Precision Tests of the Standard Model in Rare B-Meson Decays

Ahmed Ali

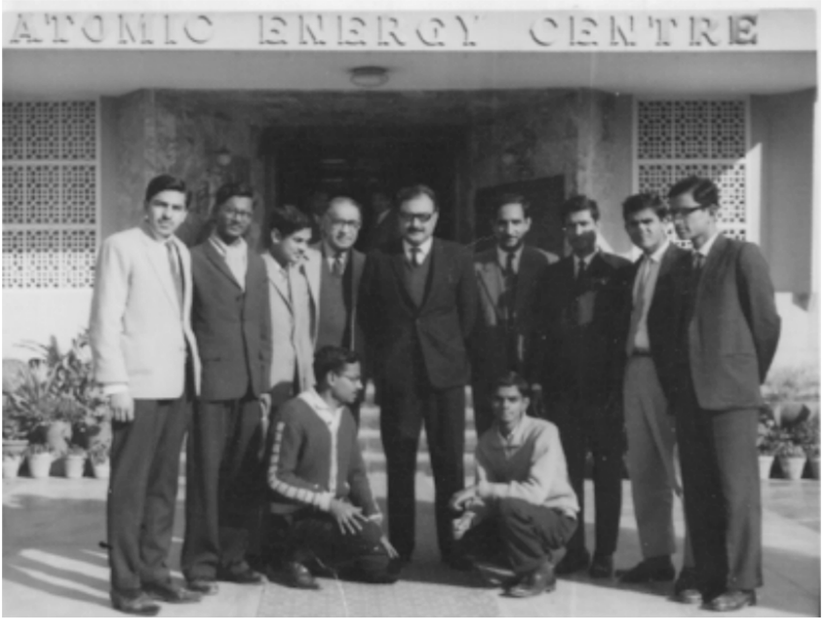
DESY, Hamburg

Jan. 25-28, 2016

Memorial Meeting for Prof. Abdus Salam's 90th. Birthday
NTU, Singapore

Professor Abdus Salam (circa 1965)

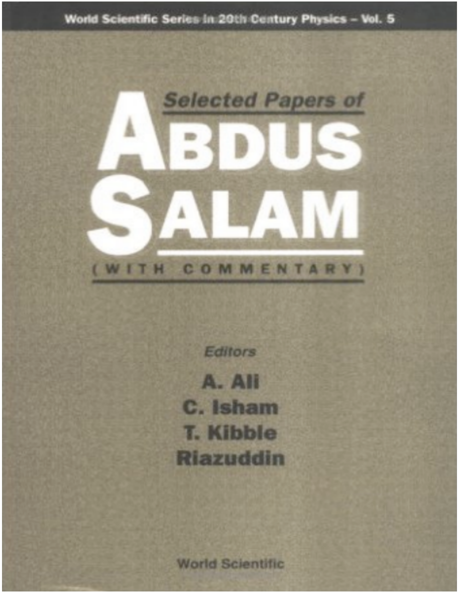
First meeting with Professor Abdus Salam



Pakistan Institute of Nuclear Science and Technology, Nilore

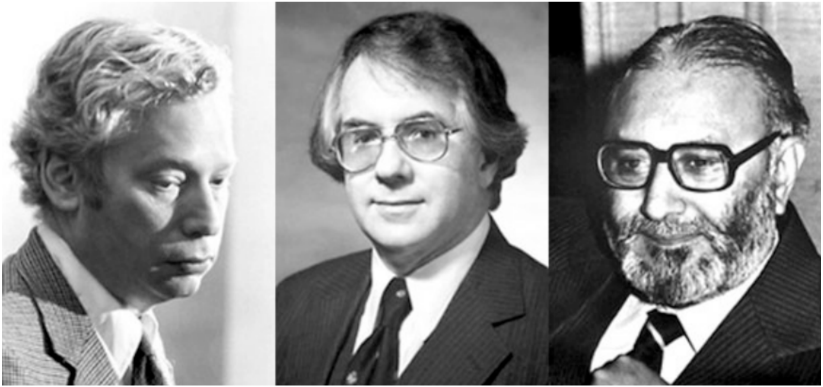
A Milestone in Pakistan's Nuclear Ambitions

Professors Abdus Salam and Riazuddin


World Scientific Series in 20th Century Physics – Vol. 5
Selected Papers of
ABDUS SALAM
(WITH COMMENTARY)
Editors
A. Ali
C. Isham
T. Kibble
Riazuddin
World Scientific

Founding Fathers of the Standard Model

S. Weinberg *S. Glashow* *A. Salam*



Rare B-decays in the Standard Model

- The Standard Candle in Rare B-Decays: $\mathbf{B} \to X_s \gamma$
- Electroweak Penguins: $\mathbf{B} \to X_s \ell^+ \ell^-$
- Exclusive Decays $\mathbf{B} \to (K, K^*, \pi) \ell^+ \ell^-$
- Current Frontier of Rare B Decays: $\mathbf{B}_s \to \mu^+ \mu^-$ & $\mathbf{B}_d \to \mu^+ \mu^-$
- Summary and Outlook

The Standard Candle: $B \to X_s\gamma$

• Interest in the rare B-decay $B \to X_s\gamma$ transcends B Physics!

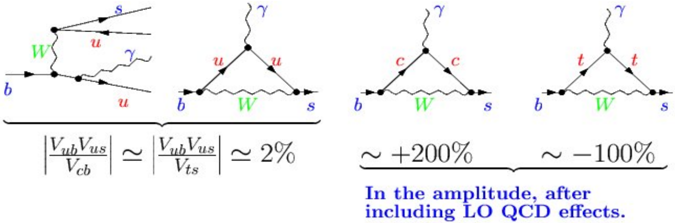
- First measurements by CLEO (1995);
  well measured at the B-factories by Belle and BaBar;
  more precise measurements anticipated at KEK (Belle-II)

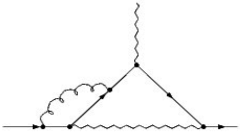
- A monumental theoretical effort has gone in improving the perturbative precision $\Longrightarrow B \to X_s\gamma$ in next-to-next-to leading order in α_s
  - First estimate of $\mathcal{B}(B \to X_s\gamma)$: M. Misiak et al., Phys. Rev. Lett. 98:022002 (2007); T. Becher and M. Neubert, Phys. Rev. Lett. 98:022003 (2007)
  - Updated in 2015: M. Misiak et al., Phys. Rev. Lett. 114 (2015) 22, 221801

