Spin structure of the “forward” nucleon charge-exchange reaction $n + p \to p + n$ and the deuteron charge-exchange breakup

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The structure of the nucleon charge-exchange process $n + p \to p + n$ is investigated basing on the isotopic invariance of the nucleon-nucleon scattering. Using the operator of permutation of the spin projections of the neutron and proton, the connection between the spin matrices, describing the amplitude of the nucleon charge-exchange process at zero angle and the amplitude of the neutron elastic scattering on the proton in the “backward" direction, has been obtained. Due to the optical theorem, the spin-independent part of the differential cross-section of the process $n + p \to p + n$ at zero angle for unpolarized particles is expressed through the difference of total cross-sections of unpolarized proton-proton and neutron-proton scattering. Meantime, the spin-dependent part of this cross-section is proportional to the differential cross-section of the deuteron charge-exchange breakup $d + p \to (pp) + n$ at zero angle at the deuteron momentum $\mathbf{k}_d = 2\,\mathbf{k}_n$ ($\mathbf{k}_n$ is the initial neutron momentum). Analysis shows that, in the wide range of neutron laboratory momenta $k_n > 700\,MeV/c$, the main contribution into the differential cross-section of the process $n + p \to p + n$ at zero angle is provided namely by the spin-dependent term.

1 Isotopic structure of NN-scattering

Taking into account the isotopic invariance, the matrices of amplitudes of proton-proton elastic scattering, neutron-proton elastic scattering and charge transfer process $n + p \to p + n$ are connected by the following relation:

$$\hat{f}_{np\to pn}(\mathbf{p}, \mathbf{p}') = \hat{f}_{pp\to pp}(\mathbf{p}, \mathbf{p}') - \hat{f}_{np\to np}(\mathbf{p}, \mathbf{p}')\,. \qquad (1)$$

Here $\mathbf{p}$ and $\mathbf{p}'$ are the initial and final momenta in the c.m. frame, the directions of $\mathbf{p}'$ are defined within the solid angle in the c.m. frame, corresponding to the front hemisphere.

It should be stressed that the differential cross-section of the charge-exchange reaction, defined in the front hemisphere $0 \leq \theta \leq \frac{\pi}{2}$, $0 \leq \phi \leq 2\pi$ (here θ is the angle between the momenta of initial neutron and final proton, ϕ is the azimuthal angle), should coincide with the differential cross-section of the elastic neutron-proton scattering into the back hemisphere by the angle $\widetilde{\theta} = \pi - \theta$ at the azimuthal angle $\widetilde{\phi} = \pi + \phi$ in the c.m. frame. Due to the antisymmetry of the state of two fermions with respect to the total permutation, including the permutation of momenta ($\mathbf{p}' \to -\mathbf{p}'$), permutation of spin projections and permutation of isotopic projections ($p \leftrightarrows n$), the following relation between the amplitudes $\hat{f}_{np\to pn}(\mathbf{p},\mathbf{p}')$ and $\hat{f}_{np\to np}(\mathbf{p},-\mathbf{p}')$ holds [1–4] :

$$\hat{f}_{np\to pn}(\mathbf{p},\mathbf{p}') = -\hat{P}^{(1,2)}\hat{f}_{np\to np}(\mathbf{p},-\mathbf{p}')\,, \tag{2}$$

where $\hat{P}^{(1,2)}$ is the operator of permutation of spin projections of two particles with equal spins; the matrix elements of this operator are [5]:
$\langle m_1' m_2' \mid \hat{P}^{(1,2)} \mid m_1 m_2\rangle = \delta_{m_1' m_2}\delta_{m_2' m_1}$. For particles with spin 1/2 [1–5]

$$\hat{P}^{(1,2)} = \frac{1}{2}(\hat{I}^{(1,2)} + \hat{\boldsymbol{\sigma}}^{(1)}\hat{\boldsymbol{\sigma}}^{(2)}), \tag{3}$$

where $\hat{I}^{(1,2)}$ is the four-row unit matrix, $\hat{\boldsymbol{\sigma}}^{(1)}$, $\hat{\boldsymbol{\sigma}}^{(2)}$ – vector Pauli operators. It is evident that $\hat{P}^{(1,2)}$ is the unitary and Hermitian operator.

