single field quintessence models [22]: the dilaton field velocity remains close to zero due to the Hubble friction term dominating the equation of motion. The second stage is reached, when the trajectory re-enters the domain of acceleration in approaching the late time attractor. If J is located inside the domain, accelerated expansion will continue forever.

In the third case, when the saddle point F is dynamically relevant, the universe is either dominated by matter or by the AD system scaling like matter, $w_{\text{eff}} = w_{\text{AD}} = 0$, with $\Omega_{\text{AD}} < 1$. When the connecting trajectory leaves the vicinity of F, the equation of state of the scalar energy density, w_{AD}, instantly starts to oscillate around the attractor value $w_{\text{AD}}(J)$. However, due to the finite contribution of matter, we now have – in contrast to the previous cases – $w_{\text{eff}} > w_{\text{AD}}$ along the connecting trajectory. Hence, recurrent acceleration is not implied. In particular, any dynamical model with $w_{\text{eff}}(J) \geq -1/3$ does not admit accelerated expansion at all. For numerical examples we refer to [121]. We conclude that recurrent acceleration is no longer a generic feature of axion-dilaton cosmology, once a perfect fluid contribution to the energy density dominates intermediate stages of cosmic evolution. The dynamical behaviour of the scalar fields themselves is significantly altered due to the existence of a saddle point scaling solution.

For the sake of completeness, let us briefly comment on the viability of the $F \to J$ scenario for dark energy model building. Any evolutionary scenario, where the regime of accelerated expansion is preceded by a matter dominated scaling solution, has a distinctively nice feature: The intermediate scaling regime washes out any dependence on initial conditions. However, the fixed point J does not correspond to the present state of the universe, but to an asymptotic de Sitter state. Hence it remains a pure coincidence, that the transition from the saddle point scaling regime toward the spiral focus began just recently enough to fit our present observations. On the other hand, J being a spiral focus, the prediction of significant oscillations in the dark energy equation of state can easily be falsified by future cosmological observations.

In order to match the currently favored value of the dark energy equation of state, which is indistinguishable from the case of a cosmological constant, one has to refer to large values of the parameter γ: For the scalar contribution to be subdominant at the saddle point, we require at least $\lambda = \sqrt{6}$. Then

$$w_{\mathrm{DE}} > -1 + \frac{2}{1 + \gamma/\sqrt{6}}\,, \tag{4.4.6}$$

where the value on the RHS corresponds to the late time attractor, which we require to match $w_\Lambda = -1$ within ten percent accuracy. Hence, we need $\gamma > 19\sqrt{6}$ in order to get a model which is not already excluded by observations [144]. Needless to say, such values of γ are incompatible with a scalar kinetic Lagrangean corresponding to (4.3.2). With $n_Z \geq 1$, we are restricted to $\gamma \leq 2\sqrt{2}$. Hence, even if we ignore the problem of gravitational strength couplings, it is impossible to realize a working model of dynamical dark energy in terms of moduli fields with non-standard kinetic terms of the type (4.3.2).

4.5 Summary

In this chapter, we developed a systematical treatment of the cosmological evolution of a complex scalar field with exponential potential and a non-standard axion kinetic term. Models of this type generically emerge from higher-dimensional supergravity, with a moduli type Kähler potential

$$K = -n_Z \ln(Z + \bar{Z})\,, \tag{4.5.1}$$

and a scalar potential

$$V_F = e^K V_0\,, \tag{4.5.2}$$

where V_0 results from integrating out heavier fields. Using methods of dynamical systems analysis, we characterized any possible model in the prescribed class by a set of three parameters, associated with the steepness of the potential, the kinetic coupling between the two real scalar degrees of freedom, and the coupling to the matter sector. Our general results are summarized in table 4.3.1. The cosmological dynamics of each model is determined by a unique stable attractor solution, supplemented by saddle points in phase-space, which may be relevant during intermediate evolutionary stages. In particular, we discussed two different applications, corresponding to two puzzles of modern cosmology, the cosmological moduli problem and the issue of dark energy.

Based on the earlier work of [141], we investigated the possibility of recurrently accelerating solutions, both in a scenario with only the complex scalar field contributing to the energy density of the universe, and with a background of perfect fluids. Recurrent acceleration generically occurs in the regime of a spiral focus, but only in absence of matter and radiation. Interestingly enough, the corresponding attractor allows for slow-roll solutions even with a steep exponential potential. The Hubble friction, which counteracts the accelerating effect of the potential in the scalar field's equation of motion, is enhanced due to the non-canonical kinetic term, whenever the axion field value evolves. These solutions, however, require large values of the kinetic coupling parameter γ, corresponding to $n_Z < 1$, which is highly unlikely to be obtained in higher-dimensional supergravity model building. Moreover, it is impossible to realize dynamical dark energy in terms of a geometrical modulus field that couples with gravitational strength. Models of this type are excluded by fifth force experiments. On the other hand, the possibility to realize accelerated expansion by means of derivative couplings, even if the scalar potential is steep, may be of relevance for inflationary model building. However, in order to realize a sufficient number of e-folds, one again needs the attractor solution itself to be quasi-de Sitter, which requires an apparently unnatural value of γ. On the other hand, a generalization of the setting of [141] toward more general kinetic terms, cf. [145], and non-constant γ would be a promising direction for further research.

Motivated by our discussion in chapter 3, we also considered a stage of fast-roll preceding the stabilization of a modulus field. Given a radion potential dominated by repulsive Casimir energy at small distances, we were able to show that the field will generically roll downward its potential slope, dynamically tracking a cosmological scaling solution. The corresponding fixed point of the evolutionary system is radiation dominated. The equation of state, which characterizes the energy density contributed by the scalar field, adjusts itself exactly to the dominant cosmological fluid. As a consequence, the radion enters the oscillatory regime with an excess of kinetic energy. Due to the shallow barrier, which separates the low energy minimum from the runaway solution, the radion field is likely to overshoot, leading to decompactification.

However, if the initial field value were already close to the position of the low energy vacuum, the radion would *sit and wait*, remaining frozen due to the Hubble friction, until the Hubble rate drops below the radion mass. If the radion is light enough to be stable compared to the lifetime of the universe, its coherent oscillations around the vacuum value contribute to the dark matter density in the universe. Based on gamma ray observations of our galaxy, we were able to constrain this contributions to be $\Omega_{\mathrm{rad}} \sim \mathcal{O}(10^{-6})\,\Omega_{\mathrm{DM}}$ for a radion mass $\mathcal{O}(1\,\mathrm{MeV})$. The amplitude of the coherent oscillations is then allowed to be $\mathcal{O}(10^{7}\,\mathrm{GeV})$ at most.

