TRACKING STUDIES IN HERA

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INTRODUCTION:

HERA is a large electron proton colliding beam machine. 820 GeV protons are collided with 30 GeV electrons in 4 interaction points in the middle of 4 straight sections. The circumference of the ring is 6336 m, just 2 3/4 times the length of PETRA. As can be seen at the topographic view of Fig.1, HERA is located on a site adjoining the DESY site. The tunnel is lying 10 to 20 m under ground. In the arcs the diameter of the tunnel is 3.2 m and the proton ring is above the electron ring (Fig.2).

In the straight sections the p-ring is bend to the level of the e-ring as can be seen on the side view of the interaction region in Fig.3.

The arcs are built of a FODO structure (Fig.4), in the present status of total 104 proton cells and 312 electron cells. They are not all normal cells, some of them are used for dispersion supression. The proton cell has 3 bending magnets per half cell and beside each quadrupole there is a correction package with steering coils, chromaticity correcting sextupoles and quadrupole and sextupole correction elements. There are still other cells in discussion. For instance a longer cell with 4 bending magnets per half cell and cells, where the correction elements are integrated in the adjacent bending magnets.

COMPUTER CODE RACETRACK:

The RACETRACK computer program was used for the investigations. It is a combination of a linear and nonlinear program, which input parameters are the positions, lengths and strengths of linear lattice elements, which are composed into a block structure
consisting of a minimum number of different blocks, where each block is represented by its transformation matrix only. The nonlinear elements are treated in thin lens approximation. Constant energy tracking or tracking with synchrotron oscillations can be performed. In the later case, one or more positions in the structure can be defined as cavity positions. The energy can be changed following a harmonic variation or according to the nonlinear synchrotron motion. After each energy change the energy dependent single element matrices are recalculated and the composition into transformation blocks is repeated (this is a very fast operation compared to the revolution time).

Preparatory calculations: Each tracking is preceded by preparatory calculations which can be activated by introducing their data or a switch in the input stream.

- the closed orbit displacements at the starting point are calculated in an iterative way, for all nonlinear elements included
- random dipole errors can be scaled to give desired orbit rms deviations, measured at positions which can be defined
- the so produced orbit distortions can be corrected using a correction routine
- beta and alfa values are calculated at the starting point for the optics along the closed orbit (that means the quadrupole contributions due to orbit errors are included)
- the linear optics can be calculated (this is very helpful for input control and error detection if one starts with a new lattice)
- the working points can be tuned to desired values by using 2 quadrupole families or combinations of that
- the chromaticities can be adjusted to desired values by tuning 2 sextupole circuits
- the nonlinear variation of tune with amplitude and energy can be calculated and plotted.
Initial coordinates: After these preparatory calculations the maximum stable amplitude for one single particle over one revolution is found by the program and used as starting amplitude.

There are several possibilities to set the initial coordinates. In most the cases for which results are shown here, the following arrangement was used:

\[ A_{\text{init}} = \sqrt{E \beta_{\text{orb}}} \]

In both phase planes the particles are equally distributed on the phase ellipses with areas proportional to the squared initial amplitude, and each trajectory vector of the horizontal plane is combined with every vector of the vertical plane. Thus the total number of tracked particles was given by the product of the trajectory coordinates of each plane (for pure sextupole tracking only the upper half of the vertical phase ellipse must be filled, since there is a mirror symmetry of the motion with respect to the median plane).

The coupling, or emittance ratio was kept constant for all particles.

Stability: An ensemble of particles, assigned by the horizontal amplitude was found to be stable, if all particles over all revolutions remained within the physical aperture. The aperture is represented in the program by aperture limiting insertions. They can be placed at any point in the structure and may also have different aperture values:

rectangular: \[ x_i < A_{px} \land z_i < A_{pz} \]

round: \[ x_i^2 + z_i^2 < R^2 \]
For the results shown here, at each chromaticity correcting sextupole in the regular cell structure the aperture property was checked for all particles. If at least one particle hit the wall, the amplitude was called unstable. To find the maximum value, the amplitude of a stable ensemble was incremented, of an unstable ensemble decremented and the transformation started again. The total number of such steps is defining the accuracy of the resulting maximum stable amplitude and the computer time.

