Precision QCD measurements at HERA

Daniel Britzger
for the H1 and ZEUS collaborations

Determination of the Fundamental Parameters in QCD Workshop
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Deep-inelastic scattering

Kinematic variables
- virtuality of exchanged boson
  \[ Q^2 = -q^2 = -(k - k')^2 \]
- Bjorken scaling variable
  \[ x = \frac{Q^2}{2 \vec{p} \cdot q} \]

Neutral current scattering
\[ ep \to e'X \]

Charged current scattering
\[ ep \to \nu_e X \]

Factorization in ep collisions
\[ \sigma_{ep \to eX} = \int_{p \to i} \otimes \hat{\sigma}_{ei \to eX} \]
\[ xf_{p \to i} = \text{quark/gluon momentum density in proton:} \]
\[ \text{Parton density functions (PDFs)} \]

PDFs are not observables – only structure functions are
Measuring these cross sections allows indirect access to the universal PDFs, which are also valid for pp collisions
Structure functions

\[
\frac{d\sigma_{NC}^\pm}{dx dQ^2} = \frac{2\pi\alpha^2}{x} \left[ \frac{1}{Q^2} \right]^2 \left[ Y_+ \tilde{F}_2 \mp Y_- x \tilde{F}_3 - y^2 \tilde{F}_L \right]
\]

\[
\frac{d\sigma_{CC}^\pm}{dx dQ^2} = \frac{G_F^2}{4 \pi x} \left[ \frac{M_w^2}{M_w^2 + Q^2} \right]^2 \left[ Y_+ \tilde{W}_2^\pm \mp Y_- x \tilde{W}_3^\pm - y^2 \tilde{W}_L^\pm \right]
\]

\[Y_\pm = 1 \pm (1 - y)^2\]

**DIS cross sections are expressed in terms of structure functions**

Dominant contribution from $F_2$ structure function

\[\tilde{F}_2 \propto \sum (xq_i + x\bar{q}_i)\]

Relevant at high $Q^2 \sim M_Z^2$

\[x\tilde{F}_3 \propto \sum (xq_i - x\bar{q}_i)\]

Sensitive at low $Q^2$ and high $y$

\[\tilde{F}_L \propto \alpha_s \cdot x g(x, Q^2)\]

**Measured cross sections are reduced cross sections**

Measurement is a direct determination of the structure functions
HERA kinematic plane

**HERA data cover a wide kinematic region of x, Q^2**

**NC measurements**
- F_2 dominates most of Q^2 reach
- xF_3 contributes to EW regime
- F_L contributes only at highest y

**CC measurements**
- W_2 and xW_3 contribute equally
- W_L only at high y

**LHC: largest mass states at large x**
For central production \( x_1 = x_2 \)
- \( M = x^* \sqrt{s} \)
i.e. \( M > 1 \text{ TeV} \) probes \( x > 0.1 \)
High-x predictions rely on
- data (DIS / fixed target)
- sum rules
- behaviour of PDFs as \( x \to 1 \)
HERA operation

**HERA-I operation 1993-2000**
- $E_e = 27.6$ GeV
- $E_p = 820 / 920$ GeV
- $\sqrt{s} = 301 \& 318$ GeV
- int. Lumi. $\sim 110$ pb$^{-1}$ per experiment

**HERA-II operation 2003-2007**
- $E_e = 27.6$ GeV
- $E_p = 920$ GeV
- $\sqrt{s} = 318$ GeV
- int. Lumi. $\sim 330$ pb$^{-1}$ per experiment
- Longitudinally polarised leptons

**Low-Energy Run 2007**
- $E_e = 27.6$ GeV
- $E_p = 575 \& 460$ GeV
- $\sqrt{s} = 225 \& 251$ GeV
- Dedicated $F_L$ measurement
H1 and ZEUS

Two multi-purpose collider experiments: H1 and ZEUS

High statistics
• Luminosity: approx. 0.5 fb\(^{-1}\) per experiment

Excellent control over experimental uncertainties
• Overconstrained system in DIS
• Electron measurement: 0.5 – 1\% scale uncertainty
• Jet energy scale: 1\%
• Trigger and normalization uncertainties: 1-2 \%
• Luminosity: 1.8 – 2.5\%
# HERA structure function data