Chapter 5

Conclusion

Geometric moduli fields are an unavoidable by-product of string inspired model building. If moduli stabilization is associated with the breaking of supersymmetry, their mass is generically of the same order as the gravitino mass, which is the order parameter of SUSY breaking in supergravity models. Particle physics considerations lead to the expectation of a low SUSY breaking scale, corresponding to a gravitino mass in the range of a few TeV or less. In this mass range, geometric moduli decay late, since they couple only with gravitational strength. Their decay products reheat the universe and thereby spoil the success of Big Bang nucleosynthesis.

We presented a class of models, in which the mass of a volume modulus, the radion, is generically suppressed *wrt* the gravitino mass. As a consequence, the mass can be low enough to render the radion stable on cosmological time scales. If this is the case, the radion field value undergoes coherent oscillations around its vacuum value, thereby contributing to the dark matter density in the universe. The viability of such a model depends crucially on initial conditions. If the radion field value is initially situated in the steep part of its potential, it is likely to overshoot the shallow minimum. This is due to a cosmological scaling regime, which we showed to be an attractor solution of the radion dynamical system. During the radiation era, the radion energy density mimics the dominant cosmological fluid of background photons, and reaches the vicinity of the potential minimum with an excess of kinetic energy. On the other hand, if the initial field value is already sufficiently close to the low energy vacuum value, the radion oscillations contribute only a small fraction to the dark matter density, in agreement with recent astrophysical and cosmological data.

We showed that the aforementioned suppression of the radion mass is inevitable in almost no-scale models for radion stabilization. In this framework, the radion potential exactly vanishes at tree level, and a potential is generated by perturbative corrections to the Kähler potential, after supersymmetry is broken. A generic contribution to the radion potential is the Casimir energy of bulk fields, which scales with $(LM)^{-1}$ compared to the leading order terms. In order to fix the size L of the compact extra-dimensional space, one needs to introduce an independent physical mass scale. We investigated a five-dimensional toy model, where the relevant mass scale emerges from brane-localized

Fayet-Iliopoulos terms, and is of order M_{GUT}.

GUT scale extra dimensions are strongly motivated by the idea of supersymmetric local grand unification [6, 7]. Combined with the assumption of low energy SUSY breaking and a gravitino mass of order 100 GeV, our example favors a radion mass of a few MeV, without refering to large extra dimensions in the way of [129]. Hence, the radion is cosmologically stable and, on the other hand, heavy enough not to cause variations of fundamental constants; a significant evolution of the radion background field value would be in conflict with BBN physics. Moreover, our toy model is predictive: The monochromatic line resulting from radion decays into two photons would be a clear signature in galactic and extragalactic gamma ray spectra.

Higher-dimensional supergravity model building can be motivated by the idea of anisotropic string compactifications [13, 66]. In the context of heterotic orbifolds, six-dimensional models allow for a quasi-realistic particle spectrum and richer phenomenology than 5D ones. 6D supergravity compactified on a flat torus orbifold gives rise to three complex moduli fields in the low energy effective action. We identified the moduli supermultiplets, explicitly including the fermionic partners, and derived the corresponding SUSY transformations. The resulting Kähler potential exhibits no-scale structure. However, the Casimir contribution to the Kähler correction only depends on a specific combination of the dilaton and the Kähler modulus. The dilaton has to be stabilized by non-perturbative effects, and the moduli potential is no longer of the almost no-scale type. However, in the presence of a SUSY breaking bulk field it is possible to adjust the parameters of the superpotential, such that Kähler and shape moduli remain exactly flat directions of the scalar potential at leading order. In a second step, these fields can then be stabilized via perturbative Kähler corrections.

On the other hand, the 5D set-up provides a working toy model for heterotic string compactifications with a universal Kähler modulus. Suppose that all other moduli fields are stabilized at higher energy scales by means of tree level and non-perturbative effects. Then all these fields can be consistently integrated out from the low energy effective action. The resulting Kähler potential exactly coincides with the radion Kähler potential at tree level. Although stringy perturbative corrections are not yet completely understood, any successful attempt to stabilize the Kähler modulus by means of such effects will inevitably be of the almost no-scale type. Hence, our results apply, and the universal Kähler modulus will turn out to be parametrically lighter than the gravitino.

In summary, we presented and investigated a self-consistent scenario of moduli stabilization, reconciling GUT scale extra dimensions and low energy SUSY breaking with cosmological constraints on light scalar fields. Our five-dimensional set-up can be viewed as a toy model for heterotic string compactifications with a universal Kähler modulus. The modulus is stabilized by means of perturbative Kähler corrections, leading to a mass of a few MeV, and may contribute a small fraction to the dark matter density in the present universe.

Appendix A

A.1 SUSY transformations, spinor decomposition and gamma matrix conventions

The bosonic supersymmetry transformations of the 6D N=2 theory [146], up to fermion bilinears, are given by

$$\delta E_M^A = -\frac{i}{\sqrt{2}M}\bar{\epsilon}\Gamma^A\Psi_M\,, \tag{A.1.1}$$

$$\delta\phi = \frac{1}{\sqrt{2}M}\bar{\epsilon}\chi\,, \tag{A.1.2}$$

$$\delta B_{MN} = -\frac{ie^{-\phi}}{2}\bar{\epsilon}\left(2\sqrt{2}A_{[M}\delta A_{N]} + 2\Gamma_{[M}\Psi_{N]} - i\Gamma_{MN}\chi\right)\,, \tag{A.1.3}$$

$$\delta A_M = -\frac{ie^{-\phi/2}}{\sqrt{2}}\bar{\epsilon}\Gamma_M\lambda\,, \tag{A.1.4}$$

and their fermionic counterparts, again up to fermion bilinears, read

$$\delta\Psi_M^i = \sqrt{2}M\,D_M\epsilon^i - \frac{e^{\phi}}{24}G_{NPR}\Gamma^{NPR}\Gamma_M\epsilon^i\,, \tag{A.1.5}$$

$$\delta\chi^i = -\frac{iM}{\sqrt{2}}\,\partial_M\phi\Gamma^M\epsilon^i - \frac{ie^{\phi}}{12}G_{NPR}\Gamma^{NPR}\epsilon^i\,, \tag{A.1.6}$$

$$\delta\lambda^i = -\frac{e^{\phi/2}}{2\sqrt{2}}\,F_{MN}\Gamma^{MN}\epsilon^i\,, \tag{A.1.7}$$

where $i = 1, 2$ runs over the fundamental of $Sp(1)$. The spinors are 8-component symplectic Majorana-Weyl, with the gravitini Ψ_M^i and the gaugini λ^i being right-handed, and the dilatini χ^i being left-handed. We shall work with complex-Weyl spinors instead, which are obtained by defining $\epsilon \equiv \epsilon^1 + i\epsilon^2$ *etc.*