HERA TRACKING PARAMETERS:

The acceptance of the HERA proton ring will mainly be influenced by
- normal chromaticity correcting sextupoles
- field errors in superconducting magnets
- persistent current sextupoles in the superconducting magnets at low energies

Chromaticity correcting sextupoles: - were placed at any position where the betavalue are sufficiently decoupled and the dispersion is large enough. Only two sextupole families were used for chromaticity compensation. Since the energy spread of the beam is rather small ($\pm 1.6\%$), it may not be necessary to compensate energy dependent nonlinear effects by using more families.

Field errors: - and persistent sextupole strengths were taken from the FERMILAB magnets:

\[
\pm 2.5 \times 10^{-4} \text{ for normal and skew multipoles}
\pm 6.0 \times 10^{-4} \text{ for normal sextupoles}
\pm 2.0 \times 10^{-4} \text{ for skew sextupoles and all higher multipoles up to } 18\text{-poles}
\]

(These values are the relative strengths referred to normal dipole strength at 1 inch)

For the simulation one third of these maximum field errors was taken as rms value of a gaussian distribution.
**Persistent sextupole:** For 40 GeV, the injection energy of HERA, the persistent sextupole is

\[-2,0 \times 10^{-3}\]

To avoid problems we decided to compensate the average contribution of persistent sextupole by a commonly powered coil in each magnet. Only a random fluctuation, which was assumed to be 10% of the normal strength was taken as rms value.

**Aperture limitation:** Only aperture limitations in the regular part of the normal cell structure were taken into account. Outside the normal cell the aperture will be made large enough to follow the beam size. The horizontal and vertical half aperture assumed here was 30 mm.

**HERA TRACKING RESULTS:**

To get a feeling how the two nonlinearities, the normal cell sextupoles and the multipole errors in the bending magnets superimpose, the pure sextupole case was treated separately. For the on energy case the influence of small tune changes on acceptance was investigated to find the optimum tune. Then for the optimum tune the maximum stable initial amplitude was calculated for different constant momentum deviations. Synchrotron oscillations were not taken into account. This was a question of available computer time. Since \(Q_s\) is very small (0.008) and you need at least a few oscillation periods, the number of revolutions would increase to several hundreds compared to hundred used here. Synchrotron oscillations would increase the beam size, but since the change of optics within the energy spread of the beam is small, they would not affect the results very much.

The effect of synchrotron oscillations can be roughly estimated, at least those effects, which may influence the short time tracking results.
Due to the energy change in the cavity, the twiss parameters are changed. Also in the linear case without sextupoles and field errors you will get a growth of beam size due to the synchrotron oscillations.

This effect can also produce satellite resonances spaced by the half synchrotron frequency around each half integer resonance. But for HERA $Q_s$ is very small and we are far away from these resonances. Thus the maximum beta beat must be a measure for the beam growth due to synchrotron oscillations. In HERA the maximum beta variation is about 10% for the maximum energy deviation in the beam. Since the emittance growth is approximately given by

$$\tilde{\varepsilon} \approx \varepsilon_0 \left(1 + \frac{\Delta \beta}{\beta} \right)$$

and the amplitude is proportional to the square root of the emittance, all calculated amplitudes should be corrected, that means reduced by about 5%.

The influence of small tune changes on the dynamical aperture is shown in Fig.5. The height of the mountain range is proportional to the maximum stable initial amplitude at the starting point, which is a horizontal focusing quadrupole in the normal cell.

The decreasing amplitude at the right hand side comes from the intrinsic 3rd integer sextupole resonance $3Q_H=100$. Sho Ohnuma calculated the driving terms and found, that they are indeed very strong. By eliminating sextupoles in the straight sections he could reduce it by a factor of 4.
For the optimum tunes of this rough optimization the maximum stable initial amplitude was calculated for different constant energy deviations. The results are shown in Fig. 6. The amplitudes resulting from the calculation were scaled by the square root of the undistorted on momentum beta value divided by the actual beta. Thus the stable amplitude shown in the pictures is always proportional to the square root of the maximum stable emittance.