### Data combination

- **H1 & ZEUS have published all datasets**
  - HERA-I
  - HERA-II at high $Q^2$
  - HERA-II at reduced centre-of-mass energies

### Data points

- 41 datasets are combined
  - NC & CC cross sections
  - $e^+p$ & $e^-p$ scattering
  - 4 values of $\sqrt{s}$
- 2927 input data points
- 1307 combined points
- Data points are swum to common $(x,Q^2)$-grid points:

\[
\sigma(x_{\text{grid}}, Q^2_{\text{grid}}) = \frac{\sigma_{\text{model}}(x_{\text{grid}}, Q^2_{\text{grid}})}{\sigma_{\text{model}}(x_{\text{meas}}, Q^2_{\text{meas}})} \cdot \sigma_{\text{meas}}(x_{\text{meas}}, Q^2_{\text{meas}})
\]

### The usage of different reconstruction techniques and the differences in the strengths of the detector components of the two experiments lead to a substantial reduction of the systematic uncertainties of the combined cross sections.
H1 & ZEUS data combination

Combination of all H1 and ZEUS datasets
- 2927 data points → 1307 combined points
- HERAverager package used
- Correlations of systematic uncertainties fully considered
- Minimisation procedure based on $\chi^2$ definition

$$\chi^2_{\text{exp.,ds}}(m, b) = \sum_i \frac{\left[ m^i - \sum_j \gamma_j^i m^i b_j - \mu^i \right]^2}{\delta_{i,\text{stat}}^2 \mu^i (m^i - \sum_j \gamma_j^i m^i b_j) + (\delta_{i,\text{uncor}} m^i)^2} + \sum_j b_j^2$$

Combination results
- $\chi^2$ of combination: 1687 for 1620 degrees of freedom
- Pull values well distributed around zero with RMS $\sim$ 1
- Great confirmation of consistency of datasets!

Procedural uncertainties
- Multiplicative vs. additive nature
- Correlation in photoproduction background
- Large pulls in corr. syst. uncert.
Combined NC DIS cross sections

**Combined HERA data based on approx. 1fb⁻¹**

- Only 6 and 4 selected x-bins shown here for \( \sqrt{s} = 318 \) GeV
- High precision reached over large kinematic range:
  Better than 1.3% for \( Q^2 < 400 \text{ GeV}^2 \)
CC DIS and low-$Q^2$ cross sections

**Combined charged current DIS cross sections for $\sqrt{s} = 318$ GeV**
- Large improvement in statistical limitations of individual datasets

**Further kinematic regions**
- Great improvements also for $\sqrt{s} = 225, 251$ and $301$ GeV
- Very low-$Q^2$ and low-x data for $\sqrt{s} = 301$ and $318$ GeV
  - $Q^2 > 0.045$ GeV$^2$ and $x_{Bj} > 6 \times 10^{-7}$
  - Interesting for dipole and saturation models
**PDF extraction from data: HERAPDF2.0**

**HERAPDF approach**
- Final combined $e^\pm p$ NC and CC data are very precise, so to allow the extraction of the parton densities
- DGLAP Analysis based only on HERA data
- PDFs parameterised at arbitrary starting scale $Q^2_0 = 1.9 \text{ GeV}^2$

\[
xg(x) = A_g x^B_g (1 - x)^C_g - A'_g x^{B'_g} (1 - x)^{C'_g},
\]
\[
xu_v(x) = A_{u_v} x^{B_{u_v}} (1 - x)^{C_{u_v}} \left(1 + E_{u_v} x^2\right),
\]
\[
xd_v(x) = A_{d_v} x^{B_{d_v}} (1 - x)^{C_{d_v}},
\]
\[
x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}} (1 - x)^{C_{\bar{U}}} (1 + D_{\bar{U}} x),
\]
\[
x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1 - x)^{C_{\bar{D}}}.
\]

**Minimise $\chi^2$ function with respect to PDF parameters**
- Perturbative QCD evolution allows PDFs to be determined at any other scale $Q^2$
- Calculate theory cross section at given $x,Q^2$ of measurement
- Usage of momentum/counting sumrules help to constrain parameter space

**The use of a single consistent data sample allows a more rigorous treatment of the experimental uncertainties**
- No fixed target data, therefore no need for heavy-target/deuterium corrections
HERAPDF2.0 NLO and NNLO

