Light-Cone Distribution Amplitudes of Bottom Baryons

Alexander Parkhomenko

P. G. Demidov Yaroslavl State University, Yaroslavl, Russia

A discussion of the three-quark light-cone distribution amplitudes (LCDAs) for the ground state heavy baryons with the spin-parities $J^P = 1/2^+$ and $J^P = 3/2^+$ in QCD in the heavy-quark limit is presented. Simple models for the bottom-baryon distribution amplitudes are analyzed with account of their scale dependence.

1 Introduction

The B-meson factories at SLAC and KEK, after approximately a decade of their operation, have made a great impact on a clarification of CP-violation origin in the quark sector of the Standard Model (SM). Study of heavy-light hadrons, in particular mesons and baryons containing the b-quark, at the LHC can serve as an additional test of the Kobayashi-Maskawa mechanism. Specific processes with bottom baryons, such as rare decays involving flavor-changing neutral currents (FCNC) transitions, are potential sources of new physics beyond the SM. In a difference to B-mesons, a non-zero spin of baryons allows also an experimental study of spin correlations. The spectrum of heavy bottom baryons have been enlarged substantially thanks to the effort done by the CDF and D0 Collaborations at the Tevatron collider and is presented in Table 1. During the LHC Run I, the majority of the bottom-baryon states has been confirmed and several new ones were observed. Unlike these progress, study of the FCNC motivated decays of bottom baryons remains to be statistically limited. A grater effort is expected during the next LHC run where heavy baryons will be copiously produced, and their weak decays may be measured precisely enough to provide important clues on physics beyond the Standard Model.

The theory of bottom baryon decays into light hadrons is more complicated compared to the B-meson decays and, hence, was receiving less attention. Calculations of heavy-baryon decays into light particles based on the heavy quark expansion, see e. g. [1], or using sum rules of the type proposed in [2, 3, 4] require the primary non-perturbative objects — the distribution amplitudes of heavy baryons. For a long period, the only existed models of heavy-baryon distribution amplitudes [5, 6] have been motivated by quark models and not consistent with QCD constraints. In the paper [7], the complete classification of three-quark light-cone distribution amplitudes (LCDAs) of the Λ_b-baryon in QCD in the heavy quark limit was given and the scale-dependence of the leading-twist LCDA is discussed. In addition, simple models of the LCDAs were suggested and their parameters were fixed based on estimates of the first few moments by the QCD sum rules method. The analysis of [7] has been extended on all the ground state b-baryons with the spin-parity both $J^P = 1/2^+$ and $J^P = 3/2^+$. The basic steps and main results of such an analysis are summarized in this lecture and all the details are presented in our papers [8, 9].

Baryon	$I(J^P)$	j^p	Experiment	HQET	Lattice QCD
Λ_b	$0(1/2^+)$	0^+	5619.4 ± 0.7	5637^{+68}_{-56}	$5641 \pm 21^{+15}_{-33}$
Σ_b^+	$1(1/2^+)$	1^+	5811.3 ± 1.9	5809^{+82}_{-76}	$5795 \pm 16^{+17}_{-26}$
Σ_b^-	$1(1/2^+)$	1^+	5815.5 ± 1.8	5809^{+82}_{-76}	$5795 \pm 16^{+17}_{-26}$
Σ_b^{*+}	$1(3/2^+)$	1^+	5832.1 ± 1.9	5835^{+82}_{-77}	$5842 \pm 26^{+20}_{-18}$
Σ_b^{*-}	$1(3/2^+)$	1^+	5835.1 ± 1.9	5835^{+82}_{-77}	$5842 \pm 26^{+20}_{-18}$
Ξ_b^-	$1/2(1/2^+)$	0^+	5791.1 ± 2.2	5780^{+73}_{-68}	$5781 \pm 17^{+17}_{-16}$
Ξ_b^0	$1/2(1/2^+)$	0^+	5788 ± 5	5903^{+81}_{-79}	$5903 \pm 12^{+18}_{-19}$
Ξ_b'	$1/2(1/2^+)$	1^+		5903^{+81}_{-79}	$5903 \pm 12^{+18}_{-19}$
$\Xi_b'^{*0}$	$1/2(3/2^+)$	1^+	5945.0 ± 2.8	5903^{+81}_{-79}	$5950 \pm 21^{+19}_{-21}$
Ω_b^-	$0(1/2^+)$	1^+	6071 ± 40	6036 ± 81	$6006 \pm 10^{+20}_{-19}$
Ω_b^*	$0(3/2^+)$	1^+		6063^{+83}_{-82}	$6044 \pm 18^{+20}_{-21}$

Table 1: Experimental measurements [10] and theoretical predictions based on HQET [11] and Lattice QCD [12] for masses of ground-state bottom baryons (in units of MeV). The mass of the $\Xi_b'^{*0}$-baryon was measured by the CMS Collaboration recently [13]. The LHCb Collaboration [14] have measured the masses of the Λ_b-, Ξ_b^--, and Ω_b^--baryons in agreement with the SM expectations.

