THEORETICAL PREDICTION OF HEAD TAIL TUNE SHIFT IN PETRA

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Abstract

The shift in frequency of the fundamental m = 0 head tail mode in a storage ring can be calculated from the optics and the wake potentials. Such calculations have been performed for various beam pipe components in PETRA and are summarized here. Comparison with measurements shows good agreement for long bunches and that the theoretical description of the impedance is incomplete for short bunches (high frequencies).

List of Symbols

q	charge
$w_{\delta\perp}(s)$	wake potential of a point charge at distance s behind it
$w_{\lambda\perp}(s)$	wake potential inside a bunch of longitudinal line charge density $\lambda(s)$
$\lambda(s)$	line charge density
β	beta function at the location of the impedance
$k_\perp$	transverse kick parameter
E_0	nominal particle energy
$\Delta p_\perp$	transverse momentum
c	speed of light
f_z	vertical betatron frequency
f_x	horizontal betatron frequency

Introduction

The shift in oscillation frequency of the head tail bunch modes in a storage ring results from wake field forces excited in many components such as rf cavities, separator tanks and vacuum chamber joints. The wake field force, integrated over the transit time, is a wake potential that describes the distribution of momentum change inside a bunch of particles. If the beam offset from the ideal orbit is small, the transverse wake potential gives the transverse kick imported by one particle to a second one at distance s:

$$c \cdot \Delta p_\perp^{1\rightarrow 2} = q_1 \cdot r_1 \cdot w_{\lambda\perp}(s) \qquad s > 0$$
$$c \cdot \Delta p_\perp^{1\rightarrow 2} = 0 \qquad s < 0$$

The kick experienced by the test particle #2 does not depend on its own radial position (i.e. offset). Taking r_1 as the bunch offset and q_1 as the total charge in a bunch we obtain a kick distribution inside the bunch given by:

$$c \cdot \Delta p_\perp(s) = q_1 \cdot r_1 \cdot w_{\lambda\perp}(s)$$

The dipole moment of the total kick per unit offset r_1 and per unit charge q_1 inside a bunch with longitudinal density $\lambda(s)$ can be calculated as

$$k\ (\sigma) = \frac{\int_{-\infty}^{\infty} \lambda(x)\, w_\lambda\ (x)\, dx}{\left[\int_{-\infty}^{\infty} \lambda(x)\, dx \right]^2}$$

This "transverse kick parameter" $k_\perp$ (which is directly calculated by the TBCI program [2]) has its equivalent in the case of higher order mode losses where one uses the loss parameter k.
The betatron tune shift of the m = 0 mode due to quadrupole error is given by [1]:

$$\Delta Q = -\frac{1}{4\pi} \oint_{\text{circum-ference}} \Delta k(s)\, \beta(s)\, ds$$

Since the wake force behaves like a quadrupole ($\vec{F} \propto \vec{r}$), we can obtain the tune shift from the wake potential as [2]:

$$\Delta Q = -\frac{e}{4\pi E_0} < \beta > \int_{\text{bunch}} \lambda(x)\, w_{\lambda\perp}(x)\, dx$$

$$= -\frac{e}{4\pi E_0} < \beta > \cdot\, k_\perp \cdot\, q$$

For all objects of cylindrical symmetry we can use TBCI to obtain correct values [3] for the transverse kick parameters. For nonsymmetric objects we need a fully three-dimensional BCI program [5]. Since this program has nearly reached its final shape it is possible to compare partly results of 3D calculation with formely obtained results by having used cylindrical models of 3D objects.

Calculations and Measurements [4]

Figure 1 shows the layout of all cavity types used in PETRA while figure 2 shows the cross section of PETRA vacuum chamber joints. The vacuum chamber joints are modeled by circular ones having the chamber height as radius. Figure 2 also shows the quadrupole chamber joints and separator tanks in PETRA.
In table I we list the transverse kick parameter $k_\perp$ for each of these components at various bunch lengths (assuming Gaussian shape) obtained from TBCI calculations. In order to simplify the use of table I we give the formula for the tune shift in PETRA as:

$$\frac{\Delta f}{H_z} = \frac{0.01135}{(E_0/7\ \text{GeV})} \left(\frac{I_b}{\text{mA}}\right) \left(\frac{<\beta>}{\text{m}}\right) \left(\frac{k_\perp}{\text{V/pCm}}\right)$$

We will consider two examples and compare them with measurements.

Example 1: PETRA at injection energy with MI100 optics and
- 60 five cell cavities 500 MHz
- 232 vacuum chamber joints type E
- 48 vacuum chamber joints type F
- 8 separator tanks.

Example 2: More recently using the optics MI100 G

- 56 five cell cavities 500 MHz
- 56 seven cell cavities 500 MHz
- 16 six cell cavities 1000 MHz
- 8 seven cell cavities 1000 MHz
- 288 vacuum chamber joints type E
- 56 vacuum chamber joints type F
- 8 separator tanks

Results of these comparisons are shown in figures 3, 4 (example 1) and figures 5, 6 (example 2). The contribution to the tune shift of each component is shown separate.
The difference between measured and calculated vertical tune shift is shown for both optics in figure 7.

