

# Generation of sub-fs Electron Beams at few-MeV Energies

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## Abstract

Time resolved electron diffraction is an alternative approach to FEL based X-ray experiments for the study of structural dynamics of matter on the relevant timescales. The required electron beam parameters are demanding in terms of emittance and bunch length and require the operation at charges typically well below 1 pC. Moreover the energy is low – a few MeV only. The longitudinal compression of the bunches can be realized with a simple longitudinal focusing scheme in a drift. In this paper the question of what limits the bunch length in this parameter regime is addressed by means of numerical simulations. Beside emittance increasing space charge effects also rf-curvature and nonlinear compression are identified as limiting factors.

*Keywords:* electron beam, ultra short bunches, electron diffraction, space charge

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## 1. Introduction

Electron bunches with a temporal duration in the fs range gain increasing interest as probes e. g. for investigations of the structural dynamics of matter. The focus of interest is on FEL applications which require GeV electron beams with typical bunch charges larger than 100 pC. A complementary approach is time resolved electron diffraction where low electron beams of hundreds of keV up to a few MeV are utilized and the bunch charge can be below 100 fC. In the first case the bunch compression is performed by means of velocity bunching in a string of cavities [1] or by magnetic bunch compressors at higher energies [2] while in the first case bunches have to be either produced directly with the required length [3] or a simple longitudinal focusing scheme (also called ballistic bunching) can be applied [4].

This contribution concentrates on the last case and the question what ultimately limits the achievable bunch length. For the study – primarily based on tracking simulations – the Relativistic Electron Gun for Atomic Exploration REGAE [6] serves as baseline setup.

REGAE is a  $\sim 5$  MeV electron linac build at DESY in a cooperation of the Max-Planck Society, the University of Hamburg and DESY. The primary goals of REGAE are time resolved electron diffraction experiments for studies of structural dynamics of matter. Besides it serves as test bed for accelerator developments e. g. in the field of beam dynamics, diagnostics, synchronization, and photo cathodes. Furthermore a plasma experiment is in preparation for which the REGAE bunches will be injected into a laser driven plasma to probe the fields inside the plasma [6].

Shortest electron bunches are mandatory both for time resolved electron diffraction as well as for the external injection experiment at REGAE. It should be noted however that from an experimental point of view limitations arise due to diagnostics problems and due to synchronization limits which restrict the usability of bunches below 10 fs. The focus of this study is hence on the general beam dynamics rather than on fully worked-out accelerator design.

## 2. REGAE

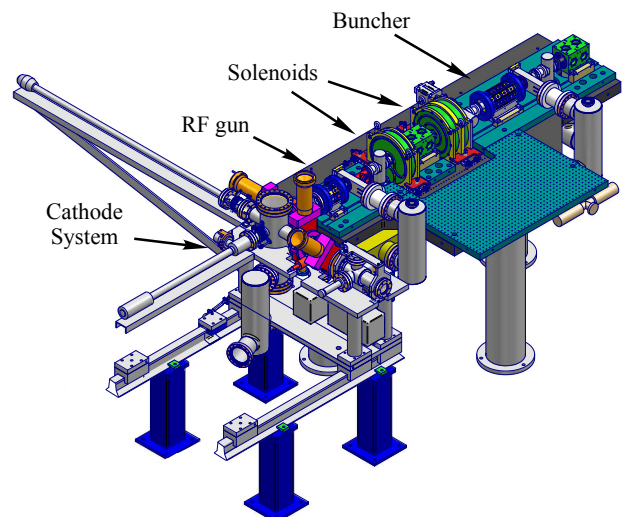


Figure 1: The first section of the REGAE facility.

Fig. 1 presents a schematic overview of the first section of REGAE. The electron bunches are photo emitted

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inside the 1.6 cell rf gun operated at 3 GHz. The beam reaches the final energy of  $\sim 5$  MeV already at the exit of the gun, so that the downstream buncher cavity is operated off-crest to introduce the required energy spread for the bunching in the following drift. For the electron diffraction experiments extremely small emittances are required which can only be reached at bunch charges well below 1 pC.