- Non-perturbative effects calculated using Heavy Quark Effective Theory

- Sensitivite to virtual new physics effects; hence constrains parameters of the BSM models such as the 2HDMs and Supersymmetry

Examples of leading electroweak diagrams for $B \to X_s \gamma$





- QCD logarithms $\alpha_s \ln \frac{M_W^2}{m_b^2}$ enhance BR($B \to X_s\gamma$) more than twice
- Effective field theory (obtained by integrating out heavy fields) used for resummation of such large logarithms

The effective Lagrangian for $B \to X_s \gamma$ and $B \to X_s \ell^+ \ell^-$

$$\mathcal{L} = \mathcal{L}_{QCD \times QED}(q, l) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i$$
$$(q = u, d, s, c, b,\ l = e, \mu, \tau)$$

$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_i b), & i = 1,2, \quad |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma'_i q), & i = 3,4,5,6, \quad |C_i(m_b)| < 0.07 \\ \frac{e m_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, \quad C_7(m_b) \sim -0.3 \\ \frac{g m_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}, & i = 8, \quad C_8(m_b) \sim -0.15 \\ \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{l}\gamma^\mu(\gamma_5) l), & i = 9, (10) \quad |C_i(m_b)| \sim 4 \end{cases}$$

Three steps of the calculation:

Matching: Evaluating $C_i(\mu_0)$ at $\mu_0 \sim M_W$ by requiring equality of the SM and the effective theory Green functions

Mixing: Deriving the effective theory RGE and evolving $C_i(\mu)$ from μ_0 to $\mu_b \sim m_b$

Matrix elements: Evaluating the on-shell amplitudes at $\mu_b \sim m_b$

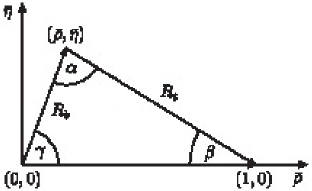
The Cabibbo-Kobayashi-Maskawa Matrix

$$V_{\mathbf{CKM}} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Customary to use the handy Wolfenstein parametrization

$$V_{\mathbf{CKM}} \simeq \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda(1+iA^2\lambda^4\eta) & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2\left(1+i\lambda^2\eta\right) & 1 \end{pmatrix}$$

- Four parameters: $A,\ \lambda,\ \rho,\ \eta;\ \bar{\rho} = \rho(1-\lambda^2/2),\ \bar{\eta} = \eta(1-\lambda^2/2)$
- The CKM-Unitarity triangle $[\phi_1 = \beta;\ \phi_2 = \alpha;\ \phi_3 = \gamma]$



Wilson Coefficients in the SM

Wilson Coefficients of Four-Quark Operators

	$C_1(\mu_b)$	$C_2(\mu_b)$	$C_3(\mu_b)$	$C_4(\mu_b)$	$C_5(\mu_b)$	$C_6(\mu_b)$
LL	-0.257	1.112	0.012	-0.026	0.008	-0.033
NLL	-0.151	1.059	0.012	-0.034	0.010	-0.040

Wilson Coefficients of other Operators

	$C_7^{\text{eff}}(\mu_b)$	$C_8^{\text{eff}}(\mu_b)$	$C_9(\mu_b)$	$C_{10}(\mu_b)$
LL	-0.314	-0.149	2.007	0
NLL	-0.308	-0.169	4.154	-4.261
NNLL	-0.290		4.214	-4.312

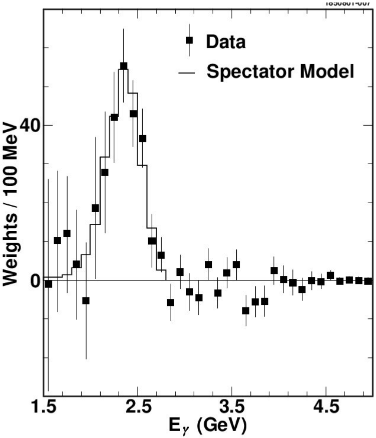
- Obtained for the following input:

$$\mu_b = 4.6 \text{ GeV} \qquad \bar{m}_t(\bar{m}_t) = 167 \text{ GeV}$$

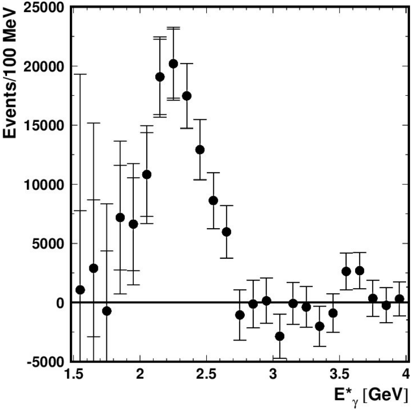
$$M_W = 80.4 \text{ GeV} \qquad \sin^2\theta_W = 0.23$$

Photon Energy Spectrum in $B \to X_s \gamma$

Spectator Model: Greub, AA; PLB 259, 182 (1991)



CLEO
PRL 87 (2001) 251807



BELLE
PRL 93 (2004) 061803

$\mathcal{B}(B \to X_s \gamma)$: Experiment vs. SM & BSM Effects

[Misiak et al., PRL 114 (2015) 22, 221801

- Expt.: CLEO, Belle, BaBar [HFAG 2014]: ($E_\gamma > 1.6$ GeV):

$$\mathcal{B}(B \to X_s \gamma) = (3.43 \pm 0.21 \pm 0.07) \times 10^{-4}$$

- SM [NNLO]: $\mathcal{B}(B \to X_s \gamma) = (3.36 \pm 0.23) \times 10^{-4}$

- Expt./SM $= 1.02 \pm 0.08$
- Excellent agreement; restricts most NP models

- BSM effects can be parametrized as additive contributions to the Wilson Coeffs. of the dipole operators C_7 and C_8

$$\mathcal{B}(B \to X_s \gamma) \times 10^4 = (3.36 \pm 0.23) - 8.22\Delta C_7 - 1.99\Delta C_8$$

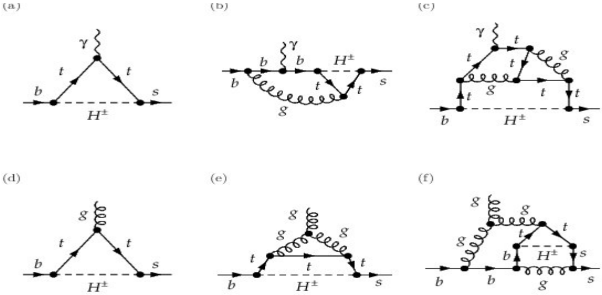
- In 2HDM, $\mathcal{B}(B \to X_s\gamma)$ puts strict bounds on M_{H^+}