2 Light-Cone Distribution Amplitudes

Light-cone distribution amplitudes (LCDAs) of heavy baryons are the transition matrix elements from the baryonic state to vacuum of non-local light-ray operators built off an effective heavy quark and two light quarks. The content of such operators supports a similarity in the construction of the heavy-baryon LCDAs to both the B-meson (within the HQET) [15, 16] and the nucleon (within QCD) [17, 18] LCDAs descriptions. An important simplifying feature of the operators containing one or more heavy quarks is an existence of the Heavy Quark Symmetry (HQS) which results into the decoupling of the heavy-quark spin from the system dynamics in the limit $m_Q \to \infty$, where m_Q is the heavy-quark mass. So, to understand the properties of heavy baryons in this limit, it is enough to switch off the heavy-quark spin and to introduce a total set of two-particle LCDAs corresponding to the light-quark system, called diquark, which quantum numbers completely determine a number of LCDAs and their asymptotic behavior.

In this simplified picture, there are the $SU(3)_F$ antitriplet of "scalar baryons" with the $J^P = 0^+$ spin-parity determined by the diquark spin-parity $j^p = 0^+$ and the $SU(3)_F$ sextet of "axial-vector baryons" with the $J^P = 1^+$ spin-parity which follows from the diquark spin-parity $j^p = 1^+$. It is reasonable to start with the description of the "scalar baryons" and then to generalize the procedure on the "axial-vector baryons". The changes originated by an account of the heavy-quark spin can be done after the total sets of the non-local operators and corresponding LCDAs are introduced in the decoupling limit. All these steps are discussed briefly in this section.

2.1 "Scalar Baryons"

The "scalar baryons" are combined into the $SU(3)_F$ antitriplet with $J^P = 0^+$ in which the light diquark states are also the scalar states with $j^p = 0^+$.

The set of the LCDAs is determined by the matrix elements between the baryonic state and vacuum of the four independent non-local light-ray operators [7, 8, 9]:

$$\begin{aligned}
\epsilon^{abc}\langle 0|\left(q_1^a(t_1 n)C\gamma_5 \not{n} q_2^b(t_2 n)\right) h_v^c(0)|H(v)\rangle &= f_H^{(2)}\,\Psi_2(t_1,t_2) \\
\epsilon^{abc}\langle 0|\left(q_1^a(t_1 n)C\gamma_5 q_2^b(t_2 n)\right) h_v^c(0)|H(v)\rangle &= f_H^{(1)}\,\Psi_3^s(t_1,t_2) \\
\epsilon^{abc}\langle 0|\left(q_1^a(t_1 n)C\gamma_5 i\sigma_{\bar{n}n} q_2^b(t_2 n)\right) h_v^c(0)|H(v)\rangle &= 2f_H^{(1)}\,\Psi_3^\sigma(t_1,t_2) \\
\epsilon^{abc}\langle 0|\left(q_1^a(t_1 n)C\gamma_5 \not{\bar{n}} q_2^b(t_2 n)\right) h_v^c(0)|H(v)\rangle &= f_H^{(2)}\Psi_4(t_1,t_2)
\end{aligned} \qquad (1)$$

where $q_i(x) = u(x)$, $d(x)$, $s(x)$ are the light-quark fields, $h_v(0)$ is the static heavy-quark field situated at the origin of the position-space frame, C is the charge conjugation matrix, n^μ and $\bar{n}^\mu$ are two light-like vectors normalized by the condition $(n\bar{n}) = 2$. In addition, the frame is adopted where the heavy-meson velocity is related to the light-like vectors as follows: $v^\mu = (n^\mu + \bar{n}^\mu)/2$. The light-quark fields on the light cone are assumed to be multiplied by the Wilson lines:

$$q(tn) = [0,tn]\,q(tn) = \mathrm{P}\exp\left\{-ig_{\rm st} t\int_0^1 d\alpha n^\mu\, A_\mu(\alpha t n)\right\} q(tn) = \sum_{N=0}^{\infty} \frac{t^N}{N!}\,(n^\mu D_\mu)^N\, q(0),$$

where the following definition of the covariant derivative $D_\mu = \partial_\mu - ig_{\rm st}A_\mu$ is accepted. The similar definition can be used for the gluonic field: $G_{\mu\nu}(tn) = [0,tn]\,G_{\mu\nu}(tn)$, where the Wilson line is determined in the adjoint representation of the color $SU(3)$-group.

The static heavy-quark field living on the light cone also includes the Wilson line but of the other type with the time-like link [19]:

$$h_v(0) = \mathrm{P}\exp\left\{ig_{\rm st}\int_{-\infty}^0 d\alpha v^\mu\, A_\mu(\alpha v)\right\}\phi(-\infty),$$

with which it is connected with the sterile field $\phi(-\infty)$.