Fig. 2 presents simulation results for input parameters as compiled in Tab. 1. All simulations are performed with the tracking program ASTRA [11]. The parameters of the bunch at the cathode are not readily achieved but are chosen to explore the limits of the setup. Challenging is not only the assumed perfect cylindrical shape but already the small transverse size of the beam is difficult to be realized due to the unavoidable distance of the last focusing lens to the cathode. It is assumed that for the generation of the low charge a photo cathode with low quantum efficiency and corresponding low kinetic energy of the electrons is employed.

Table 1: Main simulation parameters.

charge	80 fC
rms beam size at cathode	$7 \mu\text{m}$
rms pulse length of laser	500 fs
pulse shape	perfect cylinder
initial kinetic energy	0.1 eV
max. gradient on cathode	110 MV/m
emission phase	35 degree
buncher voltage	2.7 MV

While the transverse beam size increases strongly right after emission from the cathode before being focused to a constant size the emittance increases due to space charge nonlinearities slowly from the initial 3 nm up to about 10 nm along the beam line. When the bunch passes the buncher cavity at  $z \approx 1.5$  m, a correlated energy spread is introduced, such that particles in the tail of the bunch gain a higher energy and hence travel at higher velocity than particles in the head of the bunch. In the following drift the bunch length decreases by a factor 63 from the initial  $132 \mu\text{m}$  (440 fs) to a minimum of  $2 \mu\text{m}$  (7 fs) at a longitudinal position of  $\sim 5.5$  m – the position of the target chamber – before it increases again.

### 3. Longitudinal focus and space charge

Space charge forces counteract any desired focusing of a beam both in the transverse and in the longitudinal direction. Since in the longitudinal focus the bunch is however compressed only in one dimension as compared to the two dimensions in a round transverse focus the effect of the space charge force is vastly different.