Fits performed in LO, NLO and NNLO

- NLO: $\chi^2/\text{ndf} = 1357 / 1131$
- NNLO: $\chi^2/\text{ndf} = 1363 / 1131$

Differences between NLO and NNLO fit

- **gluon** ceases to raise at low-x
- **sea** at low-x somewhat steeper w.r.t. NLO
HERAPDF2.0 uncertainties

**Experimental Uncertainties**
Hessian method uses 14 eigenvector pairs
Standard definition $\Delta x^2 = 1$ for 68% CL error bands

**Model Assumptions**
Variation of charm and bottom quark masses $M_c, M_b$
Variation of $Q^2$ minimum cut used on input data $Q^2_{\text{min}}$
Variation of strange quark fraction $f_s$

**Parameterisation Uncertainties**
Variation of $Q_0^2$
Variation of fit using additional 15th parameter

<table>
<thead>
<tr>
<th>Variation</th>
<th>Standard Value</th>
<th>Lower Limit</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q^2_{\text{min}}$ [GeV$^2$]</td>
<td>3.5</td>
<td>2.5</td>
<td>5.0</td>
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<td>$Q^2_{\text{min}}$ [GeV$^2$] HiQ2</td>
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<td>$M_c$ (NLO) [GeV]</td>
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<td>$M_b$ [GeV]</td>
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<td>4.25</td>
<td>4.75</td>
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<tr>
<td>$f_s$</td>
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<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>$\alpha_s(M_Z^2)$</td>
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<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\mu_f$ [GeV]</td>
<td>1.9</td>
<td>1.6</td>
<td>2.2</td>
</tr>
</tbody>
</table>

$\alpha_s(M_Z^2)$ fixed but series of PDFs provided for large range: 0.110 to 0.130

Flavor breakdown of sea distribution
NC cross sections & HERAPDF2.0

Neutral Current $e^\pm p$

- HERA NC $e^- p$ 0.4 fb$^{-1}$
- HERA NC $e^+ p$ 0.5 fb$^{-1}$
- $\sqrt{s} = 318$ GeV
- Fixed Target
- HERAPDF2.0 $e^- p$ NLO
- HERAPDF2.0 $e^+ p$ NLO

Precision 1.3% for $Q^2 < 400$ GeV$^2$
→ factor 2 reduction in error wrt HERA-I

Statistics limited at higher $Q^2$ and high $x$

Extended reach at high $x$ compared to H1 preliminary data

This $x$ region is the ‘sweet spot’
High precision with long $Q^2$ lever arm
$x$-range relevant for Higgs production

Combination of high $Q^2$ data
HERA-1 and HERA-II

Larger HERA-II luminosity
→ improved precision at high $x / Q^2$

HERAPDF2.0 provides good description
High $Q^2$ NC & CC Cross Sections

**High $Q^2$, high-x cross sections**
- Difference in NC at high-x for $e^+$ and $e^-$ due to $xF_3$ and Z-boson exchange
- CC $e^+p$ suppressed at high-x due to $(1-y)^2$ helicity suppression of quarks
- CC $e^-p$ unaffected as helicity suppression applies to anti-quarks

**Image Description**
- **Neutral Current $e^+p$**
  - HERA $\sqrt{s} = 318$ GeV
  - HERAPDF2.0 NLO $\sqrt{s} = 318$ GeV
  - H $x_1$ = 0.02 (s575)
  - $x_H = 0.05$ (s270)
  - $x_H = 0.08$ (s170)
  - $x_H = 0.13$ (s6)
  - $x_H = 0.25$ (s2)
  - $x_H = 0.40$

- **Charged Current $e^+p$**
  - HERA $\sqrt{s} = 318$ GeV
  - HERAPDF2.0 NLO $\sqrt{s} = 318$ GeV
  - $x_H = 0.008$ (s15000)
  - $x_H = 0.013$ (s30000)
  - $x_H = 0.032$ (s700)
  - $x_H = 0.08$ (s170)
  - $x_H = 0.13$ (s20)
  - $x_H = 0.25$ (s2)
  - $x_H = 0.40$

**Result**
- HERAPDF2.0 describes high-x data well for both NC and CC channels
$\chi^2$ and $Q^2_{\text{min}}$ study and heavy flavors