The covariant derivative acts on the Killing spinor as

$$D_M\epsilon = \partial_M\epsilon + \frac{1}{4}\omega_M^{AB}\Gamma_{AB}\epsilon\,, \tag{A.1.8}$$

with

$$\omega_M^{AB} = E^{AN}\partial_{[M}E^B_{N]} - \left(\partial_{[M}E^A_{N]}\right)E^{NB} + E^{AN}E^C_M\left(\partial_{[P}E_{N]C}\right)E^{PB}\,. \tag{A.1.9}$$

The 6D gamma matrices are built from the 4D ones,

$$\Gamma^\alpha = \begin{pmatrix} \gamma^\alpha & 0 \\ 0 & \gamma^\alpha \end{pmatrix}\,, \quad \Gamma^a = \begin{pmatrix} 0 & x^a\gamma_5 \\ y^a\gamma_5 & 0 \end{pmatrix}\,, \tag{A.1.10}$$

with $x^{\dot 5} = y^{\dot 5} = i, x^{\dot 6} = 1 = -y^{\dot 6}$. We use dotted numbers as Lorentz indices. Products of 6D gamma matrices with an even (odd) number of internal indices are block-(off-) diagonal. Observing the 6D chiralities of the fermions, with the chirality matrix written as

$$\Gamma_7 = \begin{pmatrix} \gamma_5 & 0 \\ 0 & -\gamma_5 \end{pmatrix}\,, \tag{A.1.11}$$

we decompose the 6D spinors into N=1 Dirac spinors as follows,

$$\begin{aligned} \epsilon &= \begin{pmatrix} \epsilon_L^+ \\ \epsilon_R^- \end{pmatrix}\,, \quad \Psi_\mu = \begin{pmatrix} \Psi_{L\mu}^+ \\ \Psi_{R\mu}^- \end{pmatrix}\,, \quad \Psi_m = \begin{pmatrix} \Psi_{Lm}^- \\ \Psi_{Rm}^+ \end{pmatrix}\,, \\ \chi &= \begin{pmatrix} \chi_R^+ \\ \chi_L^- \end{pmatrix}\,, \quad \lambda = \begin{pmatrix} \lambda_L^+ \\ \lambda_R^- \end{pmatrix}\,. \end{aligned} \tag{A.1.12}$$

The $\pm$ refer to the orbifold parity.

Finally, we collect a set of useful gamma matrix identities, which we frequently apply in the calculations presented in the main text.

$$\{\Gamma^\mu, \Gamma^n\} = 0\,, \tag{A.1.13}$$

$$\{\Gamma^m, \Gamma^{\nu\pi\rho}\} = 6\, g^{m[\nu}\Gamma^{\pi\rho]} = 0\,, \tag{A.1.14}$$

$$[\Gamma^m, \Gamma^{np}] = -4g^{m[n}\Gamma^{p]}\,, \tag{A.1.15}$$

$$\gamma^{\nu\pi\rho} = i\epsilon^{\nu\pi\rho\sigma}\gamma_\sigma\gamma_5\,, \tag{A.1.16}$$

$$\epsilon_{\nu\pi\rho\lambda}\gamma^{\nu\pi\rho}{}_\mu = 6i\, g_{\lambda\mu}\gamma_5\,. \tag{A.1.17}$$

A.2 Metric decomposition, spin connection and Ricci scalar

We use mostly minus signature and MTW conventions for the curvature tensors. With the even parity degrees of freedom of the metric given as follows,

$$g_{MN} = \begin{pmatrix} r^{-2} g_{\mu\nu} & \\ & r^2 g_{mn} \end{pmatrix}, \quad \text{where} \quad g_{mn} = -\frac{1}{\tau_2} \begin{pmatrix} 1 & \tau_1 \\ \tau_1 & \tau_1^2 + \tau_2^2 \end{pmatrix}, \tag{A.2.1}$$

we write the corresponding vielbeins as

$$E_M^A = \begin{pmatrix} r^{-1} e_\mu^\alpha & e_m^\alpha \\ 0 & r e_m^a \end{pmatrix}, \quad E_m^a = -\frac{1}{\sqrt{\tau_2}} \begin{pmatrix} 1 & \tau_1 \\ 0 & -\tau_2 \end{pmatrix}. \tag{A.2.2}$$

The inverse vielbeins are then given by

$$E_A^M = \begin{pmatrix} r e_\alpha^\mu & e_a^\mu \\ 0 & r^{-1} e_a^m \end{pmatrix}, \quad E_a^m = -\frac{1}{\sqrt{\tau_2}} \begin{pmatrix} \tau_2 & \tau_1 \\ 0 & -1 \end{pmatrix}. \tag{A.2.3}$$

At the position of the fixed brane,[1] the spin connection coefficients are evaluated as follows,

$$\omega_\mu^{\alpha\beta} = \omega_\mu^{\alpha\beta}\left[e_\mu^\alpha\right] - \left(e_\mu^\alpha e^{\nu\beta} - e^{\alpha\nu} e_\mu^\beta\right) \frac{\partial_\nu r^2}{2r^2}, \tag{A.2.4}$$

$$\omega_\mu^{\alpha b} = 0, \tag{A.2.5}$$

$$\omega_\mu^{ab} = -\epsilon^{ab} \left[\frac{\partial_\mu \tau_1}{2\tau_2} - r^{-3}\, e_{\mu\alpha}\, \partial_{[5} e_{6]}^\alpha\right], \tag{A.2.6}$$

$$\omega_m^{\alpha\beta} = 0, \tag{A.2.7}$$

$$\omega_m^{a\beta} = r^2 \tilde{\omega}_m^{a\beta} + \frac{1}{2} e_m^a\, \partial_\nu r^2\, e^{\nu\beta} - r^{-1} e^{an} \partial_{[n} e_{m]}^\beta, \tag{A.2.8}$$

$$\omega_m^{ab} = 0, \tag{A.2.9}$$

where we defined

$$\tilde{\omega}_5^{\dot{5}\beta} = \frac{1}{2\tau_2^{3/2}}\, \partial_\nu \tau_2\, e^{\nu\beta}, \tag{A.2.10}$$

$$\tilde{\omega}_6^{\dot{5}\beta} = \frac{1}{2\tau_2^{3/2}} \left(\tau_1 \partial_\nu \tau_2 - \tau_2 \partial_\nu \tau_1\right) e^{\nu\beta}, \tag{A.2.11}$$

$$\tilde{\omega}_5^{\dot{6}\beta} = \frac{1}{2\tau_2^{3/2}}\, \partial_\nu \tau_1\, e^{\nu\beta}, \tag{A.2.12}$$

$$\tilde{\omega}_6^{\dot{6}\beta} = \frac{1}{2\tau_2^{3/2}} \left(\tau_2 \partial_\nu \tau_2 + \tau_1 \partial_\nu \tau_1\right) e^{\nu\beta}. \tag{A.2.13}$$

[1] We set the $\mathbb{Z}_2$ odd, off-diagonal components of the vielbein to zero, but kept their internal derivatives, since they contribute at the brane position.

