$1/N_c$ corrections to the baryon axial vector current in large-N_c chiral perturbation theory

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DOI: http://dx.doi.org/10.3204/DESY-PROC-2014-04/23

We study the effects of the decuplet-octet mass difference for the baryon axial vector current at one-loop order in large-N_c baryon chiral perturbation theory, where N_c is the number of color. The baryon axial vector current is considered within the combined framework of large-N_c baryon chiral perturbation theory and the baryon axial vector couplings are extracted. We extend the g_A analysis by including all effects that are suppressed by $1/N_c^2$ relative to the tree level value, which includes taking into account the nonvanshing decuplet-octet mass difference.

1 Introduction

The generalization of quantum chromodynamics (QCD) from $N_c = 3$ to $N_c \gg 3$ color charges, called large-N_c QCD, has opened a path to substantial progress in understanding strong interactions at both the formal and phenomenological levels. Formal successes spring from the fact that large-N_c QCD exhibits a well-defined limit, meaning that the renormalization group equations remain finite and nontrivial as $N_c \to \infty$. Phenomenological successes build on these formal $1/N_c$ power-counting results, but add one extra ingredient: Observables calculated to appear at relative orders $\mathcal{O}(1/N_c)$, $\mathcal{O}(1/N_c^2)$, and so on, which is precisely the origin of the $1/N_c$ expansion [1].

In the large-N_c limit a spin-flavor symmetry emerges for baryons and this symmetry can be used to classify large-N_c baryon states and matrix elements [1, 2], which has led to remarkable insights into the understanding of the nonperturbative QCD dynamics of hadrons. Applications of this formalism to the computation of static properties of baryons range from masses, couplings to magnetic moments [3, 4], to name but a few. In particular, in this work we will describe the baryon axial-vector couplings, and as a result we obtain corrections at relative orders $\mathcal{O}(1/N_c)$ and $\mathcal{O}(1/N_c^2)$. This work is organized as follows. In Section 2, the renormalization of the baryon axial vector current is presented, and contains a detailed numerical analysis, our conclusions are presented in Section 3.

2 Renormalization of the baryon axial vector current

The baryon axial vector current A^{kc} is renormalized by the one-loop diagrams displayed in Fig. 1. These loop graphs have a calculable dependence on the ratio m_Π/Δ, where m_Π is the meson mass and $\Delta \equiv M_T - M_B$ is the decuplet-octet mass difference. The contribution from Fig. 1(a,b,c) contains the full dependence on the ratio Δ/m_Π and can be written as [5]

$$\begin{aligned} \delta A^{kc} &= \frac{1}{2}\left[A^{ja},\left[A^{jb},A^{kc}\right]\right]\Pi^{ab}_{(1)} - \frac{1}{2}\left\{A^{ja},\left[A^{kc},\left[\mathcal{M},A^{jb}\right]\right]\right\}\Pi^{ab}_{(2)} \\ &+ \frac{1}{6}\left(\left[A^{ja},\left[\left[\mathcal{M},\left[\mathcal{M},A^{jb}\right]\right],A^{kc}\right]\right] - \frac{1}{2}\left[\left[\mathcal{M},A^{ja}\right],\left[\left[\mathcal{M},A^{jb}\right],A^{kc}\right]\right]\right)\Pi^{ab}_{(3)} + \dots . \end{aligned} \quad (1)$$

The baryon axial vector current A^{kc} is a spin-1 object, an octet under $SU(3)$, and odd under time reversal. Its $1/N_c$ expansion reads

$$A^{kc} = a_1 G^{kc} + \sum_{n=2,3}^{N_c} b_n \frac{1}{N_c^{n-1}}\mathcal{D}_n^{kc} + \sum_{n=3,5}^{N_c} c_n \frac{1}{N_c^{n-1}}\mathcal{O}_n^{kc}, \quad (2)$$

where the unknown coefficients a_1, b_n, and c_n have expansions in powers of $1/N_c$ and are order unity at leading order in the $1/N_c$ expansion. At $N_c = 3$ the series (2) can be truncated as

$$A^{kc} = a_1 G^{kc} + b_2 \frac{1}{N_c}\mathcal{D}_2^{kc} + b_3 \frac{1}{N_c^2}\mathcal{D}_3^{kc} + c_3 \frac{1}{N_c^2}\mathcal{O}_3^{kc}. \quad (3)$$

