Spectra of Sigma Models on symmetric and semi-symmetric spaces

Alessandra Cagnazzo

DESY theory group

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Plan of the talk

1. Introduction
2. Symmetric coset sigma model
3. Instabilities
4. Semisymmetric coset sigma model
5. Conclusions and future directions

Sigma models on coset spaces are very important integrable models.

Symmetric	Semisymmetric
$\mathbb{Z}_2$	$\mathbb{Z}_4$

Sigma models on coset spaces are very important integrable models.

Symmetric	Semisymmetric
$\mathbb{Z}_2$	$\mathbb{Z}_4$
$\downarrow$	$\downarrow$
Supersphere	AdS spaces
$S^{2S+1\|2S}$	$AdS_n \times S^n$

The study of non linear sigma models becomes difficult if we go in the strong curvature regime. For the study of this regime are widely used dualities (strong/weak coupling).

AdS/CFT

Symmetric spaces $\frac{G}{H}$

$$\mathfrak{g} = \mathfrak{g}_0 + \mathfrak{g}_1 \tag{1}$$

$\sigma : G \to G$ leaves invariant $\mathfrak{h}$ elements. So $\mathfrak{h} = \mathfrak{g}_0$

$$[T_0, T_0] \subset \mathfrak{g}_0 \tag{2}$$

$$\sigma^2 = 1 \tag{3}$$

$$[T_1, T_1] \subset \mathfrak{g}_0, \qquad [T_1, T_0] \subset \mathfrak{g}_1 \tag{4}$$

Semi-Symmetric spaces $\frac{G}{H}$

$$\mathfrak{g} = \mathfrak{g}_0 + \mathfrak{g}_1 + \mathfrak{g}_2 + \mathfrak{g}_3 \quad (1)$$

$\sigma : G \to G$ leaves invariant $\mathfrak{h}$ elements. So $\mathfrak{h} = \mathfrak{g}_0$

$$[T_0, T_0] \subset \mathfrak{g}_0 \quad (2)$$

$$\sigma^4 = 1 \quad (3)$$

$$[T_A, T_B] \subset \mathfrak{g}_{A+B \text{ mod } 4} \quad (4)$$

Action

$$\jmath = g^{-1}dg \qquad g \in G \tag{5}$$

$$S = \frac{1}{2R^2}\int_\Sigma \frac{d^2z}{\pi}\sum_{A=1}^{2N-1}(p_A - iq_A)\,\mathrm{Str}\big(\jmath_A \bar{\jmath}_{A'}\big)\ , \quad p_0 = 0,\ q_0 = 0 \tag{6}$$

Kagan and Young '05

$$A' = 2N - A\ \mathrm{mod}2N \tag{7}$$

Symmetric $N = 1$

$$\jmath = \jmath_0 + \jmath_1 \tag{8}$$

$$p_1 = 1, q_1 = 0 \tag{9}$$

Semi-symmetric $N = 2$

$$\jmath = \jmath_0 + \jmath_1 + \jmath_2 + \jmath_3 \tag{10}$$

$$p_A = 1\ , \quad q_A = 1 - \frac{A}{2}\ , \quad \text{for } A = 1, 2, 3 \text{ hybrid model} \tag{11}$$

Symmetric $\mathbb{Z}_2$ sigma model

$$S = \frac{1}{2R^2} \int_\Sigma d^2\sigma \mathrm{Str}\left(\sqrt{h} h^{\mu\nu} \jmath_\mu \jmath_\nu\right) \tag{12}$$

$$J \in \mathfrak{m} = \frac{\mathfrak{g}}{\mathfrak{h}} \tag{13}$$

or in terms of vielbein:

$$S = \frac{1}{2R^2} \int_\Sigma d^2\sigma \mathrm{Str}\left(\sqrt{h} h^{\mu\nu} E_\mu E_\nu\right) \tag{14}$$

Semi-symmetric $\mathbb{Z}_4$ sigma model

Hybrid model:

$$S = \frac{1}{2R^2}\int_\Sigma d^2\sigma \mathrm{Str}\left(\sqrt{h}h^{\mu\nu}\mathcal{J}_{2\mu}\mathcal{J}_{2\nu} + \sqrt{h}h^{\mu\nu}\mathcal{J}_{1\mu}\mathcal{J}_{3\nu} + \frac{i}{2}\epsilon^{\mu\nu}\mathcal{J}_{1\mu}\mathcal{J}_{3\nu}\right) \quad (15)$$

$$J_A \in \mathfrak{g}_A \quad (16)$$

Green-Schwarz:

$$S = \frac{1}{2R^2}\int_\Sigma d^2\sigma \mathrm{Str}\left(\sqrt{h}h^{\mu\nu}\mathcal{J}_{2\mu}\mathcal{J}_{2\nu} + i\epsilon^{\mu\nu}\mathcal{J}_{1\mu}\mathcal{J}_{3\nu}\right) \quad (17)$$