The beam size is defined for 2.2 sigmas according to a measurement at the ISR, where the aperture was limited to this value and the lifetime was still 18.5 hrs.

The number of revolutions used for these calculations was 100. Since the increase of stable revolutions near the stability limit of amplitude is a very steep function, as shown in Fig. 7, this seems to be a reasonable number. An increase of revolutions would not considerably improve the accuracy of the results. The curve in Fig. 7 is not always such nonlinear, especially if field errors are introduced. But even then 100 revolutions seem to be more than sufficient.

So far only sextupoles have been considered. Multipole errors were introduced in the following way:

For an arbitrary set of random multipoles the same tune optimization as before was performed. Then for the optimum values 20 different sets of fluctuations were calculated and one set out of the central part of the resulting distribution was used for the subsequent calculations. These results are shown in Fig. 8A.

Orbit distortions of 1 mm rms value measured at positions of the normal cell sextupoles are included. This seems to be a reasonable number after orbit correction. From the experiences with PETRA we know, that rms values of 0.5 mm can be normally achieved.

In these calculations for each run the tunes were always re-
adjusted to their original values and the dipole errors were
scaled to give 1 mm rms orbit error.
In Fig. 8B the distribution for a "bad tune" (which was optimal
for the pure sextupole case) is shown. Without tune optimization
before, the range of stable amplitude for different statistics
is increased.

For a random fluctuation out of the central region of the
distribution in Fig. 8A, the momentum dependence of the maximum
stable initial amplitude is shown in Fig. 9.
The single point at zero momentum deviation shows the result
for multipoles only and chromaticity correcting sextupoles
switched off (this can only be done for on momentum, since the
tune shifts due to chromatical effects are large). Thus the
effects of sextupoles and multipoles on dynamical aperture are
more or less comparable.

SAGITTA:
As mentioned before, a half aperture of 30 mm was assumed in
the regular cell structure. But for straight magnets there is
a sagitta effect.
The total sagitta of a straight magnet is given by

\[ 2d = \frac{l^2}{8g} \]

By appropriate horizontal location of the dipoles, the sagitta
can be divided into two equal parts d, which for the HERA magnets
are about 3.75 mm.
Taking the sagitta into account, the sextupole variation along
the magnet, although its integral value is zero, is not
negligable anymore. For the tracking the uniform field inside the
magnet was substituted by a thin lense in the middle. An aperture
reduction was taken into account at the ends and in the middle
of the magnet.
The sextupole variation combined with the off centered orbit leads predominately to a quadrupole distortion. The tune shifts produced by this distortion are

\[ Q_H = -0.36 \]
\[ Q_V = 0.40 \]

They were tuned back to their original values by using QF and QD of the normal cells.

Fig. 10 shows the maximum stable amplitudes with sagitta effects included. Only sextupoles are considered there. For comparison also the results without sagitta are drawn, corresponding to the dotted line.

In Fig. 11 multipoles and orbit distortions are included. The difference between sagitta and no sagitta is smaller as compared to the pure sextupole case. The increased nonlinearities comming from the multipoles are responsible for that. The particle loss is stronger influenced by the nonlinear motion. This can be demonstrated by the curves shown in Fig. 12 (they correspond to another optics with different tunes, but they can be used for showing the principle effect). For sextupoles only, corres-
ponding to the upper curve, a reduction in aperture would simultaneously lead to a decrease in stable amplitude. For multipoles included, this case is shown in the lower curve, the particles are mainly lost at nonlinear fields and a reduction in aperture would not decrease the stable amplitude, or at least not so much.

It is interesting to note, that the systematic shift of the multipole distribution does not affect the nonlinear acceptance very much.