For later use, let us add

$$\omega_\mu^{\alpha b} = -e^\alpha_{\ \mu}\,\frac{\partial_n r^2}{2r^2}\,e^{nb}\,, \tag{A.2.14}$$

$$\omega_5^{ab} = \frac{\epsilon^{ab}}{2\tau_2}\left(\tau_1\frac{\partial_5 r^2}{r^2} - \tau_1\frac{\partial_5\tau_2}{\tau_2} + 2\partial_5\tau_1 - \frac{\partial_6 r^2}{r^2} + \frac{\partial_6\tau_2}{\tau_2}\right)\,, \tag{A.2.15}$$

$$\begin{aligned}\omega_6^{ab} = \frac{\epsilon^{ab}}{2\tau_2}\Bigg[&(\tau_1^2+\tau_2^2)\frac{\partial_5 r^2}{r^2} - (\tau_1^2-\tau_2^2)\frac{\partial_5\tau_2}{\tau_2}\\ &+ 2\tau_1\partial_5\tau_1 - \tau_1\left(\frac{\partial_6 r^2}{r^2} - \frac{\partial_6\tau_2}{\tau_2}\right)\Bigg]\,,\end{aligned} \tag{A.2.16}$$

since the Riemann curvature tensor will include terms featuring double internal derivatives of even fields, which cannot be set to zero at the brane position. Moreover, it receives contributions from

$$\begin{aligned}\partial_m\omega_\mu^{\alpha a} \supset \partial_m\Big[&r^{-1}\Big(e^{\alpha\nu}e^\gamma_{\ \mu}(\partial_{[p}E_{\nu]\gamma})e^{pa} - (\partial_{[\mu}E^\alpha_{\ n]})e^{na}\Big)\\ &+ e^{\alpha\nu}e^\gamma_{\ \mu}\partial_{[\pi}\big(r^{-1}e_{\nu]\gamma}\big)e^{\pi a} - \partial_{[\mu}\big(r^{-1}e^\alpha_{\ \nu]}\big)e^{\nu a}\Big]\,,\end{aligned} \tag{A.2.17}$$

and

$$\begin{aligned}\partial_m\omega_n^{ab} \supset \partial_m\Bigg[&\frac{1}{2}\Big(-e^{a\sigma}\partial_\sigma(re^b_{\ n}) + \partial_\sigma(re^a_{\ n})e^{\sigma b}\Big) + r^{-2}e^{as}e_{n\gamma}\partial_{[r}e^\gamma_{\ s]}e^{rb}\\ &-\frac{1}{2}\Big(e^{a\sigma}e^c_{\ n}\partial_\sigma(re_{rc})e^{rb} - e^{as}e^c_{\ n}\partial_\rho(re_{sc})e^{\rho b}\Big)\Bigg]\,.\end{aligned} \tag{A.2.18}$$

Now we are ready to compute the Ricci scalar,

$$\begin{aligned}R &= E_A^{\ M}E_B^{\ N}R^{AB}{}_{MN}\\ &= 2E_A^{\ M}E_B^{\ N}\Big(\partial_{[M}\omega_{N]}^{AB} + \omega^A{}_{C[M}\omega_{N]}^{CB}\Big)\,.\end{aligned} \tag{A.2.19}$$

On the brane, only the following terms contribute,

$$\begin{aligned}\frac{1}{2r^2}R_{\rm br} = {}& e_\alpha^{\ \mu}e_\beta^{\ \nu}\Big(\partial_{[\mu}\omega_{\nu]}^{\alpha\beta} + \omega^\alpha{}_{\gamma[\mu}\omega_{\nu]}^{\gamma\beta}\Big)\\ &+r^{-2}e_\alpha^{\ \mu}e_b^{\ n}\Big(\partial_{[\mu}\omega_{n]}^{\alpha b} + \omega^\alpha{}_{\gamma[\mu}\omega_{n]}^{\gamma b}\Big)\\ &+r^{-2}e_a^{\ m}e_\beta^{\ \nu}\Big(\partial_{[m}\omega_{\nu]}^{a\beta} + \omega^a{}_{\gamma[m}\omega_{\nu]}^{\gamma\beta}\Big)\\ &+r^{-4}e_a^{\ m}e_b^{\ n}\Big(\partial_{[m}\omega_{n]}^{ab} + \omega^a{}_{\gamma[m}\omega_{n]}^{\gamma b}\Big)\,.\end{aligned} \tag{A.2.20}$$

The additional factor of r^{-2} results from the metric determinant. We will compute these contributions one by one, starting with

$$
\begin{aligned}
e^{\mu}_{\alpha} e^{\nu}_{\beta} R^{\alpha\beta}{}_{\mu\nu} &= R_4 - \frac{9}{2r^4}\partial^\rho r^2 \partial_\rho r^2 - \left[e^{\mu}_{\alpha}\partial_{[\mu} e^{\alpha}_{\nu]} g^{\nu\rho} - g^{\mu\rho}\partial_{[\mu} e^{\beta}_{\nu]} e^{\nu}_{\beta} \right] \frac{\partial_\rho r^2}{r^2} \\
&\quad + \frac{3}{r^2} g^{\mu\rho}\partial_\mu \partial_\rho r^2 + \frac{3}{2r^2} e^{\mu}_{\alpha}\left(\partial_\mu e^{\alpha\rho} + \omega_\mu^{\alpha\gamma} e^{\rho}_{\gamma} \right) \partial_\rho r^2 \\
&\quad + \frac{3}{2r^2}\left(\partial_\nu e^{\rho\beta}\right) e^{\nu}_{\beta} \partial_\rho r^2 - \frac{1}{2r^2} e^{\nu}_{\alpha}\omega_\nu^{\alpha\gamma} e^{\rho}_{\gamma}\partial_\rho r^2 \\
&= R_4 - \frac{9}{2r^4}\partial^\rho r^2 \partial_\rho r^2 + \frac{3}{r^2}\nabla^\rho \partial_\rho r^2 \,,
\end{aligned}
\tag{A.2.21}
$$

where R_4 is the 4D (Einstein frame) Ricci scalar and the last term can be integrated by parts. We used

$$
e^{\mu}_{\alpha}\left(\partial_\mu e^{\alpha\rho} + \partial_{[\sigma} e^{\alpha}_{\mu]} g^{\sigma\rho} \right) = e^{\mu}_{\alpha}\left(\partial_\mu e^{\alpha\rho} + \omega_\mu^{\alpha\gamma} e^{\rho}_{\gamma} \right) = \partial_\mu g^{\mu\nu} + \Gamma^{\mu}_{\mu\sigma} g^{\sigma\rho} \,. \tag{A.2.22}
$$