**Minimum value of $Q^2$ for data to ensure that pQCD is applicable**
- HERAPDF2.0: $Q^2_{\text{min}} = 3.5$ GeV$^2$
- Consider variation of this cut: $\chi^2$ decreases with increase of $Q^2$

- NLO and NNLO behave similarly
- Low-$Q^2$ cuts also removes low-$x$ region: Region where non-pert. effects, $\ln(1/x)$-resummation, saturation become important

- Fits for $Q^2_{\text{min}} = 10$ GeV$^2$ also released as PDF tables

**Heavy flavor scheme**
- Treating $F_L$ to $O(\alpha_S)$ (the same order as $F_2$) yields better $\chi^2$ than treating $F_L$ to $O(\alpha_S^2)$
- RT-Opt NNLO is marginally worse than NLO
- FONLL NNLO is worse than NLO
Jet production in DIS

Jet production in leading-order pQCD

Jet measurements are performed in Breit-frame
virtual boson collides head on with a parton from the proton

Inclusive jets
Count each jet of an event

Dijet and trijet
Count events with two/three jet event structure
Observable: average transverse momentum of two/three jets

Normalised jets
Normalise all jet data w.r.t. inclusive NC DIS cross section
Jet production in DIS

**Normalised and non-normalised jet data**
- Data well described by NLO theory (nlojet++)
- Data in general with smaller uncertainties than NLO from scale variations
- Differences between different PDF sets typically small

**Data used to extract strong coupling constant**
- $\chi^2$ minimisation of $\alpha_s$ in coefficient function
- Dependencies of the PDF on $\alpha_s$ considered as uncertainties

\[
\alpha_s(M_Z)|_{k_T} = 0.1165 \ (8)_{\text{exp}} \ (5)_{\text{PDF}} \ (7)_{\text{PDF set}} \ (3)_{\text{PDF}(\alpha_s)} \ (8)_{\text{had}} \ (36)_{\mu_r} \ (5)_{\mu_f}
\]
\[
= 0.1165 \ (8)_{\text{exp}} \ (38)_{\text{pdf,tho}}
\]
Charm production in DIS

Charm production at HERA
- Charm is produced in virtual photon-gluon fusion
- Charm production directly sensitive to the gluon density $xg(x)$

Combined charm cross sections
- Wealth of HERA charm data combined into common charm cross sections

$$\frac{d^2\sigma^{c\bar{c}}}{dxdQ^2} = \frac{2\pi \alpha^2(Q^2)}{xQ^4} \left( [1 + (1 - y)^2] F_2^{c\bar{c}}(x, Q^2) - y^2 F_L^{c\bar{c}}(x, Q^2) \right)$$
Extraction of charm mass running

**Extraction of charm mass**
- Simultaneous fit of combined charm data + inclusive HERA-I DIS data
- Different heavy-flavor schemes explored
- FFNS ABM scheme defines charm mass in MSbar scheme

\[ m_c(m_c) = 1.26 \pm 0.05_{\text{exp}} \pm 0.03_{\text{mod}} \pm 0.02_{\text{par}} \pm 0.02_{\text{as}} \text{ GeV} \]

**Charm mass running**
- extract \( m_c(m_c) \) separately for 6 kinematic ranges in \( \mu^2 = Q^2 + 4m_c^2 \)
- use appropriate PDF set for each mass (from inclusive DIS data only)
- fit charm data + HERA-I incl. data
- Translate back to \( m_c(\mu) \) using LO formula consistent with NLO MS QCD fit (OpenQCDrad, Alekhin et al.)
Determination of beauty mass

**Beauty cross sections**
- Measured of HF jets using secondary vertices + lifetime tag
- Good description of data by massive NLO QCD predictions

**Extraction of b-quark mass**
- QCD fit (FFNS) of HERA-I incl. data + ZEUS beauty data
- $m_b$ as free parameter

$$m_b(m_b) = 4.07 \pm 0.14 \text{ (fit)} \pm 0.07 \text{ (mod.)} \pm 0.05 \text{ (param.)} \pm 0.08 \text{ (theo.) GeV}$$
Charm and jet data in HERAPDF2.0

*Charm and bottom data used in HERAPDF2.0 QCD analysis*

- Charm and bottom data used to determine best quark-mass parameters
- Values of charm and bottom masses used DGLAP fits determined as $\chi^2$ scan of NLO and NNLO fits