Fields

At level $(h, \bar{h})$ in the spectrum we find the field:

$$\Phi_\Lambda(z, \bar{z}) = \mathrm{d}_{\lambda\mu\bar{\mu}} V_{\Lambda\lambda}\, \jmath_{\mathbf{m}\mu}((h-n)\partial, n\jmath) \otimes \bar{\jmath}_{\bar{\mathbf{m}}\bar{\mu}}(\bar{h}-m)\bar{\partial}, m\bar{\jmath})\ (z, \bar{z}) \quad (18)$$

Φ is an G multiplets, $\jmath$ transform in H multiplets.

$$\Phi_\Lambda = \mathrm{d}\, V\, \jmath_{\mathbf{m}} \otimes \bar{\jmath}_{\bar{\mathbf{m}}} \quad (19)$$

Multiplicity?

Fields

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Multiplicity? $\rightarrow$ if G compact and H compact, it can be found with a bit of representation theory. It is given by the multiplicity of the projective cover of λ in the decomposition of the projective cover of Λ

a bit of language...

Focus on Supergroups. These can be viewed as fermionic extensions of ordinary Lie groups.

$$\mathbf{g} = \mathbf{g}_0 + \mathbf{g}_1$$

One difference is that in a supergroup we have existence of reducible but indecomposable representation.
Finite dim. rep.:

$$\text{Typical (long) } \mathcal{L}_\mu = \mathcal{P}_\mu$$
$$\text{Atypical (short) } \mathcal{L}_\mu \neq \mathcal{P}_\mu$$

$\mathcal{L}_\mu$ is the irrep. and $\mathcal{P}_\mu$ is a projective cover

$$\mathbb{Z}_2$$

One-loop anomalous dimension

$$\langle\, \Phi_\Lambda(u,\bar{u}) \otimes \Phi_\Xi(v,\bar{v})\,\rangle_1 = \langle\, 2\delta\mathbf{h}\cdot \Phi_\Lambda(u,\bar{u}) \otimes \Phi_\Xi(v,\bar{v})\,\rangle_0 \,\log\left|\frac{\epsilon}{u-v}\right|^2 + \dots \tag{20}$$

$$\langle\, \Phi_\Lambda(u,\bar{u}) \otimes \Phi_\Xi(v,\bar{v})\,\rangle = \int_{G/H} d\mu \,\langle\, \Phi_\Lambda(u,\bar{u} \mid g_0) \otimes \Phi_\Xi(v,\bar{v} \mid g_0) e^{-S_{\rm int}} \,\rangle \ , \tag{21}$$

$$\jmath = e^{-\phi}\partial e^{\phi} = \left[\partial\phi - \frac{1}{2}\,[\phi\,,\,\partial\phi] + \frac{1}{6}\,[\phi\,,\,[\phi\,,\,\partial\phi]]\right] + \cdots \ . \tag{22}$$

$$S^{(1)}_{\rm int} = \int d^2z\; \Omega_4 = \int d^2z\; \frac{1}{3} Str\big(\,[\phi\,,\,\partial\phi]\,[\phi\,,\,\bar{\partial}\phi]\,\big) \quad \phi : \Sigma \to \frac{G}{H} \tag{23}$$

ϕ has grading one $\Rightarrow$ there is no term $[\phi, \partial\phi]$ in the expansion of $\jmath_1$.

One-loop anomalous dimension

$$\langle\, \phi(z,\bar z) \otimes \phi(w,\bar w)\,\rangle_0 = -\log\left|\frac{z-w}{\epsilon}\right|^2 t_i \otimes t^i \qquad \Phi_\Lambda = \mathrm{d}V\, \mathcal{J}_{\mathbf{m}} \otimes \bar{\mathcal{J}}_{\bar{\mathbf{m}}} \tag{24}$$