In a transverse focus the radial space charge field scales

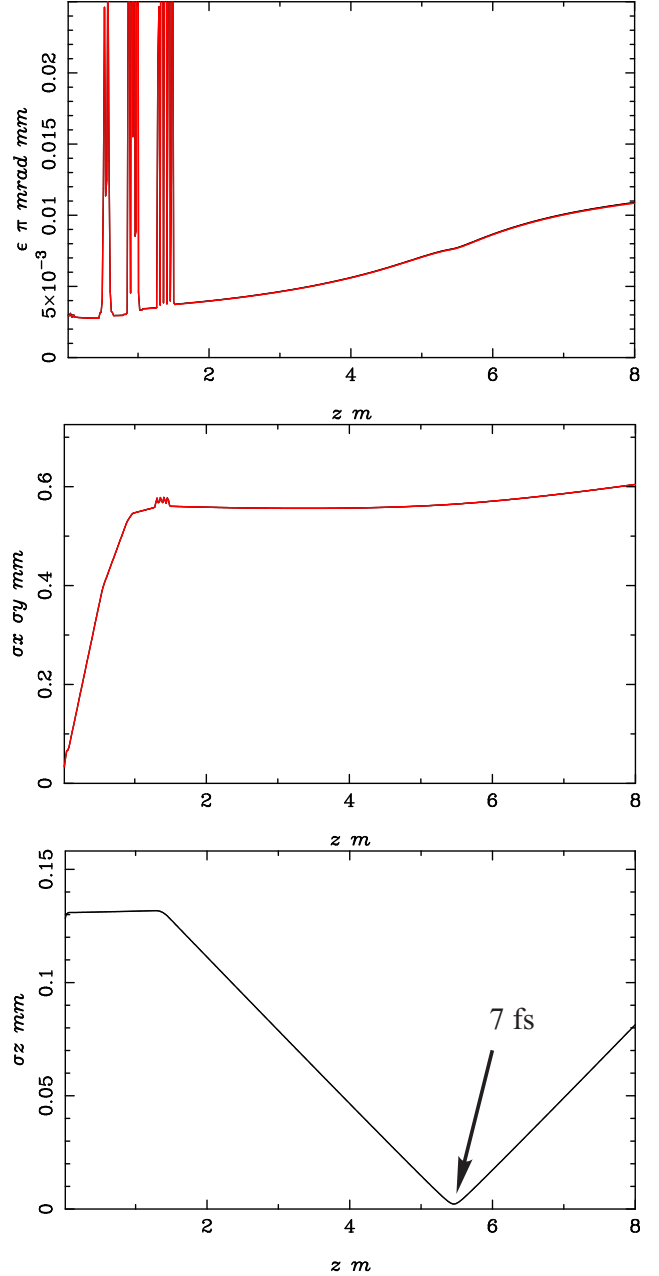


Figure 2: Emittance (top), beam size (middle) and bunch length (bottom) versus longitudinal position along the REGAE beam line for parameters as compiled in Tab. 1.

as  $E_r \propto \frac{r}{R^2}$  where  $r$  is a radial position inside the bunch while  $R$  denotes the radius of the charge distribution and a bunch with aspect ratio  $A = R/L < 1$  ( $L$  bunch length in the average rest system of the bunch) is assumed. An electron at a constant position  $r/R$  experiences hence a divergent force as the radius of the distribution decreases towards zero. As a result even a bunch with zero emittance could not be focused down to a point like spot size. Instead the inward traveling particles transfer kinetic energy into potential energy in the space charge field until they stop (in the average rest system) and gain the energy back while traveling outward. This so-called laminar focus

(Fig. 3) in which the trajectories of individual particles do not cross each other is an ideal case which is reached in the longitudinal center part of realistic distributions. Toward the head and the tail of a distribution the lower space charge field allows particles however to cross over as in a linear focus. This mixed focus leads to a strong emittance growth, which can be avoided only by focusing just modestly to shallow round foci in the transverse direction.

In the longitudinal focus, however, the space charge field

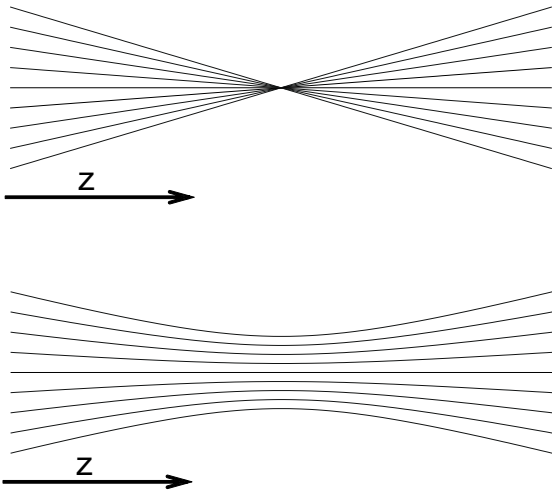


Figure 3: Linear focus without space charge (top) and laminar focus with space charge (bottom).

scales as  $E_z \propto \frac{\zeta}{L}$  where  $\zeta$  is a longitudinal position inside the bunch and an aspect ratio  $A > 1$  is assumed. The defocussing field is hence constant which allows to reach point like foci if a zero emittance beam would be available. In general all particle trajectories run through a linear focus, i. e. head and tail particles exchange their positions (cf. Fig. 3), without excessive emittance growth so that strong foci can be realized.

#### 4. Bunch compression limitations

In the REGAE setup the bunch length at the longitudinal focus cannot be reduced below 7 fs rms just by increasing the focusing strength, i. e. the buncher voltage. The longitudinal phase space at the position of the longitudinal focus displayed in Fig.4 shows structures related to rf-curvature, nonlinearities in the compression process and to longitudinal nonlinearities of the space charge field.