**COMPARISON OF 60° AND 90° CELL PHASE ADVANCE**

The sextupole strengths for chromaticity correction, using a compensation with two families only, is given by

\[ M_F (m^0) \sim S_F \cdot L_s = \frac{1}{D} \left\{ \frac{1}{4} + \frac{\ell T R}{N_c (\beta - \beta_0)} \right\} \]

- \( f \) ... focal length
- \( D \) ... dispersion in the normal cell
- \( N_c \) ... total number of cells
- \( \ell T R \) ... chromaticity of interaction regions
- \( \beta \) ... betavalues in the cell

For decreasing phase advance in the cell the focal length is increasing according to the following expression:

\[ \sin(\phi/2) = \frac{L_c}{4f} \]

\( L_c \) ... cell length

That means the denominator of the first summand in the bracket is increased. Since within the range of consideration the beta beat in the cell is also proportional to the focal length, and the dispersion is proportional to the squared focal length, the sextupole strength is reduced with the the 3rd power of focal length, going to smaller phase advances. And the sextupole strength is at least partly responsible for the reduction in nonlinear aperture.
On the other hand the beam size at the maximum beta is increased, but for the phase change from 90° to 60° it is approximately constant. Thus you would clearly expect an improvement for the 60° cell concerning the nonlinear acceptance.

When these calculations were started, a new discussion about the emittance ratio came up. So far we assumed 50% according to a measurement at the SPS. But since the emittance depends strongly on the injection history of the beam, which cannot be sufficiently estimated in the present stage, it is may be more realistic to take 100% (this also corresponds to the value they have at FERMILAB).

Then in the meantime we also got estimations for the mean values of 14- and 18-pole from magnet calculations, which have been included in the new calculations:

\[ b_7 = -2.9 \times 10^{-4} \]
\[ b_9 = -4.8 \times 10^{-4} \]

So the results are not directly comparable with the previous calculations.

In Fig.13 the maximum stable amplitude is drawn as a function of the free aperture. As can be seen, the 60° cell is clearly better, but only if there is no physical aperture limitation. At the HERA aperture the improvement is rather small.

Also for the 60° cell a similar optimization procedure as before for the 90° cell has been performed for the tune. The result is shown in Fig.14. You can clearly see the influence of the coupling resonance, which is a 4th integer, second order resonance coming from the nonlinear motion due to sextupoles. Also the structure resonance \( 3Q_H = 76 \) and a rather small influence of the intrinsic sextupole resonance \( 2Q_V + Q_H = 80 \) is visible.
For the optimum tunes the maximum stable initial amplitudes are shown in Fig. 15. Multipoles and orbit errors of 1 mm rms values are included. The solid lines are the stable amplitudes for 60° and 90° phase advance. As can be seen, for momentum deviations the 60° call is even worse as a result of the larger maximum dispersion in the cell, which is increased from 1.8 to 3.5 m.

Elliptical beam: The assumption of a rectangular beam used for all previous calculations is rather pessimistic. While for the rectangular beam the particles were equally distributed on the two phase ellipses and the emittance- or amplitude ratio was kept constant for all particles, for a four dimensional gaussian distributed beam the amplitudes are varying according to an ellipse:

\[
\begin{align*}
X^2 + (x\alpha_\lambda + x\beta_\lambda)^2 &= \varepsilon_x \beta_x \cos^2 \delta = A_x^2 \\
2^2 + (x\alpha_\lambda + x\beta_\lambda)^2 &= \varepsilon_t \beta_t \sin^2 \delta = A_t^2
\end{align*}
\]

Using an elliptical beam, the maximum stable amplitudes are increased, as shown by the dotted lines in Fig. 15.

INFLUENCE OF MEAN VALUES OF 14- AND 18-POLES:

After changing the coupling and introducing mean values for 14- and 18-poles the nonlinear acceptance was considerably reduced. To analyze how these two effects were adding up, they were investigated separately.

Fig. 16 shows the influence of mean values of 14- and 18-pole
on stable amplitude. All other nonlinearities are switched off. The solid lines correspond to calculations without aperture limitations and the dotted line (only the 14-pole is shown) corresponds to the nominal value of 30 mm half aperture. It was surprising to find, that for 30 mm half aperture and a 14-pole of $4 \times 10^{-4}$, the maximum amplitude is already limited by the nonlinear field (i.e. it is independent of the aperture). For the mean values assumed for the HERA magnet (which are of course preliminary),

$$b_7 = -2.9 \times 10^{-4}$$
$$b_9 = -4.8 \times 10^{-4}$$

(relative strengths referred to the normal dipole strength at 1 inch) the stable amplitude is far above the amplitude limited by all other effects, so they don't contribute to the nonlinear limitation. The situation may become critical for values around $2 \times 10^{-3}$.