Next, we consider

$$
e^{\mu}_{\alpha} e^{m}_{a}\left(R^{\alpha a}{}_{\mu m} + R^{a\alpha}{}_{m\mu} \right) = 2 e^{\mu}_{\alpha} e^{m}_{a}\left(2\partial_{[\mu}\omega^{\alpha a}_{m]} + \omega^{\alpha}_{\mu\gamma}\omega^{\gamma a}_{m} - \omega^{\alpha}_{mc}\omega^{ca}_{\mu} \right) . \tag{A.2.23}
$$

Although $\omega^{\alpha a}_{\mu}$ is zero on the brane, the term $\partial_m \omega^{\alpha a}_{\mu}$ requires some care. Due to our gauge choice, we have

$$
\begin{aligned}
-2\, e^{\mu}_{\alpha} e^{m}_{a}\partial_m \omega^{\alpha a}_{\mu} &\supset 2r^{-1} g^{mn} e^{\nu}_{\alpha}\partial_\nu \partial_m e^{\alpha}_{n} - 4r^{-1} e^{\mu}_{\alpha}\left(\partial_{[\nu} e^{\alpha}_{\mu]} \right) e^{m}_{a}\partial_m e^{\nu a} \\
&\qquad + 6r^{-2}(\partial_\nu r) e^{m}_{a}\partial_m e^{a\nu} \\
&= 2r^{-1} g^{mn}\left[e^{\nu}_{\alpha}\partial_\nu \partial_m e^{\alpha}_{n} + 2\, e^{\mu}_{\alpha} e^{\nu}_{\beta}\left(\partial_{[\nu} e^{\alpha}_{\mu]} \right)\partial_m e^{\beta}_{n} \right] + 6r^{-2}(\partial_\nu r) e^{m}_{a}\partial_m e^{a\nu} \\
&= 2r^{-1} g^{mn}\nabla_\alpha \partial_m e^{\alpha}_{n} - 6r^{-2} g^{mn} e^{\nu}_{\beta}(\partial_\nu r)\partial_m e^{\beta}_{n} \,,
\end{aligned}
\tag{A.2.24}
$$

where we used $\partial_m e^{\nu a} = -e^{\nu}_{\beta}(\partial_m e^{\beta}_{n}) e^{na}$. There will also be contributions with double internal derivatives of even fields. We shall treat them separately later on. With

$$
\begin{aligned}
\hat{\omega}^{\alpha}_{\mu\gamma} &\equiv \omega^{\alpha}_{\mu\gamma}[e^{\alpha}_{\mu}] - r^{-2}\left(e^{\alpha}_{\mu} e^{\nu}_{\gamma} - e^{\alpha\nu} e_{\mu\gamma} \right)\partial_\nu r^2/2 \,, \\
\omega^{\alpha a}_{m} &= r^2 \tilde{\omega}^{\alpha a}_{m} - e^{\alpha\rho}\left(\partial_\rho r^2 \right) e^{a}_{m}/2 + r^{-1}\left(\partial_{[p} e^{\alpha}_{m]} \right) e^{pa} \,,
\end{aligned}
\tag{A.2.25}
$$

we obtain for the remaining part of (A.2.23),

$$
\begin{aligned}
2\, e^{\mu}_{\alpha} e^{m}_{a}\left(\partial_\mu \omega^{\alpha a}_{m} + \hat{\omega}^{\alpha}_{\mu\gamma}\omega^{\gamma a}_{m} \right) &= 2\, e^{\mu}_{\alpha} e^{m}_{a}\left[\partial_\mu\left(r^2 \tilde{\omega}^{\alpha a}_{m} \right) + r^2 \hat{\omega}^{\alpha}_{\mu\gamma}\tilde{\omega}^{\gamma a}_{m} \right] \\
&\quad - 2\left[g^{\mu\nu}\partial_\mu \partial_\nu r^2 + e^{\mu}_{\alpha}\left(\partial_\mu e^{\alpha\nu} + \hat{\omega}^{\alpha}_{\mu\gamma} e^{\gamma\nu} \right)\partial_\nu r^2 \right] \\
&\quad + 2r^{-1} e^{\mu}_{\alpha}\left[e^{m}_{a}\partial_\mu\left(e^{ap}\partial_{[p} e^{\alpha}_{m]} \right) + \hat{\omega}^{\alpha}_{\mu\gamma} g^{mp}\partial_{[p} e^{\gamma}_{m]} \right] \\
&= 2r^2 e^{\mu}_{\alpha} e^{m}_{a}\partial_\mu \tilde{\omega}^{\alpha a}_{m} + 2(r\tau_2)^{-1} e^{\mu}_{\alpha}\partial_\mu \tau_1 \partial_{[5} e^{\alpha}_{6]} \\
&\quad - 2\nabla^\mu \partial_\mu r^2 + 3r^{-2} g^{\mu\nu}\partial_\mu r^2 \partial_\nu r^2 \,,
\end{aligned}
\tag{A.2.26}
$$

where we frequently used $e^{m}_{a}\tilde{\omega}^{\alpha a}_{m} \sim e^{m}_{a}\partial_\mu e^{a}_{m} = 0$. Finally,

$$
\begin{aligned}
-2e^{\mu}_{\alpha}e^{m}_{a}\omega^{\alpha}_{mc}\omega^{ca}_{\mu} &= 2\left(r^2 e^{\mu}_{\alpha}e^{m}_{a}\tilde{\omega}^{\alpha}_{mc}\epsilon^{ca} - 2r^{-1}\partial_{[5}e^{\alpha}_{6]}e^{\mu}_{\alpha}\right)\left[\frac{\partial_{\mu}\tau_1}{2\tau_2} - r^{-3}\partial_{[5}e^{\beta}_{6]}e_{\beta\mu}\right] \\
&= -2(r\tau_2)^{-1}e^{\mu}_{\alpha}\partial_{\mu}\tau_1\partial_{[5}e^{\alpha}_{6]} + 4r^{-4}\partial_{[5}e^{\alpha}_{6]}\partial_{[5}e_{6]\alpha}\,. \qquad \text{(A.2.27)}
\end{aligned}
$$

For later use, we record that

$$
g^{ms}g^{nr}\partial_{[m}e^{\alpha}_{n]}\partial_{[r}e_{s]\alpha} = -2\partial_{[5}e^{\alpha}_{6]}\partial_{[5}e_{6]\alpha}\,. \qquad \text{(A.2.28)}
$$

