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TRANSVERSE INSTABILITIES IN PETRA

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ABSTRACT

For a long time currents in PETRA were limited by a vertical single bunch instability. These current limitations have now been overcome by artificial bunch lengthening. Single bunch currents of 20 mA were stably stored in a machine configuration with 60 accelerating cavities installed. A theory of "head-tail-turbulence" for the observed instability was developed based on the coupling of higher head-tail modes.

THRESHOLD BEHAVIOUR AND INSTABILITY

The instability occurred above the threshold currents of about 6 mA, leading to a vertical blow-up observed on the synchrotron light TV monitors. Although the sensitivity of the pick-up loops in the ring was sufficiently large, no signals were detected indicating dipole oscillations[1].

After the beam has blown up, the vertical aperture limit is reached and the injection stops. Therefore the maximum current is nearly identical with the threshold current. The threshold current of a single bunch is independent of the number of bunches in the ring even if adjacent buckets are involved.

INSTABILITY AND MACHINE PARAMETERS

Because of the absence of dipole signals, the transverse F.B. system has no influence on the PETRA instability. The instability occurs even at vanishing vertical (horizontal) chromaticity. There is no strong dependence on this quantity but the instability is somewhat weaker at negative chromaticities.

By widening the horizontal beam size one can increase the vertical Landau damping. This has an essential influence on the fundamental head-tail mode at negative chromaticities. But such a change of Landau damping has little effect on the instability under discussion.

When a special injection optics was developed, 18 mA were stored at 6 GeV with 16 rf cavities installed, still limited by the vertical blow-up. Preparing this special optics we changed the vertical β-function at the interaction point from 20 cm to 100 cm, so that the vertical β-function in the adjacent sections was considerably reduced. When 32 rf cavities were installed, the maximum current was limited at 9 mA under the same optical conditions.

Studying the optical influence on the PETRA instability in detail, we observed that the vertical β-function in the rf sections had an essential influence on the instability. For two special optics we found the following result

average β-function in rf section	maximum single bunch current
50 m	6 mA
25 m	12 mA

After the vertical β-function in the rf sections was further reduced for the injection optics and the injection energy was changed from 6 GeV to 7 GeV 15 mA were stored in a single bunch, still limited by the vertical blow-up[2].

When nearly the total number of 60 rf cavities was installed, the single bunch current dropped to values around 4 mA. These currents increased nearly linearly with the synchrotron frequency, and 7mA at 18 kHz were reached.

At this time we were able to increase the bunch length by a change of the dampiing partition, operating at a higher radio frequency. Additional special vertical beam bumps were applied[3] leading to a further change of the damping partition, so that the bunch length was increased by more than a factor 3. By these manipulations the vertical blow-up was completely removed, and 20 mA were stored in a single bunch at 7 GeV with 60 rf cavities installed.

One of the most interesting questions during the experimental studies was to understand what is the driving mechanism of the observed instability. The conclusion drawn from the observations form "boundary" conditions for a model of the driving mechanism:

1. the instability is a single bunch instability in the vertical plane
2. the instability occurs even at zero chromaticity (and at positive and negative chromaticities)
3. no dipole oscillations are excited
4. the strength of the instability depends on the number of rf cavities and on the β-function in this area
5. Landau damping plays no important role
6. the instability is weaker at higher synchrotron frequencies
7. the instability is weaker at a larger bunch

MODEL OF "HEAD-TAIL TURBULENCE"

In the case of vanishing chromaticity the complex frequency shift of a single head-tail mode can be written as follows

$$\Delta\omega_m \sim -iI \sum_{q>0} \left\{ Z_\perp(q+\delta Q) - Z_\perp^*(q-\delta Q) \right\} \left| h_m(q) \right|^2 \qquad (1)$$

I ≙ bunch current
m ≙ mode number
$h_m(q)$ ≙ spectrum function
δQ ≙ fractional part of transverse tune
$Z_\perp$ ≙ transverse impedance

If the width of the broad-band impedance $Z_\perp$ is large as compared to the fractional part of the transverse tune the expression in the curly bracket of

(1) is small (Robinson effect). This behaviour of single bunch instabilities is connected with the properties of single bunch modes. In the frequency domain a single bunch mode is characterized by a standing wave composed of travelling waves travelling in opposite directions[4].

$$\text{mode} \langle m \rangle \leftrightarrow \sum_q \left\{ h_m(q) e^{iq\theta - i\Omega t} + h_m(-q) e^{-iq\theta - i\Omega t} \right\} \qquad (2)$$

$\theta \hat{=}$ longitudinal coordinate

$\Omega \hat{=}$ betatron frequency

These two travelling waves have opposite stability properties, so that the standing wave combination of a single bunch mode has indifferent stability. It is possible to recombine unstable travelling waves from single bunch modes

$$\alpha \langle m \rangle + \beta \langle m' \rangle \rightarrow \sum_q \tilde{h}(q) e^{iq\theta - i\Omega t} \qquad (3)$$

Since adjacent mode frequencies differ by the synchrotron frequency f_s this combination will be destroyed in a time of the order of the synchrotron period.

However, if the growth rate of travelling waves exceeds the synchrotron frequency an instability can arise. Applying the mathematical frame work of head tail mode coupling[5] the growth rate of the "head tail turbulence" instability can be written as

$$\delta = \frac{1}{2} \sqrt{4 |M(J)|^2 - \omega_s^2} \;, \quad |M| > \omega_s \qquad (4)$$

$M(I) \hat{=}$ matrive element proportional to bunch current and transverse impedance

$\omega_s = 2\pi \times$ synchrotron frequency

Since the transverse impedance is roughly determined by the longitudinal impedance the growth rate increases with the higher order mode losses. For strong focussing $M(I)$ is proportional to the β-function at the place of the driving objects. If the maximum of the transverse impedance is assumed to be above 1.3 GHz, the coupling of the first and second higher head tail modes is dominant, so that no dipole oscillations are excited.

The growth rate (4) is described by the difference of two large frequencies. Therefore the Landau damping, small as compared to the synchrotron frequency, plays no important role.

From the head tail turbulence model one can derive the following scaling law for the threshold current

$$J \sim \beta_z^{-1.4} \; Q_s^{0.7} \; \gamma^{1.4} \; \sigma_E^{0.5} \; 1/n \qquad (5)$$

$Q_s \hat{=}$ longitudinal time

σ_E ≙ relative energy spread
γ ≙ energy in rest mass units
n ≙ number of rf cavities

The observations of the PETRA instability seem to be at least in qualitative agreement with these predictions.

REFERENCES

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