The matrix elements of the space components of A^{kc} between SU(6) symmetric states yield the values of the axial vector couplings. For the octet baryons, the axial vector couplings are g_A, as defined in experiments in baryon semileptonic decays, normalized in such a way that $g_A \approx 1.27$ for neutron β decay. The other terms are spin-dependent and represent $\mathcal{M}_{hyperfine}$ introduced in the $1/N_c$ baryon chiral Lagrangian [5]

$$\mathcal{M}_{hyperfine} = \frac{m_2}{N_c} J^2. \quad (4)$$

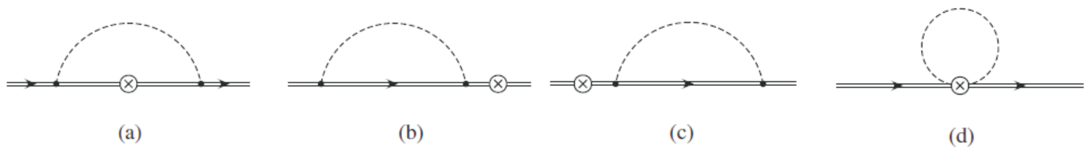

Figure 1: One loop corrections to the baryon axial vector current.

In Eq. (5), $\Pi^{ab}_{(n)}$ represents a symmetric tensor which contains meson loop integrals with the exchange of a single meson: A meson of flavor a is emitted and a meson of flavor b is reabsorbed. This tensor decomposes into flavor singlet **1**, flavor octet **8**, and flavor **27** representations as [5]

$$\Pi^{ab}_{(n)} = F_{\mathbf{1}}^{(n)}\delta^{ab} + F_{\mathbf{8}}^{(n)} d^{ab8} + F_{\mathbf{27}}^{(n)}\left[\delta^{a8}\delta^{b8} - \frac{1}{8}\delta^{ab} - \frac{3}{5}d^{ab8}d^{888}\right], \quad (5)$$

The función $F^{(n)}(m_\Pi, \Delta, \mu)$ along with its derivatives are given explicitly in [3]

$$F^{(n)}(m_\Pi, \Delta, \mu) \equiv \frac{\partial^n F(m_\Pi, \Delta, \mu)}{\partial \Delta^n}. \quad (6)$$

Operator products axial vector current and hyperfine mass	Orders $(1/N_c)$	Operator products SU(6) spin-flavor
AAA	$\mathcal{O}(1/N_c)$	$GGG, GG\mathcal{D}_2, G\mathcal{D}_2\mathcal{D}_2, GG\mathcal{D}_3$ and $GG\mathcal{O}_3$
	$\mathcal{O}(1/N_c^2)$	$\mathcal{D}_2\mathcal{D}_2\mathcal{D}_2, G\mathcal{D}_2\mathcal{D}_3$ and $G\mathcal{D}_2\mathcal{O}_3$
$AAA\mathcal{M}$	$\mathcal{O}(1/N_c)$	$GGGJ^2$ and $GG\mathcal{D}_2J^2$
	$\mathcal{O}(1/N_c^2)$	$GD_2D_2J^2$, $GG\mathcal{D}_3J^2$ and $GG\mathcal{O}_3J^2$
$AAA\mathcal{M}\mathcal{M}$	$\mathcal{O}(1/N_c)$	$GGGJ^2J^2$
	$\mathcal{O}(1/N_c^2)$	$GG\mathcal{D}_2J^2J^2$

Table 1: Relative orders $(1/N_c)$ to the operator products.

3 Results and Conclusions

In summary, we conclude that in the large-N_c limit, decuplet and octet baryon states become degenarate, the difference Δ between the SU(3) invariant masses of the decuplet and octet baryons given by $\Delta \equiv M_T - M_B \propto 1/N_c$.

The analysis was performed at one-loop order, where the corrections to the baryon axial vector coupling arise at relative orders $1/N_c$, $1/N_c^2$, and so on, which is precisely the origin of the $1/N_c$ expansion. The predicted values for g_A are listed in Table 1. Our final results referring to the degeneracy limit $\mathcal{O}(\Delta^0)$, and in the case of a nonvanishing decuplet-octet mass difference for both $\mathcal{O}(\Delta^1)$ and $\mathcal{O}(\Delta^2)$ have been analyzed in Ref. [3, 6]. In Table 1 shows the numerical values of the g_A axial vector couplings for various semileptonic processes in the $1/N_c$ expansion, individually for the flavor singlet **1**, octet **8**, and **27** contributions, the degeneracy limit (AAA), the leading ($AAA\mathcal{M}$), and the next-to-leading ($AAA\mathcal{M}\mathcal{M}$). The singlet corrections are $1/N_c$ suppressed with respect to the tree-level value. Subsequent suppressions of the octet and **27** contributions are also noticeable. The results are perfectly consistent both with the expectations from the $1/N_c$ expansion and the experimental data.