The couplings $f_H^{(i)}$ introduced in Eqs. (1) to make the LCDAs dimensionless are defined by local operators [20, 21, 22, 23]:

$$\begin{aligned}
\epsilon^{abc}\langle 0|\left(q_1^a(0)C\gamma_5 q_2^b(0)\right) h_v^c(0)|H(v)\rangle &= f_H^{(1)} \\
\epsilon^{abc}\langle 0|\left(q_1^a(0)C\gamma_5 \not{v} q_2^b(0)\right) h_v^c(0)|H(v)\rangle &= f_H^{(2)}
\end{aligned}$$

The scale dependences of these couplings are governed by the anomalous dimensions $\gamma^{(i)}$ of local operators as follows:

$$\frac{d\ln f_H^{(i)}(\mu)}{d\ln\mu} \equiv -\gamma^{(i)} = -\sum_k \gamma_k^{(i)}\, a^k(\mu), \qquad a(\mu) \equiv \frac{\alpha_s^{\overline{\rm MS}}(\mu)}{4\pi},$$

where the strong coupling is determined in the $\overline{\rm MS}$-scheme. This equation can be solved order by order in the $a(\mu)$-power expansion and in the NLO order, one can use the following relations:

$$f_H^{(i)}(\mu) = f_H^{(i)}(\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_1^{(i)}/\beta_0}\left[1 - \frac{\alpha_s(\mu_0)-\alpha_s(\mu)}{4\pi}\,\frac{\gamma_1^{(i)}}{\beta_0}\left(\frac{\gamma_2^{(i)}}{\gamma_1^{(i)}} - \frac{\beta_1}{\beta_0}\right)\right],$$

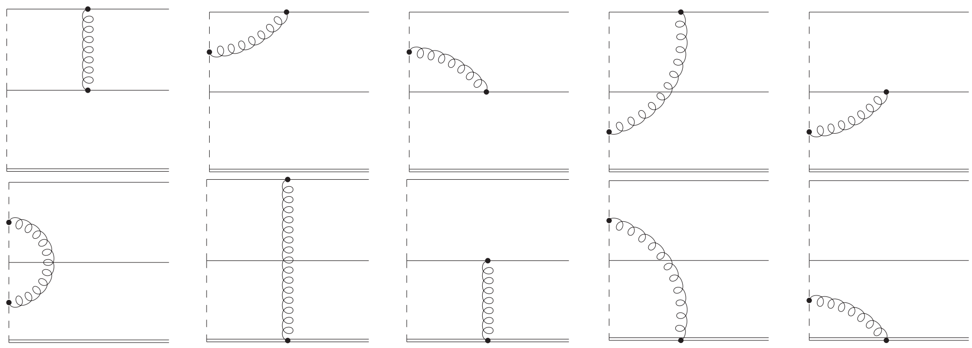


Figure 1: The complete set of the one-gluon-exchange diagrams necessary for the scale-dependence calculation of the heavy-baryon LCDAs. The normal and thick solid lines corresponds to the light and heavy quarks while dotted and wavy lines denote the Wilson lines and virtual gluons, respectively.

where $\beta_{0,1}$ are the first two coefficient in the perturbative expansion of the β-function. As the evolution to the required scale can be easily done now, one needs to know numerical values of the couplings $f_H^{(i)}(\mu)$ at some representative scale μ_0, say $\mu_0 = 1$ GeV. As this scale is rather low to use the perturbation theory, non-perturbative techniques are necessary to calculate these quantities. In particular, the QCD sum rules method in NLO for the Λ_b-baryon results the values [23]:

$$f_{\Lambda_b}^{(1)}(\mu_0 = 1\ \text{GeV}) \simeq f_{\Lambda_b}^{(2)}(\mu_0 = 1\ \text{GeV}) \simeq 0.030 \pm 0.005\ \text{GeV}^3.$$

Non-relativistic constituent-quark picture of heavy baryons H suggests that $f_H^{(2)} \simeq f_H^{(1)}$ at low scales of order 1 GeV, and this expectation is supported by numerous QCD sum rule calculations [21, 20, 22, 23]. These couplings $f_H^{(i)}(\mu)$ cannot coincide at all scales because of different anomalous dimensions $\gamma^{(i)}$ of local operators.

Similar to the couplings $f_H^{(i)}(\mu)$, the LCDAs $\Psi_i(t_1, t_2)$ introduced in Eq. (1) are also scale-dependent functions. To find their scale evolution, it is convenient to make their Fourier transform to the momentum space:

$$\Psi(t_1, t_2) = \int_0^\infty d\omega_1 \int_0^\infty d\omega_2\, e^{-it_1\omega_1 - it_2\omega_2} \psi(\omega_1, \omega_2) = \int_0^\infty \omega\, d\omega \int_0^1 du\, e^{-i\omega(t_1 u + t_2 \bar{u})}\, \widetilde{\psi}(\omega, u)$$

where $\bar{u} = 1 - u$. In the first representation $\omega_1 = u\omega$ and $\omega_2 = (1-u)\,\omega = \bar{u}\omega$ are the energies of the light quarks q_1 and q_2. The leading-order evolution equation for the $\psi_2(\omega_1, \omega_2; \mu)$ can be derived by identifying the ultra-violet singularities of the one-gluon-exchange diagrams presented in Fig. 1.