Taking into account the relations (2) and (3), the differential cross-sections of the charge-exchange process $n+p\to p+n$ and the elastic np-scattering in the corresponding back hemisphere coincide at any polarizations of initial nucleons:

$$\frac{d\sigma_{np\to pn}}{d\Omega}(\mathbf{p},\mathbf{p}') = \frac{d\sigma_{np\to np}}{d\Omega}(\mathbf{p},-\mathbf{p}'). \tag{4}$$

However, the separation into the spin-dependent and spin-independent parts is different for the amplitudes $\hat{f}_{np\to pn}(\mathbf{p},\mathbf{p}')$ and $\hat{f}_{np\to np}(\mathbf{p},-\mathbf{p}')$!

2 Nucleon charge-exchange process at zero angle

Now let us investigate in detail the nucleon charge transfer reaction $n+p\to p+n$ at zero angle. In the c.m. frame of the (np)-system, the amplitude of the nucleon charge transfer in the “forward" direction $\hat{f}_{np\to pn}(0)$ has the following spin structure:

$$\hat{f}_{np\to pn}(0) = c_1\hat{I}^{(1,2)} + c_2\,[\hat{\boldsymbol{\sigma}}^{(1)}\hat{\boldsymbol{\sigma}}^{(2)} - (\hat{\boldsymbol{\sigma}}^{(1)}\mathbf{l})(\hat{\boldsymbol{\sigma}}^{(2)}\mathbf{l})] + c_3\,(\hat{\boldsymbol{\sigma}}^{(1)}\mathbf{l})(\hat{\boldsymbol{\sigma}}^{(2)}\mathbf{l})\,, \tag{5}$$

where $\mathbf{l}$ is the unit vector directed along the incident neutron momentum. In so doing, the second term in Eq. (5) describes the spin-flip effect, and the third term characterizes the difference between the amplitudes with the parallel and antiparallel orientations of the neutron and proton spins.

The spin structure of the amplitude of the elastic neutron-proton scattering in the “backward" direction $\hat{f}_{np\to np}(\pi)$ is analogous:

$$\hat{f}_{np\to np}(\pi) = \widetilde{c}_1\hat{I}^{(1,2)} + \widetilde{c}_2\,[\hat{\boldsymbol{\sigma}}^{(1)}\hat{\boldsymbol{\sigma}}^{(2)} - (\hat{\boldsymbol{\sigma}}^{(1)}\mathbf{l})(\hat{\boldsymbol{\sigma}}^{(2)}\mathbf{l})] + \widetilde{c}_3\,(\hat{\boldsymbol{\sigma}}^{(1)}\mathbf{l})(\hat{\boldsymbol{\sigma}}^{(2)}\mathbf{l}). \tag{6}$$

However, the coefficients $\widetilde{c}$ in Eq.(6) do not coincide with the coefficients c in Eq.(5). According to Eq.(2), the connection between the amplitudes $\hat{f}_{np\to pn}(0)$ and $\hat{f}_{np\to np}(\pi)$ is the following:

$$\hat{f}_{np\to pn}(0) = -\hat{P}^{(1,2)}\hat{f}_{np\to np}(\pi)\,, \tag{7}$$

where the unitary operator $\hat{P}^{(1,2)}$ is determined by Eq.(3).