$B \to X_s \gamma$ in 2HDM

- NNLO in 2HDM [Hermann, Misiak, Steinhauser; JHEP 1211 (2012) 036] ; Updated [Misiak et al., Phys. Rev. Lett. 114 (2015) 22, 221801]

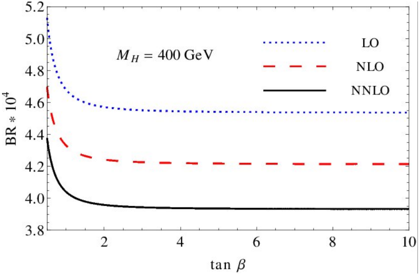
$$\mathcal{L}_{H^+} = (2\sqrt{2}G_F)^{1/2}\Sigma_{i,j=1}^{3}\bar{u}_i(A_u m_{u_i} V_{ij} P_L - A_d m_{d_j} V_{ij} P_R) d_j H^* + h.c.$$

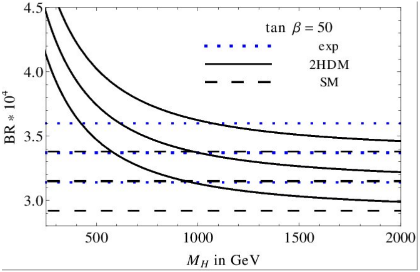
- 2HDM contributions to the Wilson coefficients are proportional to $A_i A_j^*$
  - 2HDM of type-I: $A_u = A_d = \frac{1}{\tan\beta}$
  - 2HDM of type-II: $A_u = -1/A_d = \frac{1}{\tan\beta}$



$B \to X_s \gamma$ in Type-II 2HDM

[Hermann, Misiak, Steinhauser JHEP 1211 (2012) 036]



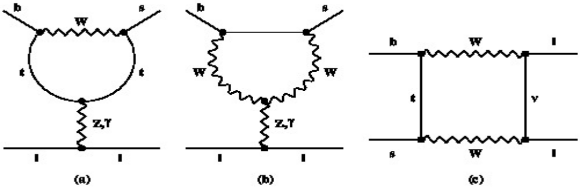


- Updated NNLO [Misiak et al., Phys. Rev. Lett. 114 (2015) 22, 221801]

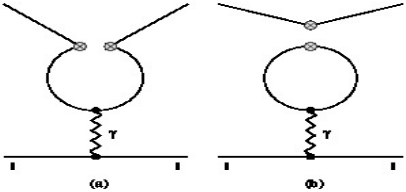
- $M_{H^+} >$ **480** GeV (at 95% C.L.)
- $M_{H^+} >$ **358** GeV (at 99% C.L.)

- Limits on 2HDM competitive to direct $H^\pm$ searches at the LHC

The decay $b \to s\ell^+\ell^-$: Leading Feynman diagram



Diagrams in the full theory



Diagrams in the effective theory

$B \to X_s l^+ l^-$

- There are two $b \to s$ semileptonic operators in SM:

$$O_i = \frac{e^2}{16\pi^2}(\bar{s}_L \gamma_\mu b_L)(\bar{l}\gamma^\mu(\gamma_5)l), \qquad i = 9, (10)$$

- Their Wilson Coefficients have the following perturbative expansion:

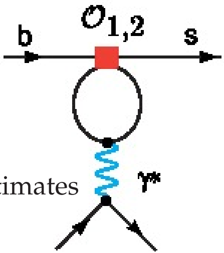
$$C_9(\mu) = \frac{4\pi}{\alpha_s(\mu)} C_9^{(-1)}(\mu) + C_9^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_9^{(1)}(\mu) + \ldots$$

$$C_{10} = C_{10}^{(0)} + \frac{\alpha_s(M_W)}{4\pi} C_{10}^{(1)} + \ldots$$

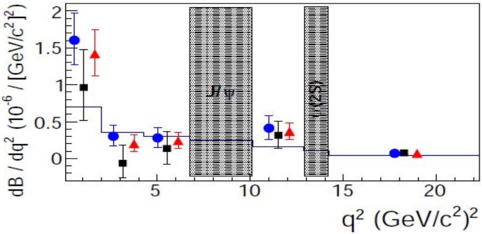
- The term $C_9^{(-1)}(\mu)$ reproduces the electroweak logarithm that originates from the photonic penguins with charm

$$\frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) = \frac{4}{9} \ln \frac{M_W^2}{m_b^2} + \mathcal{O}(\alpha_s) \simeq 2$$

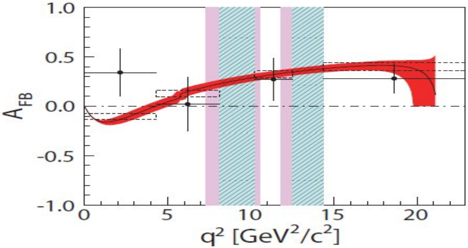
- $C_9^{(0)}(m_b) \simeq 2.2$; need to calculate NNLO for reliable estimates



Dilepton invariant mass spectrum in $B \to X_s \ell^+ \ell^-$ [BaBar 2013]



Forward-Backward Asymmetry in $B \to X_s \ell^+ \ell^-$ [Belle 2014]



Exclusive Decays $B \to (K, K^*)\ell^+\ell^-$

- $B \to K$ & $B \to K^*$ transitions involve the currents:

$$\Gamma^1_\mu = \bar{s}\gamma_\mu(1-\gamma_5)b, \;\; \Gamma^2_\mu = \bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b$$

- $\Longrightarrow$ 10 non-perturbative q^2-dependent functions (Form factors)

$$\langle K|\Gamma^1_\mu|B\rangle \supset f_+(q^2), f_-(q^2)$$

$$\langle K|\Gamma^2_\mu|B\rangle \supset f_T(q^2)$$

$$\langle K^*|\Gamma^1_\mu|B\rangle \supset V(q^2), A_1(q^2), A_2(q^2), A_3(q^2)$$

$$\langle K^*|\Gamma^2_\mu|B\rangle \supset T_1(q^2), T_2(q^2), T_3(q^2)$$

- Data on $B \to K^*\gamma$ provides normalization of $T_1(0) = T_2(0) \simeq 0.28$
- HQET/SCET-approach allows to reduce the number of independent form factors from 10 to 3 in low-q^2 domain $(q^2/m_b^2 \ll 1)$

Experimental data vs. SM in $B \to (X_s, K, K^*)\ell^+\ell^-$ Decays

Branching ratios (in units of $\mathbf{10^{-6}}$) [HFAG: 2012]