![Graphs showing $\chi^2$ vs $M_c$ and $M_b$](chart.png)
Charm and jet data in HERAPDF2.0

Additional datasets in QCD analysis
- Combined charm cross sections
- H1 norm. multijets at high $Q^2$ (HERA-II)
- H1 multijets at low $Q^2$ (HERA-I)
- H1 incl. jets at high $Q^2$ (HERA-I)
- ZEUS dijets at high $Q^2$ (HERA-II)
- ZEUS incl. jets at high $Q^2$ (HERA-I)
- Jet predictions available in NLO (nlojet++)
- Jet data significantly helps to disentangle gluon-$\alpha_s$ correlation

Determination of strong coupling
- $\alpha_s$ is additional free parameter in PDF fit
- Jet data constrain $\alpha_s$

$$\alpha_s (m_Z) = 0.1183 \pm 0.0009^{\text{exp}}_{\text{mod}} \pm 0.0012^{\text{had}}_{\text{scale}}$$

- Value mostly constrained by H1 norm. multijet data
Usage of HERA 'combined' data

**HERA combined data**
- Limit on effective quark radius
- Consider finite radius through form-factor

\[
\frac{d\sigma}{dQ^2} = \frac{d\sigma_{\text{SM}}}{dQ^2} \left(1 - \frac{R_e^2}{6Q^2}\right)^2 \left(1 - \frac{R_q^2}{6Q^2}\right)^2
\]
- Fit PDFs and 'quark radius'
- 95% C.L. on quark radius

\[-(0.47 \cdot 10^{-16} \text{ cm})^2 < R^2_q < (0.43 \cdot 10^{-16} \text{ cm})^2\]

**H1 and ZEUS data + ZEUS polarized HERA-II data**
- Study Couplings of u- and d-quarks to Z-boson
- Use in additional ZEUS polarized data

\[
\tilde{F}_{2}^{\pm} = F_{2}^{\gamma} - (v_e \pm P_{e}a_e)\chi Z F_{2}^{\gamma Z} + (v_e^2 + a_e^2 \pm 2P_{e}v_ea_e)\chi Z F_{2}^{Z},
\]
- Simultaneous fit of PDFs and axial and vector-'quark couplings'
- Values consistent with SM expectations
- Sensitivity on u-quark higher than d-quark

![Graph showing data improvements](image)
Dijets in diffractive DIS (LRG)

(Inclusive) dijets in diffractive DIS

- Diffractive events identified by 'large rapidity gap' (LRG)
- $4 < Q^2 < 100 \text{ GeV}^2$, $p_{T,\text{jet}}^{1,2} > 5.5 \ (4.0) \text{ GeV}$
- Theory: nlojet++ & H1DPDF2006 FitB
- Data used to extract strong coupling constant
  -> Fit supports concept of pQCD calculations for diff. dijets
  -> Exp. precision overshoots theoretical one

\[
\alpha_s(M_Z) = 0.119 \pm 0.004 \ (\text{exp}) \pm 0.002 \ (\text{had}) \pm 0.005 \ (\text{DPDF}) \pm 0.010 \ (\mu_r) \pm 0.004 \ (\mu_f)
\]
\[
= 0.119 \pm 0.004 \ (\text{exp}) \pm 0.012 \ (\text{DPDF, theo})
\]
**Diffr. Dijets in Photoprod. and DIS (VFPS)**

**History**
- 'Factorisation breaking' observed in diffractive events at Tevatron
- Photoproduction provides similar testing ground
- Fact. breaking observed by H1 but not by ZEUS

**Here: Simultaneous measurement of dijets in diffr. DIS and PHP**
- Use VFPS 220m from interaction point
- Calculate double-ratios: PHP/DIS

**Single differential cross sections**
- DIS data well described by NLO
- PHP NLO overshoots data
- New data with complementary method consistent with previous H1 results
- 'Suppression' shows no dependence as function of $x_\gamma$ or $E_T^{jet}$

![Graph showing ratio to NLO and $Q^2$ vs $E_T^{jet}$ comparison](image)
Study diffractive models

**Exclusive dijets in diffractive DIS**

- Study (normalised) angle between jet-plane and lepton-plane
  \[
  \frac{1}{\sigma} \frac{d\sigma}{d\phi} \propto 1 + A \cos 2\phi
  \]
- Sensitive to nature of diff. exchange: Resolved pomeron vs. two-gluon exchange model
  \[
  \beta = \frac{x_{Bj}}{x_{IP}}
  \]
- Two-gluon model is more successful in describing data than resolved Pomeron model