The evolution equation in the leading order is expressed in terms of two-particle kernels:

$$\mu \frac{d}{d\mu}\, \psi_2(\omega_1, \omega_2; \mu) = -\frac{\alpha_s(\mu)}{2\pi}\, \frac{4}{3} \left\{ \int_0^\infty d\omega_1'\, \gamma^{\text{LN}}(\omega_1', \omega_1; \mu)\, \psi_2(\omega_1', \omega_2; \mu) \right.$$
$$\left. + \int_0^\infty d\omega_2'\, \gamma^{\text{LN}}(\omega_2', \omega_2; \mu)\, \psi_2(\omega_1, \omega_2'; \mu) - \int_0^1 dv\, V(u, v)\, \psi_2(v\omega, \bar{v}\omega; \mu) + \frac{3}{2}\, \psi_2(\omega_1, \omega_2; \mu) \right\},$$

where the kernel $\gamma^{\rm LN}(\omega', \omega; \mu)$ controls the evolution of the B-meson LCDA [24] and $V(u, v)$ is the Efremov-Radyushkin-Brodsky-Lepage (ER-BL) kernel [25, 26]. The term $3\psi_2(\omega_1, \omega_2; \mu)/2$ results from the one-loop $f_H^{(2)}$ renormalization subtraction. The evolution equation above can be solved either numerically or semi-analytically [7, 9]

2.2 "Axial-Vector Baryons"

The "axial-vector baryons" are components of the $SU(3)_F$ sextet with $J^P = 1^+$ in which the light diquark states are the axial-vector states with $j^p = 1^+$. In a difference to the "scalar baryons" case, one needs to consider the baryons with the longitudinal and transverse polarizations separately.

The set of the longitudinal LCDAs is determined by the matrix elements between the baryonic state with the appropriate polarization and vacuum of the four independent non-local light-ray operators [8, 9]:

$$\begin{aligned}
\epsilon^{abc}\langle 0|\left(q_1^a(t_1) C \not{n} q_2^b(t_2)\right) h_v^c(0)|H(v,\varepsilon)\rangle &= (\bar{v}\varepsilon)\, f_H^{(2)}\, \Psi_2^{\parallel}(t_1,t_2) \\
\epsilon^{abc}\langle 0|\left(q_1^a(t_1) C\, q_2^b(t_2)\right) h_v^c(0)|H(v,\varepsilon)\rangle &= (\bar{v}\varepsilon)\, f_H^{(1)}\, \Psi_3^{\parallel s}(t_1,t_2) \\
\epsilon^{abc}\langle 0|\left(q_1^a(t_1) C\, i\sigma_{\bar{n}n} q_2^b(t_2)\right) h_v^c(0)|H(v,\varepsilon)\rangle &= 2\,(\bar{v}\varepsilon)\, f_H^{(1)}\, \Psi_3^{\parallel a}(t_1,t_2) \\
\epsilon^{abc}\langle 0|\left(q_1^a(t_1) C \not{\bar{n}} q_2^b(t_2)\right) h_v^c(0)|H(v,\varepsilon)\rangle &= -\,(\bar{v}\varepsilon)\, f_H^{(2)}\, \Psi_4^{\parallel}(t_1,t_2)
\end{aligned}$$

where $\sigma_{\bar{n}n} = i\left(\not{\bar{n}}\not{n} - \not{n}\not{\bar{n}}\right)/2$, $\bar{v}^\mu = (\bar{n}^\mu - n^\mu)/2$ is the four-vector orthogonal to the four-velocity $(v\bar{v}) = 0$ and normalized by $(\bar{v}\bar{v}) = -1$. In the LCDA definitions above, the baryonic state is assumed to have a pure longitudinal polarization $\varepsilon_\parallel^\mu = \bar{v}^\mu$ and the prefactor on the r.h.s. is simply $(\bar{v}\varepsilon) = -1$.

The similar set of the transverse LCDAs is determined as follows [8, 9]:

$$\begin{aligned}
\epsilon^{abc}\langle 0|\left(q_1^a(t_1) C\, \gamma_\perp^\mu \not{n} q_2^b(t_2)\right) h_v^c(0)|H(v,\varepsilon)\rangle &= f_H^{(2)}\, \Psi_2^{\perp}(t_1,t_2)\, \varepsilon_\perp^\mu \\
\epsilon^{abc}\langle 0|\left(q_1^a(t_1) C\, \gamma_\perp^\mu q_2^b(t_2)\right) h_v^c(0)|H(v,\varepsilon)\rangle &= f_H^{(1)}\, \Psi_3^{\perp s}(t_1,t_2)\, \varepsilon_\perp^\mu \\
\epsilon^{abc}\langle 0|\left(q_1^a(t_1) C\, \gamma_\perp^\mu i\sigma_{\bar{n}n} q_2^b(t_2)\right) h_v^c(0)|H(v,\varepsilon)\rangle &= 2 f_H^{(1)}\, \Psi_3^{\perp a}(t_1,t_2)\, \varepsilon_\perp^\mu \\
\epsilon^{abc}\langle 0|\left(q_1^a(t_1) C\, \gamma_\perp^\mu \not{\bar{n}} q_2^b(t_2)\right) h_v^c(0)|H(v,\varepsilon)\rangle &= f_H^{(2)}\, \Psi_4^{\perp}(t_1,t_2)\, \varepsilon_\perp^\mu
\end{aligned}$$