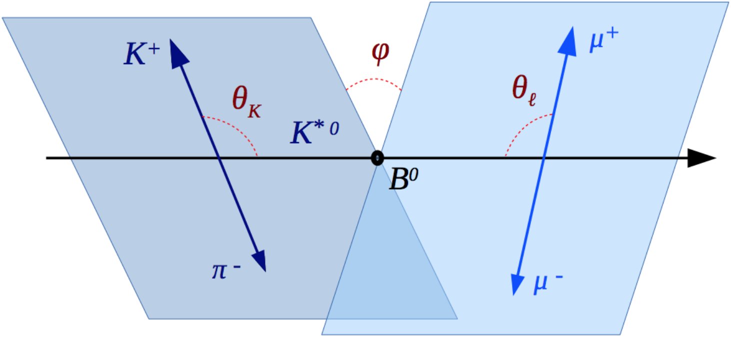
SM: [A.A., Greub, Hiller, Lunghi PR D66 (2002) 034002]

Decay Mode	Expt. (BELLE & BABAR)	Theory (SM)
$B \to K\ell^+\ell^-$	0.45 ± 0.04	0.35 ± 0.12
$B \to K^* e^+e^-$	$1.19^{+0.17}_{-0.16}$	1.58 ± 0.49
$B \to K^* \mu^+\mu^-$	$1.15^{+0.16}_{-0.15}$	1.19 ± 0.39
$B \to X_s \mu^+\mu^-$	$2.23^{+0.97}_{-0.98}$	4.2 ± 0.7
$B \to X_s e^+e^-$	$4.91^{+1.04}_{-1.06}$	4.2 ± 0.7
$B \to X_s \ell^+\ell^-$	$3.66^{+0.76}_{-0.77}$	4.2 ± 0.7

Angular analysis in $B \to K^* \mu^+ \mu^-$

$$\boldsymbol{B^0 \to K^{*0} (\to K^+ \pi^-) \mu^+ \mu^-}$$

- Decay is $P \to VV'$ (since $K^*(892)^0$ is $J^P = 1^-$).
- System fully described by q^2 and three angles $\vec{\Omega} = (\cos\theta_l, \cos\theta_K, \phi)$



Observables in $B \to K^* \mu^+ \mu^-$

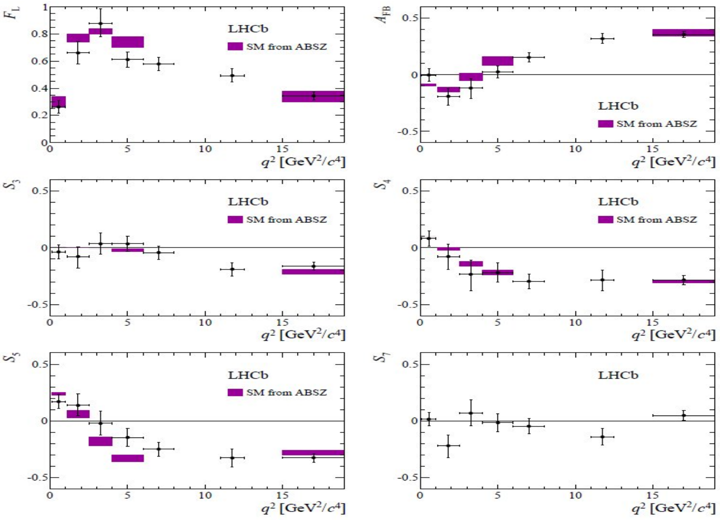
$$\frac{1}{\mathrm{d}(\Gamma+\bar{\Gamma})/\mathrm{d}q^2}\frac{\mathrm{d}^4(\Gamma+\bar{\Gamma})}{\mathrm{d}q^2\,\mathrm{d}\vec{\Omega}} = \frac{9}{32\pi}\Big[\tfrac{3}{4}(1-F_{\rm L})\sin^2\theta_K + F_{\rm L}\cos^2\theta_K$$
$$+\tfrac{1}{4}(1-F_{\rm L})\sin^2\theta_K\cos 2\theta_l$$
$$-F_{\rm L}\cos^2\theta_K\cos 2\theta_l + S_3\sin^2\theta_K\sin^2\theta_l\cos 2\phi$$
$$+S_4\sin 2\theta_K\sin 2\theta_l\cos\phi + S_5\sin 2\theta_K\sin\theta_l\cos\phi$$
$$+\tfrac{4}{3}A_{\rm FB}\sin^2\theta_K\cos\theta_l + S_7\sin 2\theta_K\sin\theta_l\sin\phi$$
$$+S_8\sin 2\theta_K\sin 2\theta_l\sin\phi + S_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi\Big]\,.$$

Optimized variables with reduced FF uncertainties

$$P_1 = 2S_3/(1-F_L);\;\; P_2 = 2A_{FB}/3(1-F_L);\;\; P_3 = -S_9/(1-F_L)$$
$$P_{4,5,6,8} = S_{4,5,6,8}/\sqrt{F_L(1-F_L)}$$

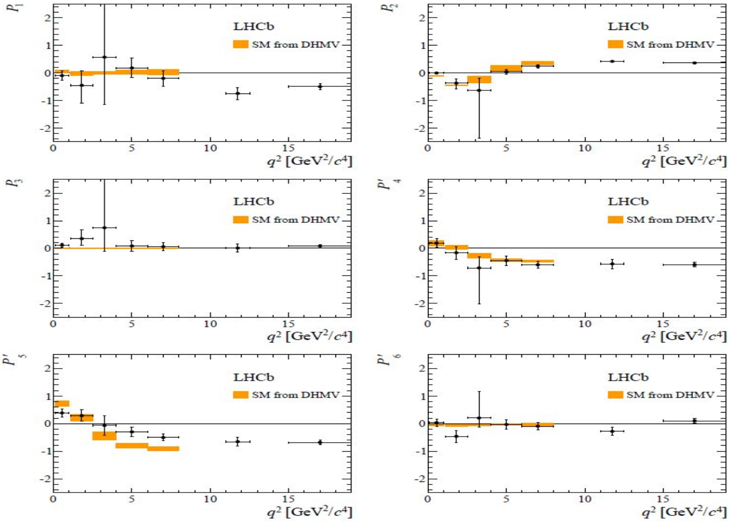
Latest Update from the LHCb: LHCb-Paper-2015-051