**Diffractive prompt isolated photons**

- Analysis extends prompt photon analysis in non-diff. PHP
  - Reminder: NLO and k_T-factorization predictions give good descriptions
  - Prompt photon variables well described by Rapgap & H1PDF2006-FitB
  - Problems at \( z_{ip} \rightarrow 1 \), where H1PDF2006-FitB was not fitted

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**ZEUS**

![Graph showing dijet production](image)
Exclusive vector-meson production

**Exclusive electroproduction of vector meson**

- Measure ratio of $\psi(2s)$ over $J/\psi(1s)$ as function of $Q^2$

  ![Diagram](image)

  - Identify VM in $\mu^+\mu^-$ decay channels
    - $30 < W < 210$ GeV, $|t| < 1$ GeV$^2$
  - Compare against various models for
    - Generating cc-dipole in photon
    - cc-dipole scattering amplitude
    - Probability to form vector charmonium

**All models perform reasonably well**

- Ratio tend to be constant vs. $W$ and $|t|$
- Spread indicate large theory uncertainty
Wealth of more QCD related measurements

New measurements, old measurements, and maybe forgotten ones...

- Search for QCD instantons to be published by H1
- Isolated photons in photoproduction
  PLB 730 C (2014) 293 & JHEP 08 (2014) 03
- Exclusive $\rho^0$ Meson Photoproduction with a Leading Neutron at HERA
- Elastic and Proton-Dissociative Photoproduction of J/psi Mesons at HERA
- Event shapes
- Numberous D* measurements
  JHEP 1509 (2015) 149, ...
- Carged particle production spectra
- ….
Conclusion

HERA inclusive DIS cross sections finalized
- One consistent dataset of all HERA structure function data
- HERAPDF2.0 as HERA-only PDF
- Baseline data for future PDF fits

Wealth of precision QCD measurements
Many topics not covered in this short talk
- Jet and photon cross sections
- Various searches and limits
- Strong diffractive DIS programme
- Many exclusive final states measured with full HERA-II statistics
- low-x and soft physics

HERA experiments still active
- Improved/new measurements can still be expected this year
Electroweak symmetry breaking

- H1 / ZEUS completed their final SF measurements
- New HERA-II data provide tighter constraints at high $x / Q^2$
- These data provide some of the most stringent constraints on PDFs
- Stress-test of QCD over 4 orders of mag. in $Q^2$
- DGLAP evolution works very well
- HERA data provide a self-consistent data set for complete flavour decomposition of the proton
- Final combination of HERA data completed
- HERAPDF2.0 QCD fit at NLO & NNLO
Valence quarks and $xF_3$

At high $Q^2$, $xF_3$ arises due to $Z^0$ effects enhanced $e^-$ cross section wrt $e^+$
Difference is $xF_3$
Sensitive to valence PDFs

$$\tilde{xF}_3 = \frac{Y^+}{2Y^-} (\tilde{\sigma}_{NC}^- - \tilde{\sigma}_{NC}^+) \approx a_e \chi_Z xF_3^{\gamma Z}$$

$$xF_3 \propto \sum (xq_i - x\bar{q}_i)$$

Measure integral of $xF_3^{\gamma Z}$ - validate sumrule:

$$\int_{0.016}^{0.725} dx \ F_3^{\gamma Z}(x, Q^2 = 1500 \text{ GeV}^2) = 1.314 \pm 0.057(\text{stat}) \pm 0.057(\text{syst})$$

LO integral predicted to be $5/3 + \mathcal{O}(\alpha_s/\pi)$
'Swimming' of data points

Data are combined onto a common $x,Q^2$ grid
Two grids used:
  - inclusive measurements $\sqrt{s}=318$ GeV
  - fine $x$ grid for $\sqrt{s}=251$ & 225 GeV

2927 data points $\rightarrow$ 1307 combined measurements

Data are translated to nearest $x,Q^2$ grid point
Iterative process using NLO QCD fit to data
Use uncombined data in first iteration
Then combined data in later iterations
No changes after 3 iterations

$$\sigma(x_{grid}, Q^2_{grid}) = \frac{\sigma_{model}(x_{grid}, Q^2_{grid})}{\sigma_{model}(x_{meas}, Q^2_{meas})} \cdot \sigma_{meas}(x_{meas}, Q^2_{meas})$$