Table I: Transverse kick parameter of various components in PETRA

A: 5 cell 500 MHz cavity

σ/cm	1.0	1.5	2.0	3.0	4.0	5.0
K /(VpC/m)	13.4	15.0	15.7	15.3	14.6	13.3

B: 7 cell 500 MHz cavity

σ/cm	1.0	1.5	2.0	3.0	4.0	5.0
K /(VpC/m)	37.0	38.2	39.1	37.3	33.2	28.7

C: 6 cell 1000 MHz cavity

σ/cm	1.0	1.5	2.0	3.0	4.0	5.0
K /(VpC/m)	19.6	21.1	21.5	20.2	17.1	13.8

D: 7 cell 1000 MHz cavity

σ/cm	1.0	1.5	2.0	3.0	4.0	5.0
K /(VpC/m)	10.9	12.1	12.8	12.6	11.1	9.2

E: Vacuum chamber joint in arc (vertical)

σ/cm	1.0	1.5	2.0	3.0	4.0	5.0
K /(VpC/m)	1.91	1.29	0.95	0.62	0.45	0.36

F: Vacuum chamber joint in straight section (vertical and horizontal)

σ/cm	1.0	1.5	2.0	3.0	4.0	5.0
K /(VpC/m)	1.14	0.83	0.65	0.44	0.33	0.26

G: Separator tank (vertical)

σ/cm	1.0	1.5	2.0	3.0	4.0	5.0
K /(VpC/m)	12.9	12.7	12.0	10.3	8.6	7.2

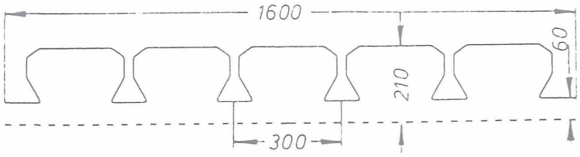
Conclusions

We find a clear discrepancy between measured and predicted tune shifts especially for short bunches (δ = 1 cm). For long bunches, however, we find good agreement and also no significant discrepancy between the tune shift in both planes, horizontal or vertical.

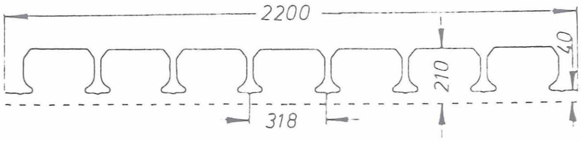
It follows that for long bunches the symmetric impedance is dominated by the cavities and that predictions are quite accurate.

For short bunches the impedance is dominated by vacuum chamber joints and other unidentified components outside the cavity region.

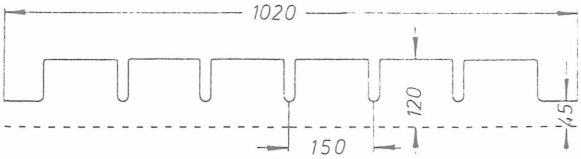
In order to complete our theoretical knowledge of this complex system we have to substitute the cylindrical models of these components by 3D calculations (which show higher contributions to the impedance for short bunches than predicted by TBCI calculation) and investigate other sources of non-geometric impedance such as aluminium oxide in the vacuum system [6].



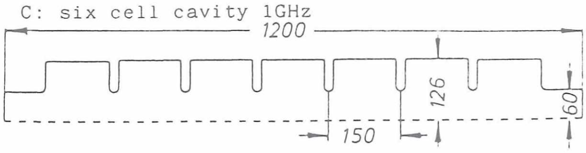
A: five cell cavity 500 MHz



B: seven cell cavity 500 MHz

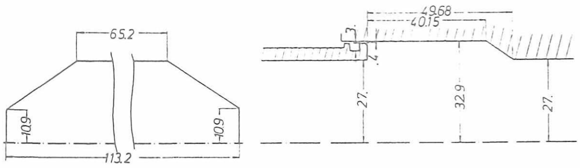


C: six cell cavity 1GHz

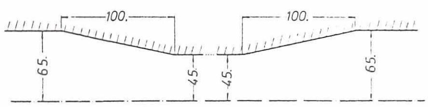


D: seven cell cavity 1 GHz

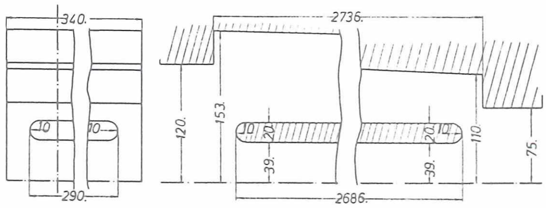
Figure 1: cavities used at PETRA



E: vacuum chamber joint, bending magnet (the left part shows a cross section ot the vacuum chamber)