While the current – the projection of the phase space onto the longitudinal axis – in this example is rather uniform over the bunch length with a small spike only in the front, a setup with a strong, central spike of  $\sim 2$  fs rms length and pronounced tails on both ends can be found with small tuning of the parameters.

A general improvement of the situation requires, however, a linearization of the space charge field in the bunch

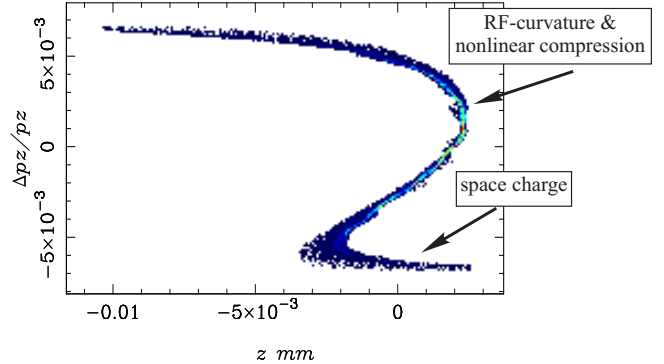


Figure 4: Longitudinal phase space at the position of the longitudinal focus ( $z = 5.5$  m).

which is achieved by starting a modified distribution. Fig. 5 displays the longitudinal phase space at the position of the longitudinal focus when a uniformly filled ellipsoid starts from the cathode (top) and when a bunch with parabolic line charge density but uniform cylindrical transverse shape is started (bottom). Both distributions feature linear longitudinal space charge fields and in fact also the uniform ellipsoid generates a parabolic line charge density. The difference is hence only in the transverse distribution. In case of the ellipsoid the space charge density is constant but the radius changes with longitudinal position, while the radius is constant but the space charge density changes in the other case. The advantage of the ellipsoidal distribution is that also the transverse space charge field is linear for all aspect ratios of the bunch; its disadvantage is that is not easily generated by a laser pulse. (The so-called blow-out concept [7, 8] is not discussed at this point for its unfavorable transverse emittance results [9, 10].) Thus for the following calculations the parabolic distribution has been chosen.

While the phase space portraits displayed in Fig. 5 clearly demonstrate the reduced space charge effects, the rms bunch length achievable with the new distributions is identical to the 7 fs achieved with the cylindrical bunch shape.

To take advantage of the improvements also the curvature of the phase space needs to be reduced. For this purpose a higher harmonic rf system can be employed[12]. The bunch needs to be decelerated with this system so that a curvature of opposite sign is added due to the phase dependence of the rf-field. The shorter the wavelength is, the stronger is the added nonlinearity, i. e. the deceleration required in total becomes less for shorter wavelengths. For the following calculations the  $3^{rd}$  harmonic is chosen.

Fig. 6 shows the result with optimized amplitude of the  $3^{rd}$  harmonic cavity. In total the beam energy is reduced by more than 1 MeV. The effect of the  $3^{rd}$  harmonic cavity and the compression process onto the longitudinal emittance is shown in Fig. 7. The first minimum of the longitudinal emittance inside the cavity at  $z = 1.65$  m indicates that the curvature is not only compensated but over compensated. The second minimum at  $z = 3.6$  m coincides