Two correlators contribute:

$$\left\langle\, V^{(1)} \otimes \mathcal{J}^{(0)}_{\mathbf{m}}(u) \otimes \bar{\mathcal{J}}^{(0)}_{\bar{\mathbf{m}}}(\bar u) \otimes V^{(1)} \otimes \mathcal{J}^{(0)}_{\mathbf{m}}(v) \otimes \bar{\mathcal{J}}^{(0)}_{\bar{\mathbf{m}}}(\bar v)\,\right\rangle \tag{25}$$

$$\int_{\mathbb{C}_\epsilon} d^2z \,\left\langle\, V^{(0)} \otimes \mathcal{J}^{(0)}_{\mathbf{m}}(u) \otimes \bar{\mathcal{J}}^{(0)}_{\bar{\mathbf{m}}}(\bar u) \otimes V^{(0)} \otimes \mathcal{J}^{(0)}_{\mathbf{m}}(v) \otimes \bar{\mathcal{J}}^{(0)}_{\bar{\mathbf{m}}}(\bar v)\Omega_4(z,\bar z)\,\right\rangle \tag{26}$$

$$\delta h = \frac{1}{2R^2}\Big(\underbrace{C_\Lambda}_{G} + \underbrace{C_\mu + C_{\bar\mu}}_{H}\Big)$$

Candu, Mitev, Schomerus '13

An example

$$\mathbb{S}^{3|2} = \frac{OSP(4|2)}{OSP(3|2)}$$

$OSP(4|2)$

$$[\underbrace{j_1}_{Sp(2)}, \underbrace{j_2\ , j_3}_{SO(4)}]$$

Highest weight representation, with j_1, j_2 and j_3 referring to the highest weight of the bosonic subalgebra. Casimir:

$$C = -4j_1(j_1 - 1) + 2j_2(j_2 + 1) + 2j_3(j_3 + 1)$$

For an atypical representation:

$$C = k^2 \qquad \text{with } k \text{ integer}$$

so the atypical rep. can be labeled by two integers

$$\Lambda_{k,l} \qquad \Gamma_k = \{\Lambda_{k,l}, l \in \mathbb{Z}\}$$

Van der Jeugt

$OSP(3|2)$

$$[\underbrace{q}_{Sp(2)}, \underbrace{p}_{SO(3)}]$$

All the $OSP(4|2)$ representation can be embedded in $OSP(3|2)$, it is not hard to find the decomposition of the $OSP(4|2)$ in $OSP(3|2)$ reps just by diagonal embedding. This will be relevant in the SM analysis.

$$C = (p + 2q)(p - 2q + 1)$$

In this case all the atypical representations have Casimir zero.

$$[0, 0] \qquad [q, 2q - 1]$$

Van der Jeugt

Spectrum

- $(h = 0, \bar{h} = 0)$: $[0, 0]$ that gives $OSP(4|2)$-multiplets $\Lambda_{l,0}$, Spherical Harmonics, $\delta^{(1)} = \frac{l^2}{R^2}$
- $(h = 1, \bar{h} = 0)$: $[\frac{1}{2}, 0]$ gives $\Lambda_{0,1}$, Noether Currents, $\delta^{(1)} = 0$
- $(h = 1, \bar{h} = 1)$: This sector is very interesting since is sensitive to the Equations of Motion of the Sigma Model

$$\Gamma_{[\frac{1}{2},0]\times[\frac{1}{2},0]} \cong \Lambda_{0,0}+2\Lambda_{0,1}+\Lambda_{0,2}+\Lambda_{1,0}+\sum_{l=2}^{\infty}(3\Lambda_{l,0}+\Lambda_{l,1}+\Lambda_{l,-1})+\text{typicals}$$

$$\delta^{(1)} = \frac{l^2}{R^2}$$

$$\Gamma_{[\frac{1}{2},0]\times[\frac{1}{2},0]} \cong \Lambda_{0,0} + 2\Lambda_{0,1} + \Lambda_{0,2} + \Lambda_{1,0} + \sum_{l=2}^{\infty}(3\Lambda_{l,0} + \Lambda_{l,1} + \Lambda_{l,-1}) + \text{typicals}$$

$$\Lambda_{0,1} = [\frac{1}{2}, 0, 0] \;\rightarrow\; [\frac{1}{2}, 0] + [0, 0] \tag{27}$$

$$\mathcal{P}_{\Lambda_{0,1}} \;\rightarrow\; \ldots + 2\mathcal{P}_{[\frac{1}{2},0]} + \ldots \tag{28}$$

Direct construction from $X = (X_A) = (x_j, \eta_1, \eta_2)$

$$
\begin{aligned}
&(0,0) \qquad F_{l,0}(X) && (29)\\
&(1,0) \qquad F_{l,0}(X)\,\partial X && (30)\\
&(1,1) \qquad F_{l,0}(X)\,\partial\bar\partial X,\; F_{l,0}(X)\,\partial X\bar\partial X && (31)
\end{aligned}
$$

Constraint:

$$
\begin{aligned}
&X^2 = 1 \,\text{and derivatives} && (32)\\
&\partial X\cdot\bar\partial X = -X\cdot\partial\bar\partial X && (33)
\end{aligned}
$$

Field counting using harmonic analysis is much more efficient.