where $\gamma_\perp^\mu = \gamma^\mu - \left(\not{\bar{n}}n^\mu + \not{n}\bar{n}^\mu\right)/2$ and $\varepsilon_\perp^\mu = \varepsilon^\mu - \varepsilon_\parallel^\mu$ is the transverse polarization of the baryon.

2.3 Real Baryons

As far as all the sets of the LCDAs are determined, it necessary to generalize their definitions to real baryons which simply means that the spin of the heavy quark should be included into the baryon wave function. In other words, the r. h. s. of matrix elements of all non-local operators must be multiplied on the Dirac spinor $U(v)$ of the heavy quark h_v, satisfying the conditions: $\not{v}\, U(v) = U(v)$ and $\overline{U}(v)\, U(v) = 1$. After these modifications, the "scalar baryons" transform to the baryons with the spin-parity $J^P = 1/2^+$ and the heavy-quark Dirac spinor $U(v)$ is nothing else but the heavy-baryon spinor $H(v)$, i. e. the spin of the heavy quark completely determines the spin structure of the heavy-baryon wave function. The case of "axial-vector baryons" is a little bit more complicated. It is well-known from quantum mechanics that the direct product

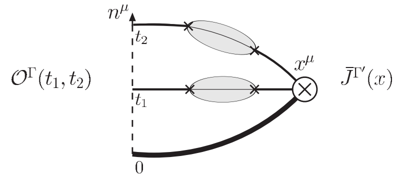


Figure 2: The two-point correlator of the local and light-ray operators in the QCD background.

of two angular momenta $j_1 = 1/2$ and $j_2 = 1$ is decomposed into two irreducible representation with the momenta $J_1 = 1/2$ and $J_2 = 3/2$. That is exactly the situation after the heavy-quark spin is switched on in the heavy baryon with the diquark in the axial-vector state $j^p = 1^+$:

$$\varepsilon_\mu U(v) = \left[\varepsilon_\mu U(v) - \frac{1}{3}(\gamma_\mu + v_\mu) \not\varepsilon U(v)\right] + \frac{1}{3}(\gamma_\mu + v_\mu) \not\varepsilon U(v) \equiv R_\mu^{3/2}(v) + \frac{1}{3}(\gamma_\mu + v_\mu) H(v).$$

As the result, there are two states with the spin-parities $J^P = 1/2^+$ and $J^P = 3/2^+$. The former one is described by the Dirac spinor $H(v)$ and for the $J^P = 3/2^+$ state the Rarita-Schwinger vector-spinor $R_\mu^{3/2}(v)$, which satisfies the relations $\not v R_\mu^{3/2}(v) = R_\mu^{3/2}(v)$, $v^\mu R_\mu^{3/2}(v) = 0$, and $\gamma^\mu R_\mu^{3/2}(v) = 0$, can be used.

3 QCD Sum Rules

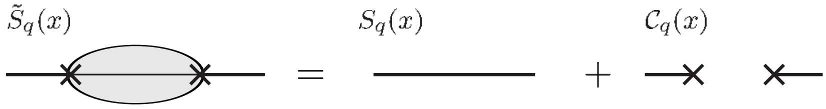
In applications to a calculation of amplitudes with heavy baryons, one needs to know realistic models for LCDAs. Such models can be obtained by matching several few moments of LCDA models and the corresponding ones calculated by some non-perturbative methods, say by the QCD sum rules. The later method requires a calculation of a two-point correlator which involve the non-local light-ray operator and a suitable local current $J^{\Gamma'}(x)$, as it is shown in Fig. 2.

The general structure of the heavy-baryon local currents can be chosen as follows:

$$\bar{J}^{\Gamma'}(x) = \epsilon^{abc}\left(\bar{q}_2^a(x)\,[A + B\,\not v]\,\Gamma' C^T \bar{q}_1^b(x)\right)\bar{h}_v^c(x),$$

where A and B are two constants with the constraint $A + B = 1$ which accounts for an arbitrariness in the choice of local currents. The variation in $A \in [0, 1]$ is adopted as a systematic error of numerical estimations. Note that the central value $A = B = 1/2$ corresponds to the a constituent quark model picture [7]. The Dirac matrix Γ' is a suitable structure determined by the spin-parity of the baryon, in particular, $\Gamma' = \gamma_5$ for baryons from the $SU(3)_F$ antitriplet ($j^p = 0^+$) and $\Gamma' = \gamma_\parallel, \gamma_\perp$ for the $SU(3)_F$-sextet baryons with $j^p = 1^+$.