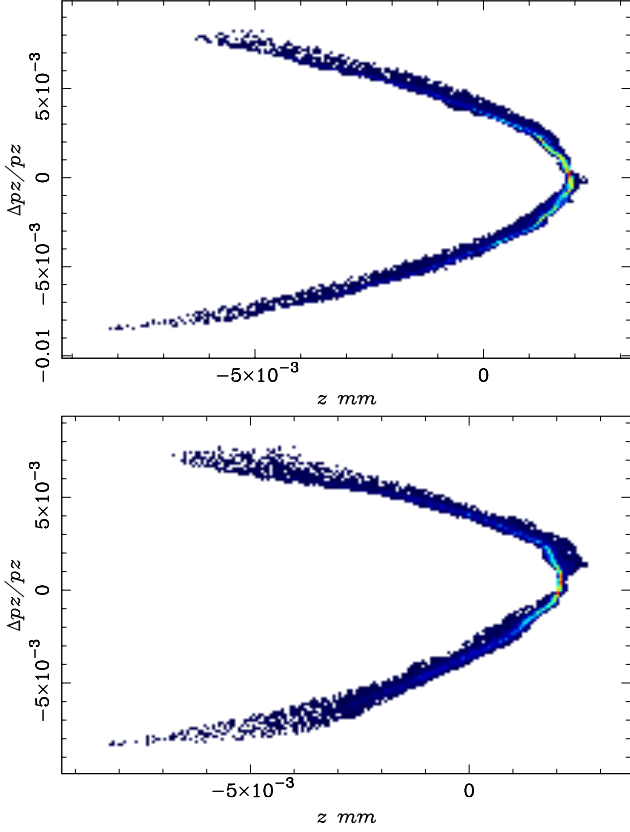


Figure 5: Comparison of the longitudinal phase space for a uniformly filled ellipsoid (top) and a uniform cylindrical transverse shape but parabolic line charge density (bottom).

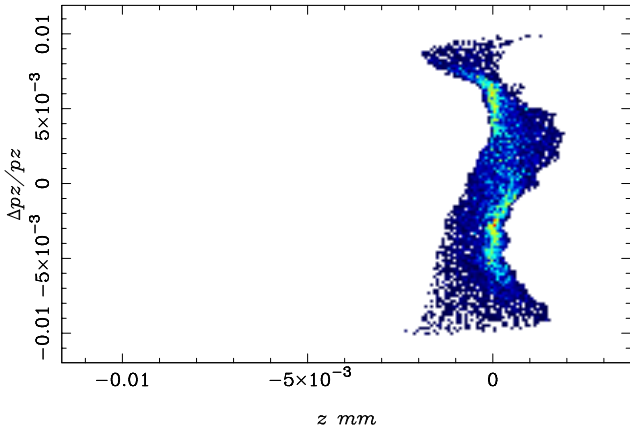


Figure 6: Longitudinal phase space after correction with a 3<sup>rd</sup> harmonic rf cavity; the scale of the abscissa corresponds to Fig. 4. The initial distribution has a parabolic line charge density; the rms bunch length is 0.6 μm (2 fs).

with the location of the longitudinal focus.

Background for this behavior is the nonlinearity of the compression process itself, because for the compression a linearly correlated velocity spread is required but in the buncher cavity a linearly correlated energy spread is introduced.

The relative longitudinal shift  $\Delta z$  of two particles in a

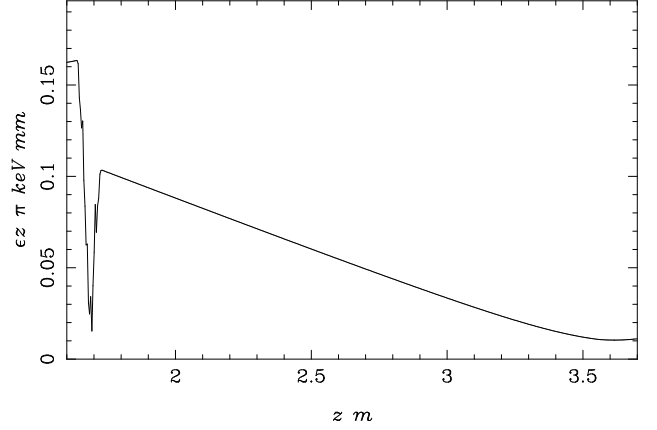


Figure 7: Development of the longitudinal emittance in the 3<sup>rd</sup> harmonic cavity ( $z = 1.65$  m) and downstream up to the longitudinal focus at  $z = 3.6$  m