To study the sigma model at strong curvature we can use dualities.

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Duality to explore:

Sigma Model on $S^{2S+1|2S}$ **?** $OSP(2S+2|2S)$ Gross-Neveu model

Candu, Saleur '08

$S = 0$ was studied, we will see it for $S = 1$.

The dual theory: Gross-Neveu model

$$S_0 = \frac{1}{2\pi}\int d^2z \left[< \Psi, \bar{\partial}\Psi > + < \bar{\Psi}, \partial\bar{\Psi} > \right]$$

$\Psi = (\psi_1, \ldots, \psi_{2S+2}, \beta_1, \ldots, \beta_S, \gamma_1, \ldots, \gamma_S)$
WZW model on

$$G = \frac{G \times G}{G} \qquad \text{with} \;\; G = OSP(2S+2|2S)$$

has $G \times G$ isometry and can be deformed in various ways, that may or may not preserve the isometry.
We want to consider a G preserving deformation

$$S_{int} = \frac{g}{\pi}\int d^2z J^a(z)\eta_{a,b}\bar{J}^b(\bar{z})$$

We want to study the example $S = 1$

Duality

We want to compare the spectra of the two theories (GN or WZW and SM).

$$g = \frac{1 - R^2}{1 + R^2} \qquad \begin{aligned} &(g \to -1) \leftrightarrow (R \to \infty) \\ &(g \to 0) \leftrightarrow (R \to 1) \end{aligned} \tag{34}$$

the WZW model is weakly coupled at $g = 0$ an the SM can be perturbatively treated at $R = \infty$. To interpolate the two sides we need to know the anomalous dimension for the operators in the two sides.

Anomalous dimension

The states in the WZW spectrum will be given by the fusion of left and right states.

$$(h, \bar{h}) \;\rightarrow\; \text{conformal dimension}$$

$$h - h_0 = \bar{h} - \bar{h}_0 = \delta \;\rightarrow\; \text{anomalous dimension}$$

$$\delta = \frac{g \;\, C_\Lambda}{2(1-g^2)} - \frac{g}{2(1+g)}(C_L + C_R)$$

This form for the anomalous dimension is valid if the diagonal representation is atypical.

Candu, Mitev, Schomerus '12

Where is the sigma model?

there are two singularities in the anomalous dimension $g = \pm 1$.

$$\delta = \frac{g \;\, m^2}{2(1-g^2)} - \frac{g}{2(1+g)}(C_L + C_R)$$

$g = 1$ everithing with casimir non zero goes to $-\infty$, everything is instable!
$g = -1$ the second part compensate and the states fulfilling the no winding condition

$$m^2 = 2(C_L + C_R) \tag{35}$$

are surviving. In this point we have the possibility to recover the sigma model.

DUALITY TEST

$(h, \bar{h})$=(0,0) Spherical harmonics

$$\Lambda_{k,0} \qquad \text{in WZW}$$

- in the chiral spectrum they first appear at $h = \frac{k^2}{2}$
- at level $(\frac{k^2}{2}, \frac{k^2}{2})$ from $\Lambda_{k,0}^2$ we get $\Lambda_{2k,0}$, if we consider the anomalous dimension at $R \to \infty$:

$$\delta = -\frac{C_\Lambda}{8} = -\frac{k^2}{2} \;\Rightarrow\; (0,0)$$

We find spherical harmonics on supersphere.

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We find spherical harmonics on supersphere.