In calculations of the correlation functions, one tacitly assumes that baryons are bound states of quarks which are not free particles inside but couple by virtue of the gluonic field. So, light quark propagators $\tilde{S}_q(x)$, being very sensitive to the influence of the background gluonic field, should be modified accordingly while for the heavy quark this effect is sub-dominant and to leading order in the heavy-quark mass m_Q expansion can be neglected. To take effects of the QCD background inside baryons into account, the method of non-local condensates [28, 29, 30] is used. In this approach the light-quark propagator can be decomposed into two parts: the perturbative $S_q(x)$ and non-perturbative $\mathcal{C}_q(x)$ ones,



and the later accumulates an information about the background inside the baryon in terms of non-local quark condensate $\langle \bar{q}(x)q(0)\rangle$:

$$S_q(x) = \frac{i\not{x}}{2\pi^2 x^4} - \frac{m}{4\pi^2 x^2} \qquad \mathcal{C}_q(x) = \frac{1}{12}\langle \bar{q}(x)q(0)\rangle$$

where color and spin indices are omitted. In both expressions the color structure is given by the identity. The factor 1/12 in $\mathcal{C}_q(x)$ is chosen in a way that the expression is normalized by taking the trace of color and spin, i. e. $\mathrm{Tr}[\mathbb{1}_{\mathrm{spin}}] = 4$ and $\mathrm{Tr}[\mathbb{1}_{\mathrm{color}}] = 3$.

The general parametrization of the non-local condensates was suggested in Refs. [28, 29]:

$$\mathcal{C}_q(x) = \langle \bar{q}q\rangle \int_0^\infty d\nu\, e^{\nu x^2/4}\, f(\nu),$$

where $\langle \bar{q}q\rangle$ is local quark condensate and the shape of the distribution is determined by the function $f(\nu)$. Among the shape models suggested, the choice have been done in favor of the following one [31, 16]:

$$f(\nu) = \frac{\lambda^{a-2}}{\Gamma(a-2)}\, \nu^{1-a}\, e^{-\lambda/\nu}, \qquad a = 3 + \frac{4\lambda}{m_0^2}, \tag{2}$$

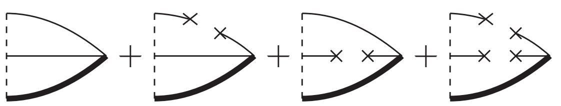
where $\lambda = \langle \bar{q}D^2 q\rangle$ is the correlation length and $m_0^2 = \langle \bar{q}g_s G^{\mu\nu}\sigma_{\mu\nu}q\rangle/\langle \bar{q}q\rangle$ is the ratio of the local mixed quark-gluon and quark condensates. If one assumes that virtualities of quarks inside the baryon are small and quarks are on the mass shell, the mixed quark-gluon condensate and the correlation length can be related (this is the usual procedure) but the smallness of such virtualities is not proven and, in general, the correlation length and the ration m_0^2 are independent.

To obtain the QCD sum rules, it is convenient to make the double Fourier transform of the correlation function:

$$\Pi_{\Gamma\Gamma'}(\omega_1, \omega_2; E) = i \int_{-\infty}^{\infty} \frac{dt_1\, dt_2}{(2\pi)^2}\, e^{i(\omega_1 t_1 + \omega_2 t_2)} \int d^4x\, e^{-iE(vx)}\, \langle 0|\mathcal{O}^\Gamma(t_1, t_2)\, \bar{J}^{\Gamma'}(x)|0\rangle$$

In the momentum space, the correlation function reads diagrammatically as follows:

$\Pi(\omega, u; E) =$



As it is well-known from the QCD-SR analysis within the HQET, the heavy-quark condensate term is suppressed by $1/m_Q$ and absent in the Heavy-Quark Symmetry limit. So, the QCD Sum Rules can be read off after the phenomenological and perturbatively calculated considerations of the correlation function are equated based on the idea of the quark-hadron duality [27]:

$$|f_H|^2\, \psi^\Gamma(\omega, u)\, e^{-\bar{\Lambda}_H/\tau} = \mathbb{B}[\Pi](\omega, u; \tau, s_0),$$

where symbol $\mathbb{B}$ means the Borel-transform, $\bar{\Lambda}_H = m_H - m_Q$ is the effective baryon mass in the HQET, and s_0 is the momentum cutoff resulting from applying the quark-hadron duality. The explicit QCD-SRs for all the baryonic non-local operators can be found in Ref. [9] and we illustrate them here by the one written for the leading-twist (twist-2) transverse LCDA:

$$\begin{aligned} f_H^{(2)} \left[A\, f_H^{(1)} + B\, f_H^{(2)} \right] \tilde{\psi}_2^{SR}(\omega, u)\, \mathrm{e}^{-\bar{\Lambda}/\tau} = \\ \frac{3\tau^4}{2\pi^4} \left[B\hat{\omega}^2\, u\bar{u} + A\,\hat{\omega}\,(\hat{m}_2 u + \hat{m}_1\bar{u}) \right] E_1(2\hat{s}_\omega)\mathrm{e}^{-\hat{\omega}} \\ - \frac{\langle \bar{q}_1 q_1 \rangle \tau^3}{\pi^2} \left[A\hat{\omega}\bar{u} + B\hat{m}_2 \right] f(2\tau\omega u)\, E_{2-a}(2\hat{s}_\kappa)\, \mathrm{e}^{-\hat{\omega}} \\ - \frac{\langle \bar{q}_2 q_2 \rangle \tau^3}{\pi^2} \left[A\hat{\omega} u + B\hat{m}_1 \right] f(2\tau\omega\bar{u})\, E_{2-a}(2\hat{s}_{\bar{\kappa}})\, \mathrm{e}^{-\hat{\omega}}. \end{aligned}$$

To simplify the presentation, the following auxiliary function was introduced:

$$E_a(x) = \frac{1}{\Gamma(a+1)} \int_0^x dt\, t^a \mathrm{e}^{-t} = 1 - \frac{\Gamma(a+1, x)}{\Gamma(a+1)}$$

where $\Gamma(a+1, x) = \int_x^\infty dt\, t^a \mathrm{e}^{-t}$ is the incomplete Γ-function. The other quantities are $\bar{\Lambda} = m_H - m_b$, $s_\omega = s_0 - \omega/2$, $\kappa = \lambda/(2u\omega\tau)$, $\bar{\kappa} = \lambda/(2\bar{u}\omega\tau)$, $\hat{\omega} = \omega/(2\tau)$, $\hat{s}_\omega = s_\omega/(2\tau)$, $\hat{m}_{1,2} = m_{1,2}/(2\tau)$, $\hat{s}_\kappa = \hat{s}_\omega - \kappa/2$, $\hat{s}_{\bar{\kappa}} = \hat{s}_\omega - \bar{\kappa}/2$. The normalizations of the symmetric LCDAs ($t = 2, 3s, 4$) can be fixed by the relation:

$$\int_0^{2s_0} \omega d\omega \int_0^1 du\, \tilde{\psi}_t^{\mathrm{SR}}(\omega, u) \equiv 1,$$

while the normalization of the antisymmetric LCDAs with $t = 3\sigma$ is different and can be fixed by the condition:

$$\int_0^{2s_0} \omega d\omega \int_0^1 du\, C_1^{1/2}(2u-1)\, \tilde{\psi}_t^{\mathrm{SR}}(\omega, u) \equiv 1,$$

where $C_n^m(x)$ are the Gegenbauer polynomials [32].

These QCD sum rules are not directly applicable for getting the LCDA shapes but can be used to constrain certain moments which are calculated based on the following definition:

$$\langle f(\omega, u) \rangle_k \equiv \int_0^{2s_0} \omega d\omega \int_0^1 du\, f(\omega, u)\, \tilde{\psi}_t^{\mathrm{SR}}(\omega, u)$$

where $t = 2, 3s, 3\sigma, 4$.

4 Numerical analysis

Numerical values of first several moments of the bottom-baryon LCDAs estimated by the QCD-SRs can be found in Ref. [9]. These moments should be matched to the corresponding moments of the model functions for the LCDAs. The general presentation of the model functions for the b-baryon LCDAs is governed by their scale evolution and can be composed of the exponential part corresponding to the heavy-light interaction and the Gegenbauer polynomials to the light-light interaction. The order of the polynomials is determined by the twist of the diquark system.

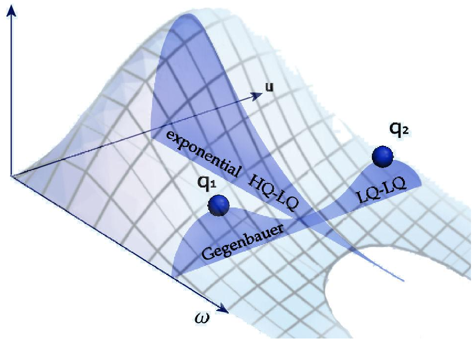


Figure 3: The general representation of the model functions for the heavy-baryon LCDAs with the ω-dependence specific for the B-meson LCDAs and the u-dependence in terms of an expansion in the Gegenbauer polynomials similar to the ones for the light mesons.