SM Estimates: Altmannshofer & Straub, EPJC 75 (2015) 382



Analysis of the optimised angular variables: LHCb-Paper-2015-051

SM Estimates: Descotes-Genon, Hofer, Matias, Virto; JHEP 12 (2014) 125



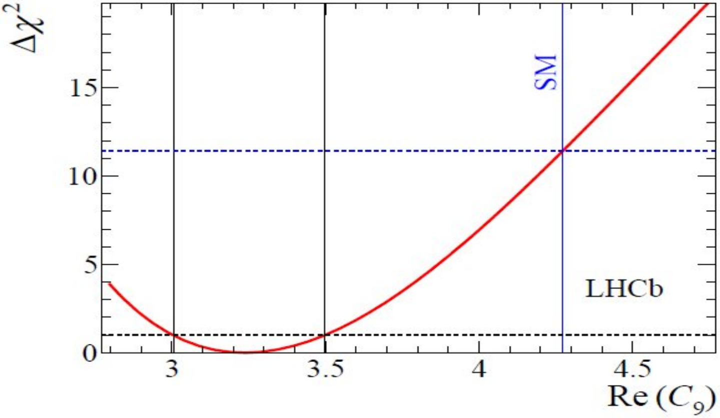
Recent Updates: Pull on the SM [Altmannshofer, Straub (2015)]

W. Altmannshofer & D.M. Straub, EPJ C75 (2015) 8, 382

Decay	obs.	q^2 bin	SM pred.	measurement		pull
$\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-$	$10^7 \frac{dBR}{dq^2}$	$[2, 4.3]$	0.44 ± 0.07	0.29 ± 0.05	LHCb	$+1.8$
$\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-$	$10^7 \frac{dBR}{dq^2}$	$[16, 19.25]$	0.47 ± 0.06	0.31 ± 0.07	CDF	$+1.8$
$\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-$	F_L	$[2, 4.3]$	0.81 ± 0.02	0.26 ± 0.19	ATLAS	$+2.9$
$\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-$	F_L	$[4, 6]$	0.74 ± 0.04	0.61 ± 0.06	LHCb	$+1.9$
$\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-$	S_5	$[4, 6]$	-0.33 ± 0.03	-0.15 ± 0.08	LHCb	-2.2
$B^- \to K^{*-}\mu^+\mu^-$	$10^7 \frac{dBR}{dq^2}$	$[4, 6]$	0.54 ± 0.08	0.26 ± 0.10	LHCb	$+2.1$
$\bar{B}^0 \to \bar{K}^0\mu^+\mu^-$	$10^8 \frac{dBR}{dq^2}$	$[0.1, 2]$	2.71 ± 0.50	1.26 ± 0.56	LHCb	$+1.9$
$\bar{B}^0 \to \bar{K}^0\mu^+\mu^-$	$10^8 \frac{dBR}{dq^2}$	$[16, 23]$	0.93 ± 0.12	0.37 ± 0.22	CDF	$+2.2$
$B_s \to \phi\mu^+\mu^-$	$10^7 \frac{dBR}{dq^2}$	$[1, 6]$	0.48 ± 0.06	0.23 ± 0.05	LHCb	$+3.1$
$B \to X_s e^+e^-$	10^6 BR	$[14.2, 25]$	0.21 ± 0.07	0.57 ± 0.19	BaBar	-1.8

Tension on the SM from $B \to K^* \mu^+ \mu^-$ measurements

- Perform χ^2 fit of the measured observables $F_L, A_{FB}, S_3, ..., S_9$
- Float the generic vector coupling, i.e., $\mathrm{Re}(C_9)$
- Best fit: $\Delta\mathrm{Re}(C_9) = \mathrm{Re}(C_9)^{\mathrm{LHCb}} - \mathrm{Re}(C_9)^{\mathrm{SM}} = -1.04 \pm 0.25$



Effective Weak $b \to d$ Hamiltonian

$$H_{\text{eff}}^{(b\to d)} = -\frac{4G_F}{\sqrt{2}}\Big[V_{tb}^* V_{td} \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu)$$
$$+V_{ub}^* V_{ud} \sum_{i=1}^{2} C_i(\mu)\Big(\mathcal{O}_i(\mu) - \mathcal{O}_i^{(u)}(\mu)\Big)\Big] + \text{h.c.}$$

- G_F (Fermi constant), $C_i(\mu)$ (Wilson coefficients), and $\mathcal{O}_i(\mu)$ (dimension-six operators) are the same (modulo $s \to d$) as in $H_{\text{eff}}^{(b\to s)}$
- CKM structure of the matrix elements more interesting in $H_{\text{eff}}^{(b\to d)}$, as $V_{tb}^* V_{td} \sim V_{ub}^* V_{ud} \sim \lambda^3$are of the same order in $\lambda = \sin\theta_{12}$
- Anticipate sizable CP-violating asymmetries in $b \to d$transitions compared to $b \to s$

Operator Basis

- Tree operators

$$\mathcal{O}_1 = \left(\bar{d}_L \gamma_\mu T^A c_L\right)\left(\bar{c}_L \gamma^\mu T^A b_L\right), \quad \mathcal{O}_2 = \left(\bar{d}_L \gamma_\mu c_L\right)\left(\bar{c}_L \gamma^\mu b_L\right)$$

$$\mathcal{O}_1^{(u)} = \left(\bar{d}_L \gamma_\mu T^A u_L\right)\left(\bar{u}_L \gamma^\mu T^A b_L\right), \quad \mathcal{O}_2^{(u)} = \left(\bar{d}_L \gamma_\mu u_L\right)\left(\bar{u}_L \gamma^\mu b_L\right)$$

- Dipole operators

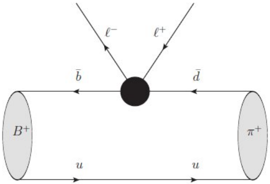
$$\mathcal{O}_7 = \frac{e\, m_b}{g_{\rm st}^2}\left(\bar{d}_L \sigma^{\mu\nu} b_R\right) F_{\mu\nu}, \quad \mathcal{O}_8 = \frac{m_b}{g_{\rm st}}\left(\bar{d}_L \sigma^{\mu\nu} T^A b_R\right) G^A_{\mu\nu}$$

- Semileptonic operators

$$\mathcal{O}_9 = \frac{e^2}{g_{\rm st}^2}\left(\bar{d}_L \gamma^\mu b_L\right)\sum_\ell \left(\bar{\ell}\gamma_\mu \ell\right), \quad \mathcal{O}_{10} = \frac{e^2}{g_{\rm st}^2}\left(\bar{d}_L \gamma^\mu b_L\right)\sum_\ell \left(\bar{\ell}\gamma_\mu \gamma_5 \ell\right)$$