Data are also translated outside of region of DGLAP fit validity $Q^2 < 3.0$ GeV$^2$
Use phenomenological “fractal” model and interpolate to DGLAP region
Other phenomenological fits tested $\rightarrow$ negligible differences
H1 & ZEUS data combination II

Overall $\chi^2/\text{ndf} = 1685 / 1620 = 1.04$

Pulls defined for each measurement difference between measured & average values after applying sys shifts $b_j$ in units of uncorrelated uncertainty

Pulls of the data points should be distributed as a unit Gaussian

Each measurement channel shows pull centred on zero & unit width

Pulls of the systematic sources $b_j$

$$p^{i,k} = \frac{\mu^{i,k} - \mu^{i,\text{ave}}(1 - \sum_j \gamma^{i,k}_j b_{j,\text{ave}})}{\sqrt{\Delta^2_{i,k} - \Delta^2_{i,\text{ave}}}}$$
PDF extraction from data: HERAPDF2.0

**HERAPDF1.0 & 1.5**
Combine NC and CC HERA-I data from H1 & ZEUS
Complete MSbar NLO fit
NLO: standard parameterisation with 10 parameters
NNLO HERAPDF 1.5 with 14p

**HERAPDF2.0**
Include additional NC and CC HERA-II combined data
Complete MSbar NLO and NNLO fit
NLO & NNLO fits require 15 parameters

\[ x_f(x, Q_0^2) = A \cdot x^B \cdot (1 - x)^C \cdot (1 + Dx + Ex^2) \]

\[ xg(x) = A_g x^{B_g} (1 - x)^{C_g} \]
\[ xu_v(x) = A_{u_v} x^{B_{u_v}} (1 - x)^{C_{u_v}} (1 + E_{u_v} x^2) \]
\[ xd_v(x) = A_{d_v} x^{B_{d_v}} (1 - x)^{C_{d_v}} \]
\[ xU(x) = A_U x^{B_U} (1 - x)^{C_U} \]
\[ xD(x) = A_D x^{B_D} (1 - x)^{C_D} \]

\[ x\bar{S} = f_s x\bar{D} \text{ strange sea is a fixed fraction } f_s \text{ of } D \text{ at } Q_0^2 \]

Apply momentum/counting sum rules:
\[ \int_0^1 dx \cdot (xu_v + xd_v + xU + xD + xg) = 1 \]
\[ \int_0^1 dx \cdot u_v = 2 \quad \int_0^1 dx \cdot d_v = 1 \]
\[ B_U = B_D \]
\[ Sea = 2(U + \bar{D}) \]
\[ A_U = A_D (1 - f_s) \]
\[ \text{ensures } xU \rightarrow x\bar{D} \text{ as } x \rightarrow 0 \]
\[ Q_0^2 = 1.9 \]
\[ Q_{min}^2 = 3.5 \text{ or } 10 \text{ GeV}^2 \]
\[ \alpha_s(M_Z^2) = 0.118 \]
\[ 2 \cdot 10^{-4} \leq x \leq 0.65 \]
High $Q^2$ charged current cross sections

Electron scattering

$$\frac{d^2\sigma_{CC}^-}{dx dQ^2} = \frac{G_F^2}{2\pi} \left( \frac{M_W^2}{M_W^2 + Q^2} \right)^2 \left[ (u + c) + (1 - y)^2 (\bar{d} + \bar{s}) \right]$$

Positron scattering

$$\frac{d^2\sigma_{CC}^+}{dx dQ^2} = \frac{G_F^2}{2\pi} \left( \frac{M_W^2}{M_W^2 + Q^2} \right)^2 \left[ (\bar{u} + \bar{c}) + (1 - y)^2 (d + s) \right]$$

Combination of high $Q^2$ CC data (HERA-I+II)

Improvement of total uncertainty

Dominated by statistical errors

Provide important flavour decomposition information

CC $e^+$ data provide strong $d_\nu$ constraint at high $x$

Precision limited by statistics: typically 3-7%

HERA-I precision of 10-15% for $e^+p$
Comparison of HERAPDF2.0 vs MMHT14, NNPDF3.0, CT10 (others use only HERA-1 combined data)