Motivated by the analysis done for the Λ_b-baryon [7], the following simple models for the LCDAs have proposed [8, 9]:

$$\begin{aligned}
\tilde{\psi}_2(\omega, u) &= \omega^2 u(1-u) \sum_{n=0}^{2} \frac{a_n^{(2)}}{\epsilon_n^{(2)\,4}} C_n^{3/2}(2u-1)\, \mathrm{e}^{-\omega/\epsilon_n^{(2)}}, \\
\tilde{\psi}_{3s}(\omega, u) &= \frac{\omega}{2} \sum_{n=0}^{2} \frac{a_n^{(3)}}{\epsilon_n^{(3)\,3}} C_n^{1/2}(2u-1)\, \mathrm{e}^{-\omega/\epsilon_n^{(3)}}, \\
\tilde{\psi}_{3\sigma}(\omega, u) &= \frac{\omega}{2} \sum_{n=0}^{3} \frac{b_n^{(3)}}{\eta_n^{(3)\,3}} C_n^{1/2}(2u-1)\, \mathrm{e}^{-\omega/\eta_n^{(3)}}, \\
\tilde{\psi}_4(\omega, u) &= \sum_{n=0}^{2} \frac{a_n^{(4)}}{\epsilon_n^{(4)\,2}} C_n^{1/2}(2u-1)\, \mathrm{e}^{-\omega/\epsilon_n^{(4)}}.
\end{aligned}$$

The qualitative behavior of the twist-2 LCDAs is presented in Fig. 3. The estimates of the parameters entering the theoretical models for the heavy-baryon LCDAs at the scale $\mu_0 = 1$ GeV can be found in Refs. [8, 9]. The dependence of the twist-2 LCDAs on the scaled energy u of the lightest quark and the diquark energy ω at the energy scales $\mu_0 = 1$ GeV are shown on the left and right plots in Fig. 4, respectively. The $SU(3)_F$-symmetry breaking in LCDAs based on taking into account the s-quark difference from the u- and d-quarks is clearly seen on these plots. The effect of the symmetry breaking is estimated to be approximately 15%.

5 Renormalization of higher twist operators

The renormalization of the heavy-light light-ray operators up to twist-three was performed in Ref. [33]. Here. both the $2 \to 2$ and $2 \to 3$ kernels were considered and the problem of the operator mixing under the renormalization has been discussed. To work out the evolution, the spinor formalism applied to QCD appears to be the most convenient. In addition, one-loop counterterms of the non-local operators were analyzed on an existence of the conformal

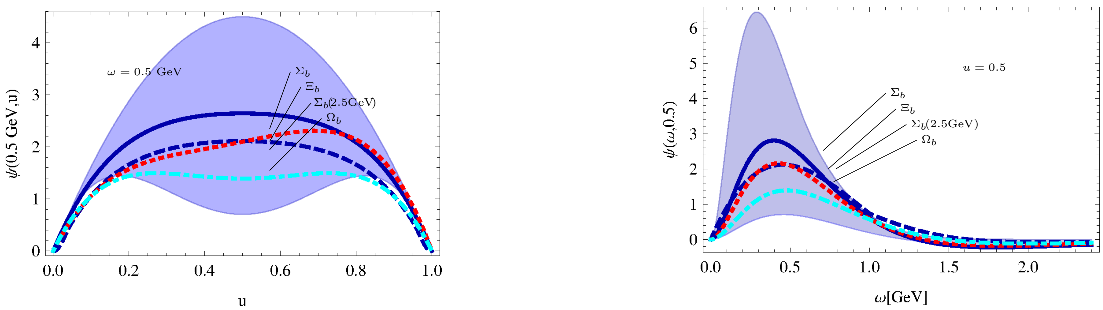


Figure 4: Twist-2 LCDAs of Σ (blue), Ξ (red) and Ω (cyan) baryons in dependence on the scaled energy u of the lightest quark (the left plot) and the diquark energy ω (the right plot) at the scale $\mu_0 = 1$ GeV estimated within the range for the most conservative error $A \in [0, 1]$.

symmetry and the main finding is that the ultra-violet renormalization of a cusp of two Wilson lines results the break down of this symmetry. As a technical output of this analysis, evolution equations for the twist-three operators were written explicitly.

The other step in working out solutions of the heavy-baryon evolution equations analytically was undertaken recently in Ref. [34]. In particular, the eigenfunctions of the Lange-Neubert evolution kernel were found and used for a systematic implementation of the renormalization-group effects for both the B-meson and Λ_b-baryon wave-function evolutions. Based on these foundations, the new strategy to construct the LCDA models in accordance with the Wandzura-Wilczek-like relations was presented. As a possible extension of the above analysis in application to baryons, the classification of the non-local baryonic operators constructed from four particles (three quarks and a gluon) is required to work out equations involving explicitly the there-particle LCDAs and the twist-four four-particle ones which should reduce to the Wandzura-Wilczek relations after four-particle LCDAs are neglected.

6 Conclusions

The total set of the non-local light-ray operators for the ground-state heavy baryons with $J^P = 1/2^+$ and $J^P = 3/2^+$ is constructed in QCD in the heavy-quark limit. Matrix elements of these operators sandwiched between the heavy-baryon state and vacuum determine the LCDAs of different twist through the diquark current. The first several moments of LCDAs are calculated within the method of QCD sum rules using the non-local light-quark condensates. Simple theoretical models for the LCDAs have been proposed and their parameters are fitted based on the QCD sum rules estimations. $SU(3)_F$ breaking effects result the correction of order 10%. The possibility to work out the LCDA evolution analytically is discussed.

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