$B \to \pi$ transition matrix elements

Momentum transfer:
$q = p_B - p_\pi = p_{\ell^+} + p_{\ell^-}$



The Feynman diagram for the $B^+ \to \pi^+\ell^+\ell^-$ decay.

$$\langle \pi(p_\pi)|\bar{b}\gamma^\mu d|B(p_B)\rangle = f_+(q^2)\left(p_B^\mu + p_\pi^\mu\right) + \left[f_0(q^2) - f_+(q^2)\right]\frac{m_B^2 - m_\pi^2}{q^2}q^\mu$$

$$\langle \pi(p_\pi)|\bar{b}\sigma^{\mu\nu}q_\nu d|B(p_B)\rangle = \frac{if_T(q^2)}{m_B + m_\pi}\left[\left(p_B^\mu + p_\pi^\mu\right)q^2 - q^\mu\left(m_B^2 - m_\pi^2\right)\right]$$

- Dominant theoretical uncertainty is in the form factors $f_+(q^2), f_0(q^2)$, $f_T(q^2)$; require non-perturbative techniques, such as Lattice QCD
- Their determination is the main focus of the theory

$B \to \pi \ell^+ \nu_\ell$ decay

$$\langle \pi | \bar{u} \gamma^\mu b | B \rangle = f_+(q^2) \left(p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2} q^\mu \right) + f_0(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q^\mu$$

- $f_0(q^2)$contribution is suppressed by m_ℓ^2/m_B^2 for $\ell = e, \mu$
- Differential decay width

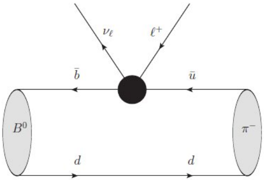
$$\frac{d\Gamma}{dq^2}(B^0 \to \pi^- \ell^+ \nu_\ell) = \frac{G_F^2 |V_{ub}|^2}{192 \pi^3 m_B^3} \lambda^{3/2}(q^2) |f_+(q^2)|^2$$

with $\lambda(q^2) = \left(m_B^2 + m_\pi^2 - q^2\right)^2 - 4 m_B^2 m_\pi^2$

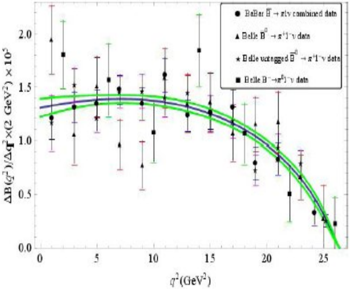
- Assuming Isospin symmetry: $f_+(q^2)$ and $f_0(q^2)$in charged current $B \to \pi \ell \nu_\ell$ and neutral current $B \to \pi \ell^+ \ell^-$decays are equal

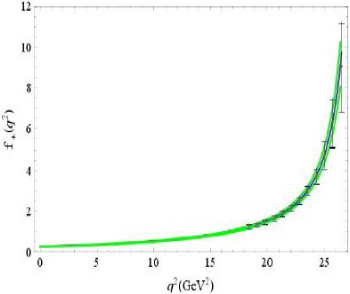
- Global fit of the CKM matrix element [PDG, 2012]

$$|V_{ub}| = (3.51^{+0.15}_{-0.14}) \times 10^{-3}$$



Fits of the data on $B \to \pi^+ \ell^- \nu_\ell$ yield $f_+(q^2)$





Heavy-Quark Symmetry (HQS) relations

- Including symmetry-breaking corrections, Heavy Quark Symmetry relates $f_+(q^2), f_0(q^2)$ and $f_T(q^2)$ (for $q^2/m_b^2 \ll 1$) [Beneke, Feldmann (2000)]

$$f_0(q^2) = \left(\frac{m_B^2 + m_\pi^2 - q^2}{m_B^2}\right) \left[\left(1 + \frac{\alpha_s(\mu) C_F}{4\pi}\left(2 - 2L(q^2)\right)\right) f_+(q^2) + \frac{\alpha_s(\mu) C_F}{4\pi} \frac{m_B^2(q^2 - m_\pi^2)}{(m_B^2 + m_\pi^2 - q^2)^2} \Delta F_\pi \right],$$

$$f_T(q^2) = \left(\frac{m_B + m_\pi}{m_B}\right) \left[\left(1 + \frac{\alpha_s(\mu) C_F}{4\pi}\left(\ln\frac{m_b^2}{\mu^2} + 2L(q^2)\right)\right) f_+(q^2) - \frac{\alpha_s(\mu) C_F}{4\pi} \frac{m_B^2}{m_B^2 + m_\pi^2 - q^2} \Delta F_\pi \right],$$

$$L(q^2) = \left(1 + \frac{m_B^2}{m_\pi^2 - q^2}\right) \ln\left(1 + \frac{m_\pi^2 - q^2}{m_B^2}\right), \quad \Delta F_\pi = \frac{8\pi^2 f_B f_\pi}{N_c m_B} \left\langle l_+^{-1} \right\rangle_+ \left\langle \bar{u}^{-1} \right\rangle_\pi$$

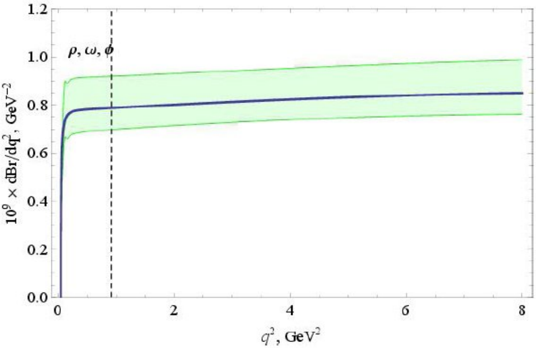
$B^\pm \to \pi^\pm \ell^+ \ell^-$ at large hadronic recoil ($q^2/m_b^2 \ll 1$)

[AA, A. Parkhomenko, A. Rusov; Phys. Rev. D89 (2014) 094021]

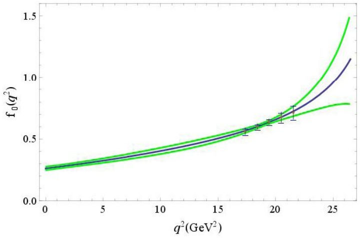
- Partially integrated branching fractions for $B^\pm \to \pi^\pm \ell^+ \ell^-$