Acknowledgments

The author would like to express their gratitude to Local Organizing Committee of PANIC2014 also acknowledge support.

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Process	Total	Tree	**1**		**8**		**27**	
			Figures 1(a)-1(c), $\mathcal{O}(\Delta^0)$					
			$\mathcal{O}(\frac{1}{N_c})$	$\mathcal{O}(\frac{1}{N_c^2})$	$\mathcal{O}(\frac{1}{N_c})$	$\mathcal{O}(\frac{1}{N_c^2})$	$\mathcal{O}(\frac{1}{N_c})$	$\mathcal{O}(\frac{1}{N_c^2})$
$n \to pe^-\bar{\nu}_e$	1.275	1.121	−0.212	−0.338	0.079	0.292	0.004	−0.001
$\Sigma^\pm \to \Lambda e^\pm \nu_e$	0.629	0.745	−0.042	−0.321	−0.005	0.147	−0.002	0.000
$\Lambda \to pe^-\bar{\nu}_e$	−0.879	−0.628	0.175	0.133	−0.058	−0.063	0.000	0.003
$\Sigma^- \to ne^-\bar{\nu}_e$	0.340	0.704	0.442	−0.783	−0.037	0.022	−0.001	0.008
$\Xi^- \to \Lambda e^-\bar{\nu}_e$	0.361	−0.117	0.077	−0.023	−0.047	0.206	0.005	−0.019
$\Xi^- \to \Sigma^0 e^-\bar{\nu}_e$	0.820	0.793	−0.150	−0.239	−0.028	−0.104	0.004	0.008
$\Xi^0 \to \Sigma^+ e^-\bar{\nu}_e$	1.160	1.121	−0.212	−0.338	−0.041	−0.145	0.005	0.012

TABLE 2: Relative orders $1/N_c$ and $\mathcal{O}(\Delta^0)$ to the coupling constants g_A.

1		**8**		**27**	
Figures 1(a)-1(c), $\mathcal{O}(\Delta^1)$					
$\mathcal{O}(\frac{1}{N_c})$	$\mathcal{O}(\frac{1}{N_c^2})$	$\mathcal{O}(\frac{1}{N_c})$	$\mathcal{O}(\frac{1}{N_c^2})$	$\mathcal{O}(\frac{1}{N_c})$	$\mathcal{O}(\frac{1}{N_c^2})$
−0.103	0.464	0.048	−0.218	0.000	−0.002
−0.026	0.065	0.012	−0.052	0.000	0.001
0.084	−0.490	−0.018	0.138	0.000	0.002
−0.029	−0.239	−0.003	0.047	0.000	−0.001
−0.032	0.398	0.009	−0.050	0.000	0.003
−0.073	0.328	−0.017	0.077	0.000	−0.002
−0.105	0.466	−0.024	0.109	0.000	−0.003

Continuing, TABLE 2: Relative orders $1/N_c$ and $\mathcal{O}(\Delta^1)$ to the coupling constants g_A.

1		**8**		**27**		**1**	**8**	**27**
Figures 1(a)-1(c), $\mathcal{O}(\Delta^2)$						Figure 1(d)		
$\mathcal{O}(\frac{1}{N_c})$	$\mathcal{O}(\frac{1}{N_c^2})$	$\mathcal{O}(\frac{1}{N_c})$	$\mathcal{O}(\frac{1}{N_c^2})$	$\mathcal{O}(\frac{1}{N_c})$	$\mathcal{O}(\frac{1}{N_c^2})$	$\mathcal{O}(\frac{1}{N_c})$	$\mathcal{O}(\frac{1}{N_c})$	$\mathcal{O}(\frac{1}{N_c})$
0.002	−0.043	0.001	−0.023	0.000	0.000	0.303	−0.101	0.002
0.002	−0.023	−0.001	0.006	0.000	0.000	0.201	−0.067	0.001
0.000	0.030	0.000	0.008	0.000	0.000	−0.170	−0.028	0.004
−0.004	−0.006	0.000	0.002	0.000	0.000	0.190	0.032	−0.004
−0.001	−0.008	0.000	−0.005	0.000	0.001	−0.032	−0.005	0.001
0.001	−0.030	0.000	−0.008	0.000	−0.001	0.214	0.036	−0.005
0.002	−0.043	0.000	0.011	0.000	−0.001	0.303	0.050	−0.007

Continuing, TABLE 2: Relative orders $1/N_c$ and $\mathcal{O}(\Delta^2)$ to the coupling constants g_A.