Fig.17 shows the variation of stable amplitude with the coupling. It can be seen, that in fact the reduction in stable amplitude compared to previous calculations was a result of the increased emittance ratio. The amplitude is considerabely reduced going from 50% to 100% coupling.

**PERSISTENT CURRENT Sextupole**

How is the stable amplitude influenced, if the chromaticity produced by the persistent sextupoles in the bending magnets is compensated by the normal lattice sextupoles. The relative strength of the persistent sextupole at 40 GeV is $-2.10^{-3}$.

The chromaticities produced by these sextupoles and the natural chromaticities are:

- **horizontally:** $\xi^h = -181$  \hspace{1cm} \text{natural:}  \hspace{1cm} \xi^h = -65$
- **vertically:** $\xi^v = +143$  \hspace{1cm} \text{natural:}  \hspace{1cm} \xi^v = -89$
If they are compensated by the normal lattice sextupoles the strengths are changed

   horizontally from $-0.069$ to $-0.1867 \ [m^2]$
   vertically from $+0.169$ to $-0.0066$

Both sextupoles got the same sign, since the vertical chromaticity is now overcorrected by the persistent sextupole.

The maximum stable initial amplitudes for this arrangement are shown in Fig.18. the reduction in amplitude is not dramatic, but since we can avoid it and keep the safety region large, we still believe to compensate the average contribution of persistent sextupole by a commonly powered line.
DESY site

Fig. 1
Cross section of the HERA tunnel (in the arc)
Fig. 3

Layout of the interaction region
Bending angle $\alpha = \frac{2\pi}{624} = 1.007 \times 10^{-2}$

Lattice for the electron and proton ring
**HERA 90 - tune variation**

Maximum stable initial amplitude vs. tune

Rectangular beam
Sixth poles only \( E_{uv} \approx 0 \)
Emittance ratio \( K = 0.5 \)
Energy deviation \( \Delta \rho / \rho = 0 \)

\( Q_u = 33 + \frac{dQ_u}{Q_u} = 35 + \frac{dQ_v}{Q_v} \)

\( N_{rev} = 100 \)

*Fig. 5*
**HERA90-energy variation**

Maximum stable initial amplitude vs. energy deviation

![Graph with axes and markers]

Rectangle beam sextupoles only \( \hat{\varepsilon}_{H,V} = 0 \)

Emittance ratio \( K = 0.5 \)

\[ Q_H = 33.15 \quad Q_V = 35.18 \]

Nev = 100

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**Definitions:**

Maximum stable initial amplitude \( \hat{A} = \hat{a} \sqrt{\frac{\beta_H (\hat{y} = 0)}{\beta_H (\hat{y} = 0)}} \)

Beam size at 40 GeV

Beam size at 820 Gev

Beam size = 2.2091 \( \times \) total energy spread

40 GeV: \( \delta_T = 0.1 \quad (\Delta \gamma)_{tot} = \pm 1.6 \% \)

820 GeV: \( \delta_T = 0.0047 \quad (\Delta \gamma)_{tot} = \pm 1.4 \% \)

**Fig. 6**
HERA90 - number of stable revolutions vs. initial amplitude

rectangular beam
sextupoles only $\xi_4 = 0$
emittance ratio $K = 0.5$

$Q_H = 33.15 / Q_v = 35.08$
energy deviation $4\pi / \beta = 0$

$N_{rev}$ number of stable revolutions

$A_i$ [mm] starting amplitude

Fig. 7
HERA90 – Variation of maximum stable amplitude with multipole statistics

rectangular beam
sextupoles and multipoles $f_{U1V1} = 0$
with closed orbit errors $C_{0.1rms} = 1mm$
emittance ratio $K = 0.5$
energy deviation $\Delta p/p = 0$
$N_{cu} = 100$