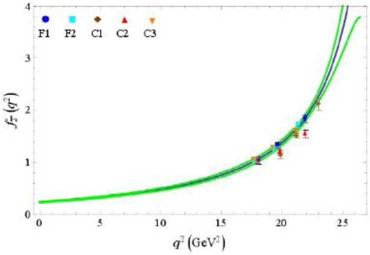
  $$\mathbf{BR_{SM}}(B^+ \to \pi^+ \mu^+ \mu^-; 1\,\mathrm{GeV}^2 \le q^2 \le 8\,\mathrm{GeV}^2) = \left(0.57^{+0.07}_{-0.05}\right) \times 10^{-8}$$

- Dimuon invariant mass spectrum at large hadronic recoil



Determination of $f_0^{B\pi}(q^2)$ and $f_T^{B\pi}(q^2)$ and comparison with Lattice QCD

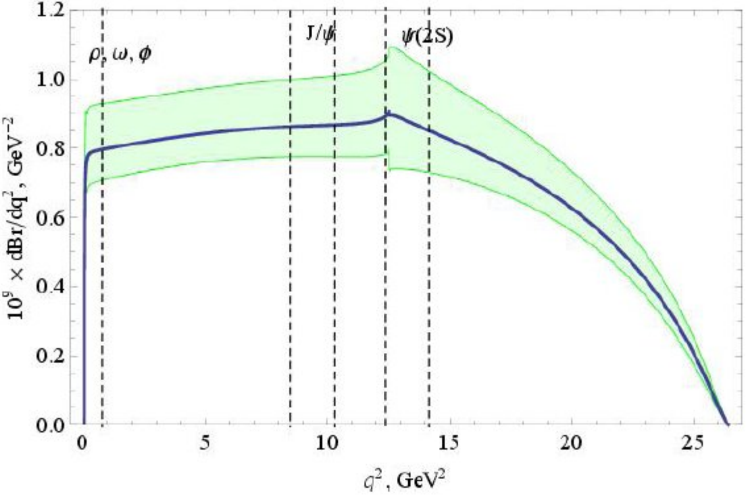




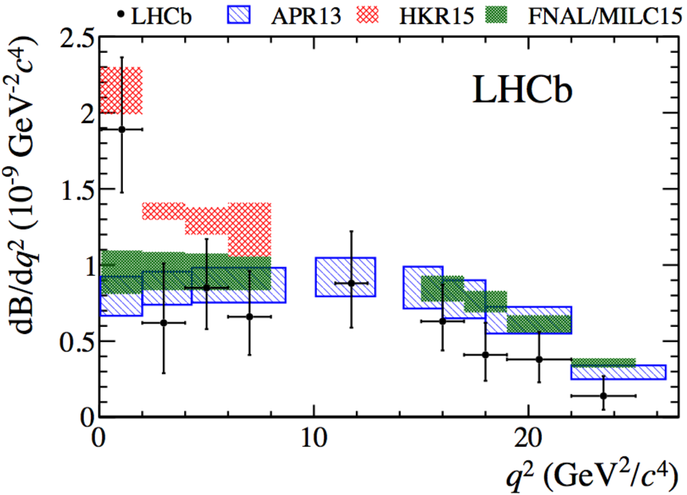
- FFs are obtained by the z-expansion [Boyd, Grinstein, Lebed] and constraints from data in low-q^2
- Lattice data (in high-q^2 are obtained by the HPQCD Collab.
  for $f_0^{B\pi}(q^2)$ from [arXiv:hep-lat/0601021]
  for $f_T^{B\pi}(q^2)$ from [arXiv:1310.3207]
- In almost the entire q^2-domain, the form factors are now determined accurately. Recent Fermilab/MILC lattice results are in agreement

$B^+ \to \pi^+\mu^+\mu^-$ in the entire range of q^2

[AA, A. Parkhomenko, A. Rusov; Phys. Rev. D89 (2014) 094021]



Dimuon invariant mass spectrum in $B \to \pi \ell^+ \ell^-$



- In excellent agreement with the APR2013 predictions, as well as with the Lattice results

SM vs. experimental data

- SM theoretical estimate of the total branching fraction
  [AA, A. Parkhomeno, A. Rusov; Phys. Rev. D89 (2014) 094021] :

$$\mathrm{BR_{SM}}(B^{\pm} \to \pi^{\pm}\mu^{+}\mu^{-}) = \left(1.88^{+0.32}_{-0.21}\right) \times 10^{-8}$$

- Uncertainty from the form factors is now reduced greatly.
  Residual theoretical uncertainty is mainly from the scale dependence and the CKM matrix elements

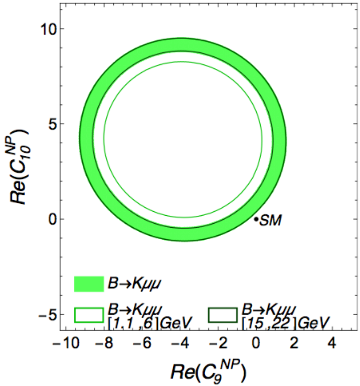
- LHCb has measured the BR and dimuon invariant mass distribution in $B^{\pm} \to \pi^{\pm}\mu^{+}\mu^{-}$) based on 3 fb^{-1} integrated luminosity
  [LHCb-PAPER-2015-035; arXiv:1509.00414] :

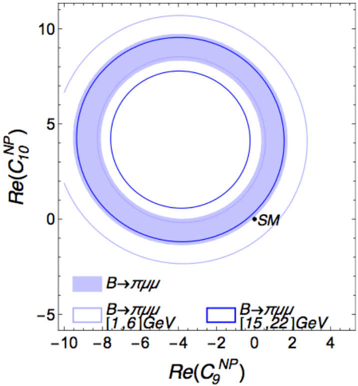
$$\mathrm{BR_{exp}}(B^{\pm} \to \pi^{\pm}\mu^{+}\mu^{-}) = (1.83 \pm 0.24(\mathrm{stat}) \pm 0.05(\mathrm{syst})) \times 10^{-8}$$

- Excellent agreement with SM-based APR2013-theory within errors, but significant improvement expected from the future analysis

Determination of Wilson Coeffs. from $B \to (\pi/K)\mu^+\mu^-$

[Fermilab/MILC, arxiv:1510.02349]





$B_s \to \mu^+\mu^-$ in the SM & BSM

- Effective Hamiltonian

$$\mathcal{H}_{eff} = -\frac{G_F \alpha}{\sqrt{2}\pi} V_{ts}{}^* V_{tb} \sum_i \left[C_i(\mu)\mathcal{O}_i(\mu) + C_i'(\mu)\mathcal{O}_i'(\mu) \right]$$