Now for the last part of R_{br}. Consider

$$
\begin{aligned}
2\,e^{m}_{a}e^{n}_{b}\partial_{[m}\omega^{ab}_{n]} \supset\; & r\,e^{m}_{a}e^{n}_{b}\Big[\big(\partial_{\sigma}e^{a}_{n}\big)\partial_{m}e^{\sigma b} + \big(\partial_{n}e^{a\sigma}\big)\partial_{\sigma}e^{b}_{m}\Big]/2 \\
&+ r\Big[g^{ms}\big(\partial_{m}e^{\rho}_{b}\big)\partial_{\rho}e^{b}_{s} + g^{nr}\big(\partial_{n}e^{\sigma}_{a}\big)\partial_{\sigma}e^{a}_{r}\Big]/2 \\
&+ 2\left[r^{-2}g^{ms}g^{nr}\big(\partial_{[m}e_{n]\gamma}\big)\partial_{[r}e^{\gamma}_{s]} - e^{m}_{a}\big(\partial_{m}e^{a\sigma}\big)\partial_{\sigma}r\right] \\
=\;& r\left[e^{n}_{a}e^{m}_{b}\big(\partial_{\sigma}e^{a}_{m}\big) + g^{nr}\big(\partial_{\sigma}e_{rb}\big)\right]\partial_{n}e^{\sigma b} \\
&+ 2\left[r^{-2}g^{ms}g^{nr}\partial_{[m}e_{n]\gamma}\partial_{[r}e^{\gamma}_{s]} + g^{mn}\big(\partial_{m}e^{\beta}_{n}\big)e^{\sigma}_{\beta}\partial_{\sigma}r\right]. \qquad \text{(A.2.29)}
\end{aligned}
$$

Again we have left out double internal derivative terms. We can further simplify,

$$
\begin{aligned}
& r^{-1}\left[e^{n}_{a}e^{m}_{b}\big(\partial_{\sigma}e^{a}_{m}\big) + g^{nr}\big(\partial_{\sigma}e_{rb}\big)\right]\partial_{n}e^{\sigma b} \\
&\quad = r^{-1}\left[e^{n}_{a}e^{m}_{b}\big(e^{a}_{r}(\partial_{\sigma}e^{r}_{c})e^{c}_{m}\big) + g^{nr}\Big(e_{pb}\big(\partial_{\sigma}e^{p}_{c}\big)e^{c}_{r}\Big)\right]e^{\sigma}_{\beta}\big(\partial_{n}e^{\beta}_{l}\big)e^{lb} \\
&\quad = r^{-1}\Big[\big(\partial_{\sigma}e^{n}_{c}\big)e^{c}_{l} + e^{nc}\partial_{\sigma}e^{l}_{c}\Big]e^{\sigma}_{\beta}\partial_{n}e^{\beta}_{l} \\
&\quad = r^{-1}e^{\sigma}_{\beta}\big(\partial_{\sigma}g^{nl}\big)\partial_{n}e^{\beta}_{l}\,. \qquad \text{(A.2.30)}
\end{aligned}
$$

Combining with (A.2.24), we get

$$
\begin{aligned}
& 2r^{-3}g^{mn}\nabla_{\alpha}\partial_{m}e^{\alpha}_{n} - 4r^{-4}g^{mn}e^{\nu}_{\beta}(\partial_{\nu}r)\partial_{m}e^{\beta}_{n} + r^{-1}e^{\sigma}_{\beta}(\partial_{\sigma}g^{nl})\partial_{n}e^{\beta}_{l} \\
&\quad = 2\nabla_{\alpha}\big(r^{-3}g^{mn}\partial_{m}e^{\alpha}_{n}\big) - r^{-1}e^{\mu}_{\alpha}\partial_{\mu}\big(r^{-2}g^{mn}\big)\partial_{m}e^{\alpha}_{n}\,. \qquad \text{(A.2.31)}
\end{aligned}
$$

The covariant divergence does not affect the equations of motion and will therefore be omitted below. We are now left with

$$
\begin{aligned}
r^{-2}e^{m}_{a}e^{n}_{b}\omega^{a}_{\gamma[m}\omega^{\gamma b}_{n]} =\;& r^2 e^{m}_{a}e^{n}_{b}\tilde{\omega}^{a}_{\gamma[m}\tilde{\omega}^{\gamma b}_{n]} + e^{m}_{a}e^{n}_{b}\left[\tilde{\omega}^{a\gamma}_{[n}e^{b}_{m]} - e^{a}_{[n}\tilde{\omega}^{\gamma b}_{m]}\right]e^{\mu}_{\gamma}r\partial_{\mu}r \\
&+ \frac{1}{2r}e^{m}_{a}e^{n}_{b}\Big[\big(\tilde{\omega}^{a\gamma}_{m}e^{bp} + e^{ap}\tilde{\omega}^{\gamma b}_{m}\big)\partial_{[p}e_{n]\gamma} - \big(\tilde{\omega}^{a\gamma}_{n}e^{bp} + e^{ap}\tilde{\omega}^{\gamma b}_{n}\big)\partial_{[p}e_{m]\gamma}\Big] \\
&+ e^{m}_{a}e^{n}_{b}\left[e^{a}_{[n}e^{b}_{m]}g^{\mu\nu}\partial_{\mu}r\partial_{\nu}r + \frac{e^{as}}{2r^4}\big(\partial_{[s}e_{n]\gamma}\partial_{[r}e^{\gamma}_{m]} - \partial_{[s}e_{m]\gamma}\partial_{[r}e^{\gamma}_{n]}\big)e^{rb}\right] \\
&+ \frac{1}{2r^2}g^{lp}\partial_{[p}e^{\alpha}_{l]}e^{\lambda}_{\alpha}\partial_{\lambda}r
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{r^2}{2} e^m_{\ a} e^n_{\ b} \tilde{\omega}^{a}_{\gamma n} \tilde{\omega}^{\gamma b}_{m} + r^{-1} e^m_{\ a} g^{np} \tilde{\omega}^{\gamma a}_{n} \partial_{[p} e_{m]\gamma} \\
&\quad - \frac{1}{4r^2} g^{\mu\nu} \partial_\mu r^2 \partial_\nu r^2 + \frac{1}{2r^4} g^{ms} g^{nr} \partial_{[s} e_{n]\gamma} \partial_{[r} e^{\gamma}_{\ m]} \,.
\end{aligned} \tag{A.2.32}
$$

An explicit calculation shows that $e^m_{\ a} g^{np} \tilde{\omega}^{\gamma a}_{n} \partial_{[p} e_{m]\gamma} = 0$. Finally, integrating by parts,

$$
\begin{aligned}
&e^m_{\ a} \left(2\, e^\mu_{\ \alpha} \partial_\mu \tilde{\omega}^{\alpha a}_{m} - e^n_{\ b} \tilde{\omega}^{a}_{\gamma n} \tilde{\omega}^{\gamma b}_{m}\right) \\
&\quad = g^{\mu\nu} \left[(\partial_\mu e^m_{\ a}) \partial_\nu e^a_{\ m} - \frac{1}{2} \left(\eta_{ab} g^{mn} - e^m_{\ a} e^n_{\ b}\right) (\partial_\mu e^b_{\ m}) \partial_\nu e^a_{\ n} \right] \\
&\quad = -\frac{1}{2\tau_2^2} g^{\mu\nu} (\partial_\mu \tau_1 \partial_\nu \tau_1 + \partial_\mu \tau_2 \partial_\nu \tau_2) \,.
\end{aligned} \tag{A.2.33}
$$