F: vacuum chamber joint, quadrupole magnet



G: separator tank

Figure 2: vacuum chamber joints and separator tanks used at PETRA

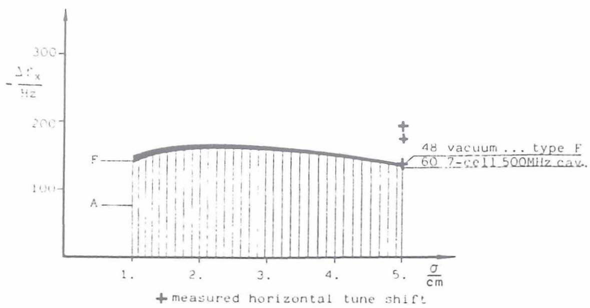


Figure 3: calculated and measured horizontal tune shift for MI100 optics in PETRA

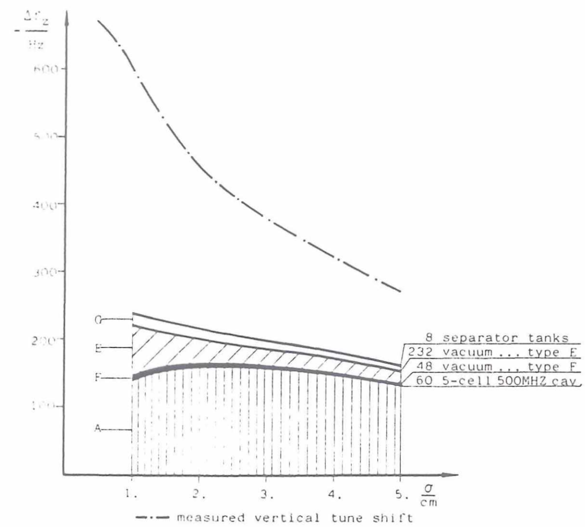


Figure 4: calculated and measured vertical tune shift for MI100 optics in PETRA

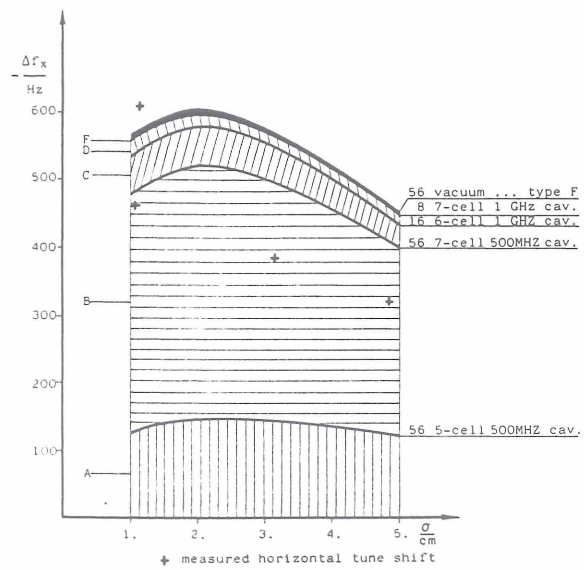


Figure 5: calculated and measured horizontal tune shift for MI100G optics in PETRA

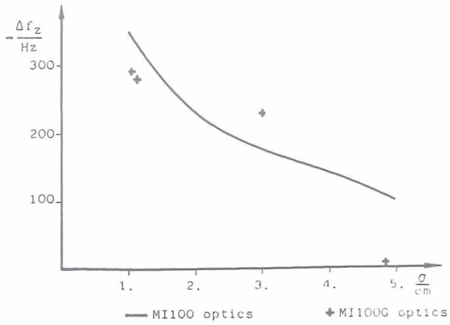


Figure 7: difference between measured and calculated vertical tune shift

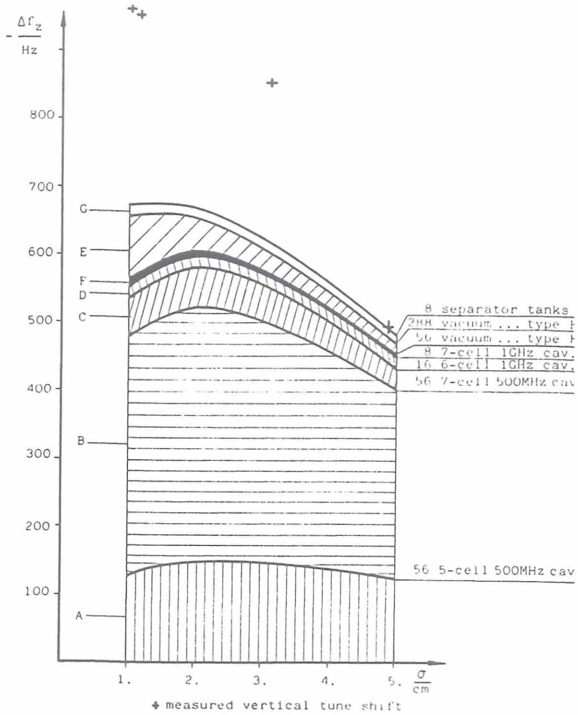


Figure 6: calculated and measured vertical tune shift for MI100G optics in PETRA

Literature

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[5] T. Weiland, Three-dimensional BCI program, not yet published

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