As a result of calculations with Pauli matrices, we obtain:

$$c_1 = -\frac{1}{2}\,(\widetilde{c}_1 + 2\widetilde{c}_2 + \widetilde{c}_3); \quad c_2 = -\frac{1}{2}\,(\widetilde{c}_1 - \widetilde{c}_3); \quad c_3 = -\frac{1}{2}\,(\widetilde{c}_1 - 2\widetilde{c}_2 + \widetilde{c}_3). \tag{8}$$

Hence, it follows from here that the "forward" differential cross-section of the nucleon charge-exchange reaction $n+p \to p+n$ for unpolarized initial nucleons is described by the expression:

$$\frac{d\sigma_{np\to pn}}{d\Omega}(0) = |c_1|^2 + 2|c_2|^2 + |c_3|^2 =$$
$$= \frac{1}{4}\,|\widetilde{c}_1 + 2\widetilde{c}_2 + \widetilde{c}_3|^2 + \frac{1}{2}\,|\widetilde{c}_1 - \widetilde{c}_3|^2 + \frac{1}{4}\,|\widetilde{c}_1 - 2\widetilde{c}_2 + \widetilde{c}_3|^2 = |\widetilde{c}_1|^2 + 2|\widetilde{c}_2|^2 + |\widetilde{c}_3|^2. \tag{9}$$

Thus, $\frac{d\sigma_{np\to pn}}{d\Omega}(0) = \frac{d\sigma_{np\to np}}{d\Omega}\,(\pi)$, just as it must be in accordance with the relation (4).

The differential cross-section of the process $n+p \to p+n$ in the "forward" direction for unpolarized nucleons can be presented in the following form, distinguishing the spin-independent and spin-dependent parts:

$$\frac{d\sigma_{np\to pn}}{d\Omega}(0) = \frac{d\sigma^{(si)}_{np\to pn}}{d\Omega}(0) + \frac{d\sigma^{(sd)}_{np\to pn}}{d\Omega}(0)\,. \tag{10}$$

In doing so, in accordance with the optical theorem, the spin-independent part $\frac{d\sigma^{(si)}_{np\to pn}}{d\Omega}(0)$ in Eq.(10) is determined by the difference of total cross-sections of the unpolarized proton-proton and neutron-proton interaction:

$$\frac{d\sigma^{(si)}_{np\to pn}}{d\Omega}(0) = |c_1|^2 = \frac{k^2}{16\pi^2}(\sigma_{pp} - \sigma_{np})^2(1+\alpha^2)\,, \tag{11}$$

where $k = |\mathbf{p}| = |\mathbf{p}'|$ is the modulus of neutron momentum in the c.m. frame of the colliding nucleons [1)], $\alpha = \mathrm{Re}\, c_1/\mathrm{Im}\, c_1$. The spin-dependent part of the cross-section of the "forward" charge-exchange process is

$$\frac{d\sigma^{(sd)}_{np\to pn}}{d\Omega}(0) = 2|c_2|^2 + |c_3|^2. \tag{12}$$

Meantime, according to Eqs. (6), (8) and (9), the spin-dependent part of the cross-section of the "backward" elastic np-scattering is

$$\frac{d\sigma^{(sd)}_{np\to np}}{d\Omega}(\pi) = 2|\widetilde{c}_2|^2 + |\widetilde{c}_3|^2. \tag{13}$$

We see that $\frac{d\sigma^{(sd)}_{np\to pn}}{d\Omega}(0) \neq \frac{d\sigma^{(sd)}_{np\to np}}{d\Omega}(\pi)$.

Further it is advisable to deal with the differential cross-section $\left.\frac{d\sigma}{dt}\right|_{t=0}$, being a relativistic invariant ($t = -(p_1 - p_2)^2 = (\mathbf{p} - \mathbf{p}')^2 - (E - E')^2 = 2k^2(1-\cos\theta)$). In this representation, the spin-independent and spin-dependent parts of the differential cross-section of the "forward"

[1)] We use the unit system with $\hbar = c = 1$.

charge transfer process $\left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0}$ are as follows: $\left.\frac{d\sigma^{(si)}_{np\to pn}}{dt}\right|_{t=0} = (\pi/k^2)\,|c_1|^2$, $\left.\frac{d\sigma^{(sd)}_{np\to pn}}{dt}\right|_{t=0}$ $= (\pi/k^2)\,(2|c_2|^2 + |c_3|^2)$.