Now let us come back to the issue of double internal derivative terms. We start with

$$
\begin{aligned}
-2 e^\mu_{\ \alpha} e^m_{\ a} \partial_m \omega^{\alpha a}_{\mu} &\supset e^\mu_{\ \alpha} e^m_{\ a} \partial_m \left(r^{-4} e^\alpha_{\ \mu} \partial_n r^2 e^{na} \right) \\
&\supset 4 r^{-4} g^{mn} \partial_m \partial_n r^2 \,.
\end{aligned} \tag{A.2.34}
$$

For the remaining terms, we obtain

$$
\begin{aligned}
2\, e^m_{\ a} e^n_{\ b} \partial_{[m} \omega^{ab}_{n]} &= 4\, \partial_{[5} \omega^{\dot{5}\dot{6}}_{6]} \\
&\supset \frac{\tau_1^2 + \tau_2^2}{\tau_2} \frac{\partial_5 \partial_5 r^2}{r^2} - \frac{\tau_1^2 - \tau_2^2}{\tau_2} \frac{\partial_5 \partial_5 \tau_2}{\tau_2} + 2\tau_1 \frac{\partial_5 \partial_5 \tau_1}{\tau_2} \\
&\quad - 2 \left(\frac{\tau_1}{\tau_2} \frac{\partial_5 \partial_6 r^2}{r^2} - \frac{\tau_1}{\tau_2} \frac{\partial_5 \partial_6 \tau_2}{\tau_2} + \frac{\partial_5 \partial_6 \tau_1}{\tau_2} \right) \\
&\quad + \frac{1}{\tau_2} \frac{\partial_6 \partial_6 r^2}{r^2} - \frac{1}{\tau_2} \frac{\partial_6 \partial_6 \tau_2}{\tau_2} \\
&= -g^{mn} \left(\frac{\partial_m \partial_n r^2}{r^2} - \frac{\partial_m \partial_n \tau_2}{\tau_2} \right) \\
&\quad + \frac{2}{\tau_2} \left(\tau_2 \partial_5 \partial_5 \tau_2 + \tau_1 \partial_5 \partial_5 \tau_1 - \partial_5 \partial_6 \tau_1 \right) \,.
\end{aligned} \tag{A.2.35}
$$

Putting everything together, we finally arrive at

$$
\begin{aligned}
E_6 R_6 \to e_4 \Bigg\{ & R_4 - \frac{1}{r^4} \partial^\rho r^2 \partial_\rho r^2 - \frac{1}{2\tau_2^2} g^{\mu\nu} (\partial_\mu \tau_1 \partial_\nu \tau_1 + \partial_\mu \tau_2 \partial_\nu \tau_2) \\
&+ \frac{1}{r^6} g^{ms} g^{nr} \partial_{[m} e_{n]\gamma} \partial_{[r} e^{\gamma}_{\ s]} - \frac{1}{r} e^\mu_{\ \alpha} \partial_\mu \left(r^{-2} g^{mn} \right) \partial_m e^\alpha_{\ n} \\
&+ r^{-2} g^{mn} \left(3 \frac{\partial_m \partial_n r^2}{r^2} + \frac{\partial_m \partial_n \tau_2}{\tau_2} \right) \\
&+ \frac{2}{r^2 \tau_2} \left(\tau_2 \partial_5 \partial_5 \tau_2 + \tau_1 \partial_5 \partial_5 \tau_1 - \partial_5 \partial_6 \tau_1 \right) \Bigg\} \,.
\end{aligned} \tag{A.2.36}
$$

Appendix B

B.1 Casimir energy

In this appendix, we give some detail on the Casimir energy of bulk fields in a 5D set-up with the extra dimension being of size L, cf. [80, 81]. A single real scalar field with supersymmetric mass M_0 and periodic boundary conditions contributes to the effective potential with

$$r^2 V_0^+ = \frac{1}{2} \sum_n \int \frac{d^4k}{(2\pi)^4} \ln\left(k^2 + M_0^2 + \frac{\pi^2 n^2}{L^2 r^2}\right) , \tag{B.1.1}$$

where the sum is taken over all Kaluza-Klein modes with masses $M_n = n^2\pi^2(Lr)^{-2}$. The prefactor of r^2 is necessary to relate the RHS to an Einstein frame scalar potential. The expression is divergent and requires regularisation. The finite part is then given by

$$r^2 V_0^+ = -\frac{3}{64\pi^2(Lr)^4}\left[\frac{4l^2r^2}{3}\mathrm{Li}_3(e^{-lr}) + 2lr\mathrm{Li}_4(e^{-lr}) + \mathrm{Li}_5(e^{-lr})\right] , \tag{B.1.2}$$

where $l \equiv M_0 L$, and the polylogarithmic functions are defined by

$$\mathrm{Li}_s\left(e^{-lr}\right) \equiv \sum_{k=1}^{\infty} \frac{e^{-klr}}{k^s} . \tag{B.1.3}$$

Taking the massless limit, one finds

$$r^2 V_0^+ = -\frac{3}{64\pi^2}\frac{\zeta(5)}{(Lr)^4} . \tag{B.1.4}$$

By counting the number of degrees of freedom of higher spin fields,[1] and taking into account the mass shifts from supersymmetry breaking, one can derive the contributions from complete multiplets. One finds

$$r^2 V^0 = -\frac{6(2+N_V)}{64\pi^2(Lr)^4}\big(\zeta(5) - \zeta_\omega(5)\big) \approx -\frac{3(2+N_V)}{64\pi^2}\frac{\zeta(3)\omega^2}{(Lr)^4} \tag{B.1.5}$$

[1] Fermions contribute with a relative sign.

from the gravity and gauge sectors with N_V massless vectormultiplets. Here

$$\zeta_\omega(s) \equiv \sum_{k=1}^{\infty} \frac{\cos(2\pi k\omega)}{k^s}\,, \tag{B.1.6}$$

and the so-called Scherk-Schwarz parameter ω can be related to F_T [147] via

$$\omega = \frac{|W|L}{M^2} \ll 1\,. \tag{B.1.7}$$

A massive hypermultiplet contributes with

$$r^2 V^m = \frac{3\omega^2}{64\pi^2 (Lr)^4}\left[\frac{l^2 r^2}{3}\mathrm{Li}_1(e^{-lr}) + lr\mathrm{Li}_2(e^{-lr}) + \mathrm{Li}_3(e^{-lr})\right]\,. \tag{B.1.8}$$

We note that this term is repulsive at short distances, while V^0 is attractive.