Differences at high x
- New HERA combined data improve precision at high x, $Q^2$
- HERAPDF uses proton target data only → no nucleon / deuterium data
- Softer gluon at high x
NC and CC measurements

**Neutral current event selection:**

- High $P_T$ isolated scattered lepton
- Suppress huge photo-production background by imposing longitudinal energy-momentum conservation

Kinematics may be reconstructed in many ways:
- energy/angle of hadrons & scattered lepton provides excellent tools for sys cross checks

Removal of scattered lepton provides a
- high stats “pseudo-charged current sample”
- Excellent tool to cross check CC analysis

Final selection: $\sim 10^5$ events per sample at high $Q^2$
- $\sim 10^7$ events for $10 < Q^2 < 100 \text{ GeV}^2$

**Charged current event selection:**

- Large missing transverse momentum (neutrino)
- Suppress huge photo-production background
- Topological finders to remove cosmic muons
- Kinematics reconstructed from hadrons
- Final selection: $\sim 10^3$ events per sample

slide by E. Rizvi
Figure 20: The dependence of $\chi^2$/d.o.f. on $Q_{\text{min}}^2$ for HERAPDF2.0 fits using a) the RTOPT [83], FONNL-B [90], ACOT [109] and fixed-flavour (FF) schemes at NLO and b) the RTOPT and FONNL-B/C [91] schemes at NLO and NNLO. The $F_L$ contributions are calculated using matrix elements of the order of $\alpha_s$ indicated in the legend. The number of degrees of freedom drops from 1148 for $Q_{\text{min}}^2 = 2.7\,\text{GeV}^2$ to 1131 for the nominal $Q_{\text{min}}^2 = 3.5\,\text{GeV}^2$ and to 868 for $Q_{\text{min}}^2 = 25\,\text{GeV}^2$. 
HERAPDF2.0 NLO vs. NNLO

$\mu_r^2 = 10$ GeV$^2$

- HERAPDF2.0 NNLO
- uncertainties:
  - experimental
  - model
  - parameterisation
- HERAPDF2.0 AG NNLO

$xg \times 0.05$

$xS \times 0.05$
HERAPDF2.0 variants

The following variants of the HERAPDF2.0 PDFs have been released and will soon be available on LHAPDF (https://lhapdf.hepforge.org)

HERAPDF2.0 (NLO,NNLO, $Q^2_{\text{min}}=3.5 \text{ GeV}^2$)  “Default PDF set”
- Data: combined HERA NC and CC inclusive cross sections
- HF Scheme: ROPT
- $\alpha_s(M_Z^2)=0.118$
- Grid with different $\alpha_s(M_Z^2)$ values (in the range [0.110-0.130] in steps of 0.01) are also released

HERAPDF2.0HiQ2 (NLO,NNLO)  “High-$Q^2$ version”
- as HERAPDF2.0 but with $Q^2_{\text{min}}=10 \text{ GeV}^2$

HERAPDF2.0AG (LO,NLO,NNLO, $Q^2_{\text{min}}=3.5 \text{ GeV}^2$)  “Alternative Gluon”
- Data: combined HERA NC and CC inclusive cross sections
- Use an alternative gluon parameterisation
- HF Scheme: ROPT
- $\alpha_s(M_Z^2)=0.130$ (LO) and $\alpha_s(M_Z^2)=0.118$ (NLO,NNLO)

HERAPDF2.0FF (NLO, $Q^2_{\text{min}}=3.5 \text{ GeV}^2$)  “FF Schemes”
- Data: combined HERA NC and CC inclusive cross sections
- HF Schemes: Use two alternative (FF3A and FF3B) Fixed-Flavour schemes
- $\alpha_s(M_Z^2)^{N_f=3}=0.106573$ equivalent to $\alpha_s(M_Z^2)^{N_f=5}=0.118$ (FF3A) and $\alpha_s(M_Z^2)=0.118$ (FF3B)

HERAPDF2.0Jets (NLO, $Q^2_{\text{min}}=3.5 \text{ GeV}^2$)  “Charm and Jets”
- Data: combined HERA NC and CC inclusive cross sections and selected HERA charm and jet production measurements
- HF Schemes: ROPT
- free $\alpha_s(M_Z^2)$ or $\alpha_s(M_Z^2)=0.118$