Now it should be noted that, in the framework of the impulse approach, there exists a simple connection between the spin-dependent part of the differential cross-section of the charge-exchange reaction $n+p\to p+n$ at zero angle $\left.\frac{d\sigma^{(sd)}_{np\to pn}}{dt}\right|_{t=0}$ (not the "backward" elastic neutron-proton scattering, that would be an error !) and the differential cross-section of the deuteron charge-exchange breakup $d+p\to(pp)+n$ in the "forward" direction $\left.\frac{d\sigma_{dp\to(pp)n}}{dt}\right|_{t=0}$ at the deuteron momentum $\mathbf{k}_d = 2\mathbf{k}_n$ ($\mathbf{k}_n$ is the the initial neutron momentum) . In the case of unpolarized particles we have [6–8]:

$$\left.\frac{d\sigma_{dp\to(pp)n}}{dt}\right|_{t=0} = \frac{2}{3}\left.\frac{d\sigma^{(sd)}_{np\to pn}}{dt}\right|_{t=0}. \tag{14}$$

In doing so, this formula remains still valid if one takes into account the deuteron D-wave state [8].

It is easy to understand also that, due to the isotopic invariance, the same relation (like Eq. (14)) takes place for the process $p+d\to n+(pp)$ at the proton laboratory momentum $\mathbf{k}_p = \mathbf{k}_n$ and for the process $n+d\to p+(nn)$ at the neutron laboratory momentum $\mathbf{k}_n$.

As a result, it follows from Eqs. (10),(11) and (14) that

$$\left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0} = \frac{3}{2}\left.\frac{d\sigma_{dp\to(pp)n}}{dt}\right|_{t=0} + \frac{1}{16\pi}(\sigma_{pp}-\sigma_{np})^2(1+\alpha^2)\,. \tag{15}$$

Thus, in principle, taking into account Eqs. (14) and (15), the modulus of the ratio of the real and imaginary parts of the spin-independent charge transfer amplitude at zero angle ($|\,\alpha\,|$) may be determined using the experimental data on the cross-sections.

The analysis shows: if we suppose that the real part of the spin-independent amplitude of charge transfer $n+p\to p+n$ at zero angle is smaller or of the same order as compared with the imaginary part ($\alpha^2 \lesssim 1$), then it follows from the available experimental data on the differential cross-section of charge transfer $\left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0}$ and the data on the total cross-sections σ_{pp} and σ_{np} that the main contribution into the cross-section $\left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0}$ is provided namely by the spin-dependent part $\left.\frac{d\sigma^{(sd)}_{np\to pn}}{dt}\right|_{t=0}$.

If the differential cross-section $\frac{d\sigma}{dt}$ is given in the units of $mbn/\left(\frac{GeV}{c}\right)^2$ and the total cross-sections are given in mbn, then the spin-independent part of the "forward" charge transfer cross-section may be expressed in the form :

$$\left.\frac{d\sigma^{(si)}_{np\to pn}}{dt}\right|_{t=0} \approx 0.0512\,(\sigma_{pp}-\sigma_{np})^2(1+\alpha^2). \tag{16}$$

Using (16) and the data from the works [9–11], we obtain – assuming $\alpha^2 \lesssim 1$ – the estimates of the ratio $\left.\frac{d\sigma^{(si)}_{np\to pn}}{dt}\right|_{t=0} \Big/ \left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0}$, for different values of the neutron laboratory momentum $k_n = (0.7 \div 2.5)\ \frac{GeV}{c}$, at the level of $(10 \div 20)\%$, i.e. the spin-dependent part provides at least $(80 \div 90)\%$ of the total magnitude of the "forward" charge transfer cross section (see also [1–4]) .

The experimental data on the differential cross-sections of "forward" deuteron charge-exchange breakup processes $d + p \to (pp) + n$ and $n + d \to (nn) + p$ (see [4] and references therein), obtained recently in Dubna (JINR, Laboratory of High Energy Physics), also confirm the conclusion about the predominant role of the spin-dependent part of the differential cross-section of the nucleon charge-exchange reaction $n + p \to p + n$ in the "forward" direction.

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