drift is given by  $\Delta z = \int z' dz = \int \frac{\Delta\beta}{\beta} dz$ , where  $\beta$  and  $\Delta\beta$  denote the average velocity and the velocity difference in units of speed of light. A Taylor expansion yields

$$\begin{aligned} \frac{\Delta\beta}{\beta} &= \frac{1}{\beta} \frac{\partial\beta}{\partial\gamma} \Delta\gamma + \frac{1}{2\beta} \frac{\partial^2\beta}{\partial\gamma^2} \Delta\gamma^2 + \frac{1}{6\beta} \frac{\partial^3\beta}{\partial\gamma^3} \Delta\gamma^3 + \dots \\ \frac{\partial\beta}{\partial\gamma} &= \frac{1}{\beta\gamma^3} \\ \frac{\partial^2\beta}{\partial\gamma^2} &= \frac{2-3\gamma^2}{\beta^3\gamma^6} \\ \frac{\partial^3\beta}{\partial\gamma^3} &= \frac{6-15\gamma^2+12\gamma^4}{\beta^5\gamma^9} \end{aligned}$$

where  $\gamma$  and  $\Delta\gamma$  denote the average energy and the energy spread in terms of the relativistic Lorentz factor.

The linear term scales as  $1/\gamma^3$  thus an increasingly longer drift section is required to achieve a given bunching ratio at higher energies. The nonlinear terms scale as  $\approx 1/\gamma^4$  and  $\approx 1/\gamma^5$ , respectively, indicating a preference for higher energies.

With the 3<sup>rd</sup> harmonic cavity in place four parameters (two amplitudes and two phases) are available to adjust the average energy plus a polynomial of third order in the energy spread. The correction of the third order is however not attempted at this point, instead the phases of the cavities stay fixed at -90 degree for the buncher and -180 degree for the 3<sup>rd</sup> harmonic cavity and only the amplitude of the 3<sup>rd</sup> harmonic cavity is optimized.

A further reduction of the bunch length requires either to reduce the 2<sup>nd</sup> order corrected longitudinal emittance or to increase the focusing strength.

A detailed study of the generation of incoherent energy spread in a photo injector is beyond the scope of this paper but some basic trends should be discussed. First the processes right at the cathode should be considered.

The dominating contribution to the incoherent energy spread finds its origin in the variation of the longitudinal space charge force with the radial position in the bunch. The

space charge force leads to an acceleration of the particles in the head of the bunch and a deceleration of the particles in the back, so that a linearly increasing energy chirp is introduced. The slope of the chirp is steepest for particles traveling in the center of the bunch and somewhat more shallow for particles traveling at large radii, see the schematic insert in Fig. 8. As a result a bow-tie like structure is generated in the longitudinal phase space with a minimum of the slice energy spread at the position where the lines cross. Details of the structure depend on the total charge and on the distribution of the particles. The bow-tie structure is most pronounced at higher charge and for a uniform particle distribution – as in typical FEL applications – but still visible for the low charge parabolic distribution as displayed in Fig. 8. Remnants of this structure are also visible as waists in the longitudinal phase space of the – now rotated – distributions at the focus, e. g. in Fig. 5, Fig. 6 or Fig. 12.