- Operators: O_i (SM) & O_i' (BSM)

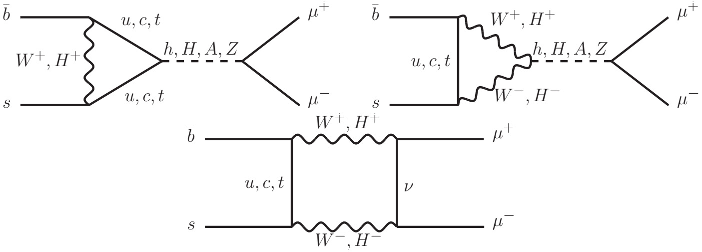
$$\mathcal{O}_{10} = \left(\bar{s}_\alpha \gamma^\mu P_L b_\alpha\right)\left(\bar{l}\gamma_\mu\gamma_5 l\right), \qquad \mathcal{O}_{10}' = \left(\bar{s}_\alpha \gamma^\mu P_R b_\alpha\right)\left(\bar{l}\gamma_\mu\gamma_5 l\right)$$
$$\mathcal{O}_S = m_b \left(\bar{s}_\alpha P_R b_\alpha\right)\left(\bar{l} l\right), \qquad \mathcal{O}_S' = m_s \left(\bar{s}_\alpha P_L b_\alpha\right)\left(\bar{l} l\right)$$
$$\mathcal{O}_P = m_b \left(\bar{s}_\alpha P_R b_\alpha\right)\left(\bar{l}\gamma_5 l\right), \qquad \mathcal{O}_P' = m_s \left(\bar{s}_\alpha P_L b_\alpha\right)\left(\bar{l}\gamma_5 l\right)$$

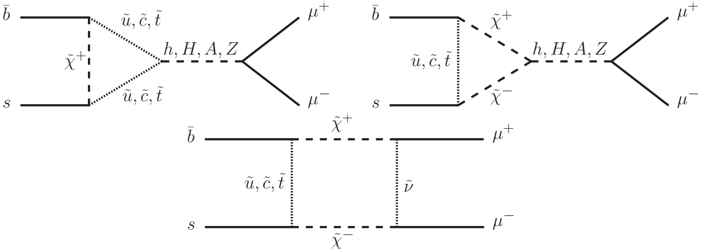
$$\mathbf{BR}\left(\bar{B}_s \to \mu^+\mu^-\right) = \frac{G_F^2 \alpha^2 m_{B_s}^2 f_{B_s}^2 \tau_{B_s}}{64\pi^3} |V_{ts}{}^* V_{tb}|^2 \sqrt{1 - 4\hat{m}_\mu^2} \times \left[\left(1 - 4\hat{m}_\mu^2\right)|F_S|^2 + |F_P + 2\hat{m}_\mu^2 F_{10}|^2\right]$$

$$F_{S,P} = m_{B_s}\left[\frac{C_{S,P} m_b - C_{S,P}' m_s}{m_b + m_s}\right], \quad F_{10} = C_{10} - C_{10}', \; \hat{m}_\mu = m_\mu / m_{B_s}$$

$$\mathbf{BR}\left(\bar{B}_s \to \mu^+\mu^-\right)_{\mathbf{SM}} = (3.23 \pm 0.27) \times 10^{-9}$$ [Buras et al.; arxiv:1208.09344]

Leading diagrams for $B_s \to \mu^+\mu^-$ in SM, 2HDM & MSSM





Compatibility of the SM with $B^0_{(s)} \to \mu^+\mu^-$ measurements

$B^0_s \to \mu^+\mu^-$

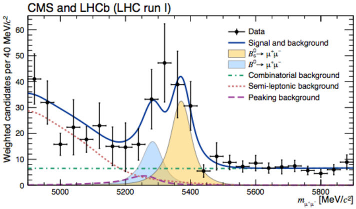
Combined analysis with CMS

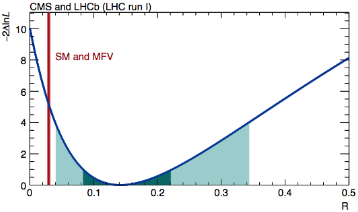
[Nature 522(2015)]

- **First observation** of $B^0_s \to \mu^+\mu^-$ and **evidence** for $B^0 \to \mu^+\mu^-$.
  - 6.2σ and 3.2σ respectively.
- Measurement of **branching fractions** and **ratio** of branching fractions.

$$\mathcal{B}\left[B^0_s \to \mu^+\mu^-\right] = 2.8^{+0.7}_{-0.6} \times 10^{-9}$$
$$\mathcal{B}\left[B^0 \to \mu^+\mu^-\right] = 3.9^{+1.6}_{-1.4} \times 10^{-10}$$

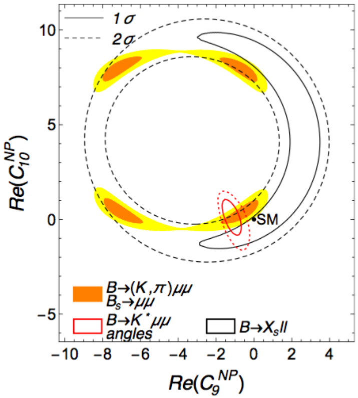
- Ratio found to be compatible with SM to 2.3σ.

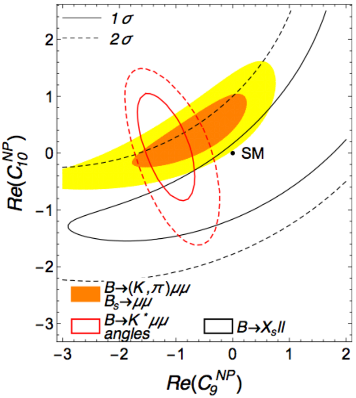




Test of the SM in Semileptonic B-decays and $B_s \to \mu^+\mu^-$

[Fermilab/MILC, arxiv:1510.02349]





Summary and outlook

- Lattice QCD, QCD sum rules, and heavy quark symmetry provide a controlled theoretical framework for B-meson physics
- Despite this impressive progress, some non-perturbative power corrections remain to be calculated quantitatively, limiting the current theoretical precision
- B-decays have been measured over 9 orders of magnitude and are found to be compatible with the SM, in general
- There is some tension on the SM in a number of rare B decays, typically 2 -3 σ; whether this is due to New Physics or QCD remains to be seen
- FCNC processes remain potentially very promising to search for physics beyond the SM, and they complement direct searches of BSM physics
- We look forward to improved theory and even more precise measurements at the LHC and the Super-B factory at KEK