A: $Q_H = 33.15/Q_v = 35.18$

$(R\Delta)^o = \text{random multipole set selected for proceeding calculations}$

B: $Q_H = 33.15/Q_v = 35.08$

Fig. 8
HERA90-energy variation

Maximum stable initial amplitude vs. energy deviation

Rectangular beam sextupoles and multipoles emittance ratio $K = 0.5$

$Q_{x} = 33.15 / Q_{y} = 33.18$

$N_{bev} = 100$

Orbit rms-error = 1 mm

$\Delta \theta [\%]$
HERA90 - energy variation with sagitta

Maximum stable amplitude vs. energy deviation

Rectangular beam

Emittance ratio $K = 0.5$

$Q_x = 33.15, Q_y = 35.08$

$N_v = 160$

Beam size at 40 GeV

Figure 10
HERA90-energy variation with sagitta

Maximum stable amplitude
vs. energy deviation

rectangular beam
 sextupoles and multipoles
emittance ratio $K = 0.5$
orbit-rms $= 1\, \text{mm}$
$\Omega_n = 23.15 / \Omega_n = 35.18$

beam size at 40 GeV

Fig. 11
Maximum stable initial betatron amplitude as a function of the geometric aperture ($\Delta p/p = 0$).
HERA90/60 - aperture variation

\[ \Phi_c = 60^\circ \]
\[ Q_H = 25.10 \]
\[ Q_V = 24.22 \]

\[ \Phi_c = 90^\circ \]
\[ Q_H = 33.15 \]
\[ Q_V = 33.08 \]

HERA aperture

\[ A_{PH} = A_{PV} \]

half aperture

rectangular beam

sixth order only

emittance ratio \( K = 1.0 \)

no energy deviation

\[ N_{euv} = 30 \]

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**Fig. 13**
HERA60 - tune variation

Maximum stable initial amplitude vs. tune

rectangular beam
 sextupoles only \( f_{x,y} = 0 \)
 emittance ratio \( \kappa = 1.0 \)
 no energy deviation
 \( Q_{x} = 2.9 + 3Q_{y}/Q_{v} = 2.9 + 3Q_{y} \)
 selected to be optimal!
\( N_{av} = 30 \)

**Fig. 14**
HERA90/60-energy variation

rectangular and elliptical beam
sextupoles and multiples
1mm orbit rms errors
emittance ratio K=1.0

\[ \Delta \phi \left[ \text{\%} \right] \]

\[ A [\text{mm}] \]

\[ \Phi_c = 60^\circ \]

\[ \Phi_c = 90^\circ \]

elliptical beam

rectangular beam

beam size at 10 GeV

\[ \beta_h = 76.0 \]

\[ \beta_v = 14.3 \]

\[ Q_h = 33.15 \]

\[ Q_v = 35.18 \]

\[ \Phi_c = 90^\circ \]

\[ \Phi_c = 60^\circ \]

25.1625
29.225

79.2 m
27.9 m

at the observation point

Fig. 15
HERA 90 - 14 and 18 pole mean value variation

- Rectangular beam
- No sextupoles
- No multipoles
- No orbit errors

Fig. 16

$A_{\text{mm}}$ vs. $b_7, b_9 \times 10^6$
HERA90 - Variation of coupling

rectangular beam
sextupoles + multipoles
1mm orbit rms errors

\( N_{se} \cdot 30 \)
\( \beta = 0 \)

\( \Delta A \) - reduction in amplitude due to increased coupling

\( K = \frac{e_w}{E_H} \)

**Fig. 17**
HERA90 - maximum amplitude vs. momentum deviation with persistent sextupoles compensated by normal lattice sextupoles

rectangular beam
Sextupoles + multipoles
I mm orbit radius ratio
emittance ratio \( K = 1.0 \)

Without persistent sextupoles

With persistent sextupoles

Chromaticities produced by persistent sextupoles:

\[
\left( \text{def. } \frac{\Delta \rho}{\Delta p/\rho} \right)
\]

\[ u = -181 \]
\[ v = +143 \]

change in normal lattice sextupole strengths:

\[ S_{h \perp} \text{[m}^2\text{]}: -0.06898 \rightarrow -0.1867 \]
\[ S_{v \perp} \text{[m}^2\text{]}: +0.16915 \rightarrow -0.0066 \]