The dependence of the 2<sup>nd</sup> order corrected longitudinal

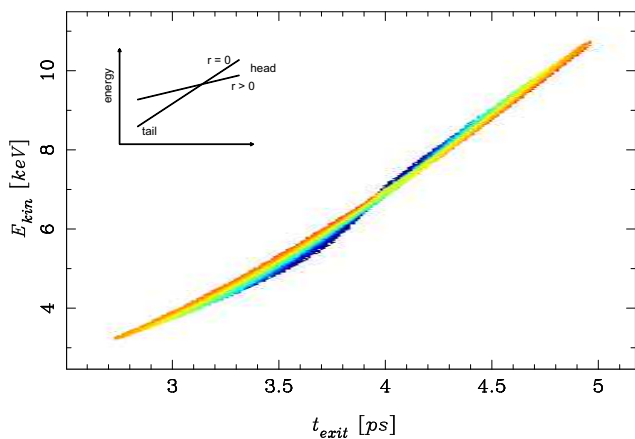


Figure 8: Energy spread vs. time just after emission from the cathode ( $z = 9 \cdot 10^{-5}$  m). The color code represents the radial position of the particles. Particles starting in the center of the distribution (blue) experience a higher space charge force than particles starting at larger radii (red).

nal emittance just after emission from the cathode on the laser pulse length is shown in Fig. 9. The emittance is the product of pulse length and incoherent energy spread. The reduction of the emittance of the parabolic distribution for increasing laser pulse length up to 2 ps is thus understood as a stronger reduction of the energy spread than the increased bunch length contributes to the emittance. The continuous increase of the emittance for the ellipsoidal distribution indicates on the other hand an over proportional increase of the energy spread.

In contrast to the laser pulse length the longitudinal emittance is fairly independent on the transverse beam size at the cathode with a small preference for smaller beam sizes.

Compared to the small initial values of the longitudinal emittance right after emission significantly larger values are found behind the buncher cavity as shown in Fig. 10.

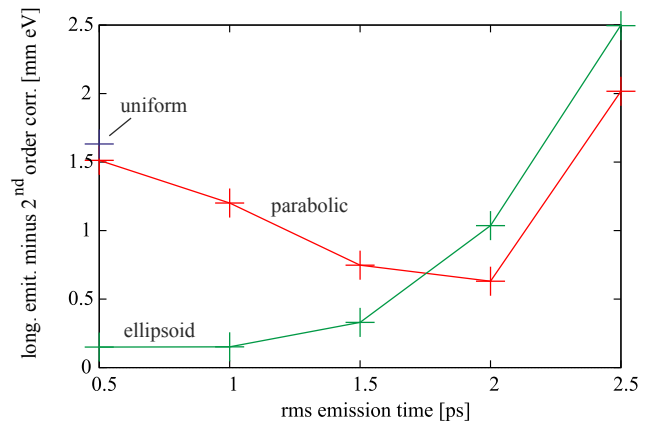


Figure 9: 2<sup>nd</sup> order corrected longitudinal emittance as function of the laser pulse length just after emission from the cathode ( $z = 9 \cdot 10^{-5}$  m). Only one data point is shown for the uniform distribution.

The dependence on the initial distribution is rather weak and the minimum of the longitudinal emittance is reached already at 1 ps for the parabolic case.

A longer pulse length is in general advantageous for the

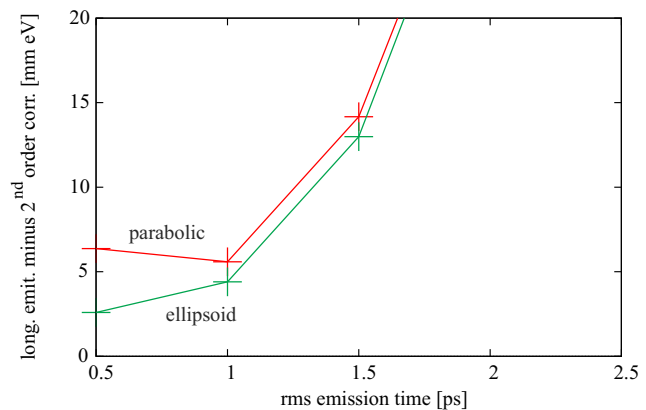


Figure 10: 2<sup>nd</sup> order corrected longitudinal emittance as function of the laser pulse length behind the buncher cavity ( $z = 1.6$  m).

compression because with the same rf voltage a larger energy spread is introduced which leads to a smaller final bunch length provided the longitudinal emittance increases not strongly with the initial bunch length. Fig. 11 shows the minimal bunch length achievable with the parabolic distribution and correction of the 2<sup>nd</sup> order distortion in the REGAE setup if only the bunch length is varied. An rms bunch length of 700 as is reached for an initial laser pulse length of 1.5 ps corresponding to a compression factor of more than 2000.