Appendix C

C.1 Dynamical systems terminology

We consider a system of n first order ordinary differential equations (ODE),

$$\dot{x}_i = f_i(x_1, ..., x_n)\,, \tag{C.1.1}$$

which is called *autonomous* if none of the n functions f_i explicitly depends on time. A solution of the system is given in terms of a trajectory in phase-space,

$$t \longmapsto X(t) := (x_1(t), ..., x_n(t))\,, \tag{C.1.2}$$

uniquely determined by choice of initial conditions $X(t_{\text{init}})$.

A point $X_s := (x_{1,s}, ..., x_{n,s})$ is said to be a *critical, stationary* or *fixed point* if

$$f_i(X_s) = 0 \qquad \forall\, i\, \leq\, n\,, \tag{C.1.3}$$

and an *attractor* if there exists a neighborhood of the fixed point such that every trajectory entering this neighborhood satisfies the following condition,

$$\lim_{t\to\infty} X(t) = X_s\,. \tag{C.1.4}$$

Now consider small perturbations around the critical point,

$$x_i = x_{i,s} + \delta x_i\,. \tag{C.1.5}$$

Linearizing the evolution equations we obtain a system of first order ODE *linear* in the perturbations,

$$\frac{d}{dt}\delta x_i = \sum_j M_{ij}\delta x_j\,, \tag{C.1.6}$$

where

$$M_{ij} \equiv \left.\frac{\partial f_i(X)}{\partial x_j}\right|_{X=X_s}\,. \tag{C.1.7}$$

The general solution of this system is given by

$$\delta x_i = \sum_{k=1}^{n} C_{ik} e^{\mu_k t} \,, \tag{C.1.8}$$

where the C_{ik} are integration constants and the μ_k eigenvalues of the Jacobi or *stability matrix* M^J, which we assume to be distinct for simplicity. Obviously, the perturbation will decay if each μ_k has negative real part.

The critical points of a dynamical system can be classified in terms of the eigenvalues of the corresponding stability matrix. An attractor is characterized by the requirement

$$Re[\mu_k] < 0 \qquad \forall\, k \leq n \,, \tag{C.1.9}$$

and called *spiral focus* if at least one pair of eigenvalues is complex, and *stable node* else. Furthermore we shall use the terminus *saddle point*, if and only if M^J has one single eigenvalue with positive real part. Otherwise we call the fixed point *unstable*.

C.2 Jacobi matrix eigenvalues

Here we list the fixed points of the dynamical system (4.3.10–4.3.13) discussed in section 4.3, together with the respective set of Jacobi matrix eigenvalues.

$$
\begin{aligned}
A:&\quad -\frac{1}{2}+Q^2\,,\ -\frac{3}{2}+Q^2\,,\ -\frac{3}{2}+Q(Q+\gamma)\,,\ \frac{3}{2}+Q(Q-\lambda)\,,\\
B_1:&\quad 1\,,\ 3-\sqrt{6}Q\,,\ \sqrt{\frac{3}{2}}\gamma\,,\ 3-\sqrt{\frac{3}{2}}\lambda\,,\\
B_2:&\quad 1\,,\ 3+\sqrt{6}Q\,,\ -\sqrt{\frac{3}{2}}\gamma\,,\ 3+\sqrt{\frac{3}{2}}\lambda\,,\\
C:&\quad 2\,,\ -1\,,\ -1\,,\ 1\,,\\
D:&\quad -1+\frac{\gamma}{2Q}\,,\ 2-\frac{\lambda}{2Q}\,,\ \frac{1}{2}\left(-1\pm\frac{\sqrt{2Q^2-3Q^4}}{Q^2}\right)\,,\\
E:&\quad 1-\frac{4Q}{\lambda}\,,\ -1+\frac{2\gamma}{\lambda}\,,\ \frac{1}{2}\left(-1\pm\frac{\sqrt{64\lambda^2-15\lambda^4}}{\lambda^2}\right)\,,\\
F:&\quad -\frac{\lambda-4Q}{2(\lambda-Q)}\,,\ \frac{3}{2}\left(-1+\frac{\gamma+Q}{\lambda-Q}\right)\,,\\
&\quad \frac{3}{4(\lambda-Q)^2}\Bigg(-(\lambda-2Q)(\lambda-Q)\\
&\qquad\qquad \pm\sqrt{(\lambda-Q)^2[24-7\lambda^2-12\lambda Q+20Q^2+\frac{16}{3}\lambda Q(\lambda-Q)^2]}\Bigg)\,,\\
G:&\quad \frac{1}{2}(\lambda^2-6)\,,\ \frac{1}{2}(\lambda(\lambda+\gamma)-6)\,,\ -3+\lambda(\lambda-Q)\,,\ \frac{1}{2}(\lambda^2-4)\,,\\
H:&\quad 1-\frac{3\gamma}{2(\gamma+Q)}\,,\ \frac{3}{2}(1-\frac{\lambda-Q}{\gamma+Q})\,,
\end{aligned}
$$

$$
\begin{aligned}
&\quad \frac{1}{4(\gamma+Q)}\left(-3\gamma \pm \sqrt{81\gamma^2 - 24\gamma Q[2(\gamma+Q)^2-3]}\right)\,,\\
J: &\quad 3\left(1-\frac{2(\gamma+Q)}{\gamma+\lambda}\right)\,,\ 1-\frac{3\gamma}{\gamma+\lambda}\,,\\
&\quad \frac{3}{2(\gamma+\lambda)}\left(-\gamma \pm \sqrt{\gamma^2+8\gamma(\gamma+\lambda)-\frac{4}{3}\gamma\lambda(\gamma+\lambda)^2}\right)\,.
\end{aligned}
$$

In the case of the reduced system (4.4.2)-(4.4.3), corresponding to the model of [141], we find the following eigenvalues,

$$
\begin{aligned}
B_1: &\quad \sqrt{\frac{3}{2}}\gamma\,, \quad 6-\sqrt{6}\lambda\,,\\
B_2: &\quad -\sqrt{\frac{3}{2}}\gamma\,, \quad 6+\sqrt{6}\lambda\,,\\
G: &\quad \frac{1}{2}(\lambda^2-6)\,, \quad \frac{1}{2}(\lambda(\lambda+\gamma)-6)\,,\\
J: &\quad \frac{3}{2(\gamma+\lambda)}\left(-\gamma \pm \sqrt{\gamma^2+8\gamma(\gamma+\lambda)-\frac{4}{3}\gamma\lambda(\gamma+\lambda)^2}\right)\,.
\end{aligned}
$$

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