The phase space portrait for a laser pulse length of 2 ps displayed in Fig. 12 shows the asymmetry as expected for a 3<sup>rd</sup> order distortion. A further reduction of the bunch length would hence require to compensate also this distortion.

Fig. 10 indicates that shorter bunches could also be possible at 1 ps pulse length but increased rf voltage in the

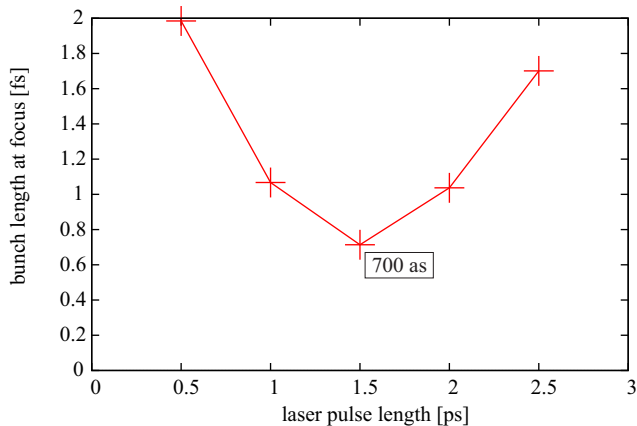


Figure 11: Minimal bunch length as function of the laser pulse length for the parabolic distribution. The longitudinal position is about  $z = 3.5$  m.

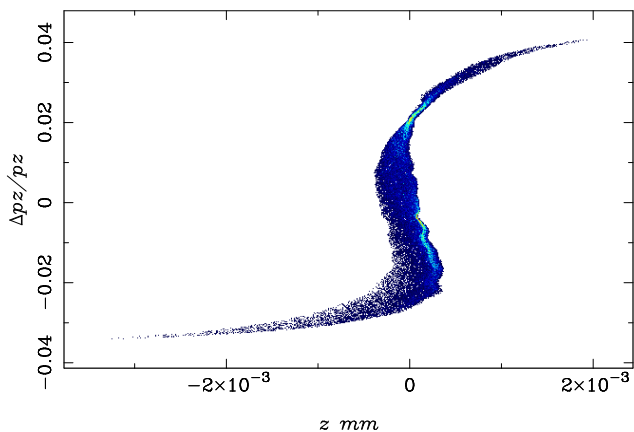


Figure 12: Longitudinal phase space at the position of the longitudinal focus for a laser pulse length of 2 ps with a 3<sup>rd</sup> order distortion preventing a further compression to shorter bunches.

buncher. While a better setting could not be realized yet, it is likely that more studies on a wider parameter range yield further improvements.

## 5. Summary

The longitudinal compression of low charge, low energy beams is limited by various factors as space charge induced phase space distortions, rf-curvature and nonlinear compression. Optimized beam parameter of the generating laser beam and a higher order cavity for the correction of nonlinearly correlated distortions can be employed to overcome these limitations and reach sub-fs bunch length. Only space charge effects and standard beam line components as rf fields are taken into account in this study. An influence of wake fields and possibly electron-electron scattering in the focus is conceivable and requires further studies.

In a first attempt only the second order distortion has been corrected with a 3<sup>rd</sup> harmonic cavity; a correction of the

third order appears to be feasible but the tuning of the parameters is more complex. Analytical models would be desirable as a guideline for the tuning process. A study of the generation of the incoherent energy spread at the cathode and during the transport could lead to further improvements. In any case, however, improvements of the synchronization and on the beam diagnostics are required before the generation of sub-fs beams for electron diffraction experiments becomes reality.

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