- (1,0)(0,1) $\Rightarrow$ MATCH in the block of the zero! (Currents)
- (1,1) $\Rightarrow$ MATCH in the block of the zero! (EoM)

Instabilities

Both in the SM and in the WZW model we see instabilities.
In fact for $R \to \infty$ in the WZW model and, already at the first loop, for $R \to 1$ in the Sigma Model there are infinitely many modes that go to $-\infty$. This means that on the way from strong to weak coupling there are infinitely many modes that becomes relevant.
WZW model ($g \to -1$):

$$\begin{aligned} \delta &= -\frac{C_\Lambda}{8} \qquad && C_\Lambda = 2(C_L + C_R) \\ \delta &= \infty && C_\Lambda < 2(C_L + C_R) \\ \delta &= -\infty && C_\Lambda > 2(C_L + C_R) \end{aligned}$$

S. Ryu,C. Mudry, A. W. W. Ludwig, and A. Furusaki 2010

One model instability free

current-current deformation of $psl(2|2)$ WZW model, that is conjectured to be dual to NLSM on $CP^{1|2}$

S. Ryu, C. Mudry, A. W. W. Ludwig, and A. Furusaki 2010

The $psl(N|N)$ model are shown to be instability free at one loop. We want to extend the result at all loops, at least for $psl(2|2)$.

representation are labelled by the two sl(2) of the bosonic superalgebra:

$$\text{non-BPS} \qquad [j, l] \tag{36}$$

$$\frac{1}{2}\text{-BPS} \quad [j],\ j = l \tag{37}$$

Anomalous dimension:

$$\delta_g = -\frac{g}{2(1+g)}(C_L + C_R). \tag{38}$$

$$\begin{aligned} C\big([j,l]\big) &= -j(j+1) + l(l+1) \\ C\big([j]\big) &= 0. \end{aligned} \tag{39}$$

first appearence in the spectrum $j \neq l$

$$h^{\min}_{g=0}\big([j,l]\big) = \begin{cases} j + (l+1)^2 & j,l \in \mathbb{Z} \\ j + (l+1)^2 + \dfrac{1}{4} & j,l \in \mathbb{Z} + \dfrac{1}{2}, \end{cases} \tag{40}$$

for atypical reps the formula is different, but is not relevant in this discussion.

Dangerous states are those with $l > j$. Among these the most dangerous are $[0, l]$-states, since they occur earlier. the anomalous dimension $\delta_g\big([0, l]\big)$ of invariant operators with $\Lambda_L = [0, l] = \Lambda_R$ is given by

$$\delta_g\big([0, l]\big) = -\frac{g}{1+g} l(l+1). \tag{41}$$

$$h^{min}_{g=0} = (l+1)^2 \; (+1/4) \tag{42}$$

This remain irrelevant $h_\infty \geq 1 + l$.

$$\mathbb{Z}_4$$

$$S = \frac{1}{2R^2}\int_\Sigma \frac{d^2z}{\pi}\sum_{A=1}^{3}(p_A - iq_A)\, Str\left(\jmath_A \bar{\jmath}_{A'}\right) \ , \quad p_0 = 0,\ q_0 = 0 \tag{43}$$

Conformal, hybrid model:

$$S = \frac{1}{2R^2}\int_\Sigma \frac{d^2z}{\pi}(\jmath_2, \bar{\jmath}_2) + (1+\frac{i}{2})(\jmath_1, \bar{\jmath}_3) + (1-\frac{i}{2})(\jmath_3, \bar{\jmath}_1) \tag{44}$$

Berkovits et al. 1999

$$S_{\text{int}} = \int \frac{d^2z}{\pi}\Omega_3 + \Omega_4 \tag{45}$$

$$AdS_2 \times S^2 \quad \text{as} \quad \frac{PSL(1,1,|2)}{U(1)\times U(1)}$$

$$\left\langle V^{(1)} \otimes \jmath^{(0)}_{\mathbf{m}}(u) \otimes \bar{\jmath}^{(0)}_{\bar{\mathbf{m}}}(\bar{u}) \otimes V^{(1)} \otimes \jmath^{(0)}_{\mathbf{m}}(v) \otimes \bar{\jmath}^{(0)}_{\bar{\mathbf{m}}}(\bar{v}) \right\rangle$$

$$\int_{\mathbb{C}_\epsilon} \frac{d^2z}{\pi} \left\langle V^{(0)} \otimes \jmath^{(0)}_{\mathbf{m}}(u) \otimes \bar{\jmath}^{(0)}_{\bar{\mathbf{m}}}(\bar{u}) \otimes V^{(0)} \otimes \jmath^{(0)}_{\mathbf{m}}(v) \otimes \bar{\jmath}^{(0)}_{\bar{\mathbf{m}}}(\bar{v}) \Omega_4(z,\bar{z}) \right\rangle$$

Old New

$$\left\langle V^{(0)} \otimes \jmath^{(1)}_{\mathbf{m}}(u) \otimes \bar{\jmath}^{(0)}_{\bar{\mathbf{m}}}(\bar{u}) \otimes V^{(0)} \otimes \jmath^{(1)}_{\mathbf{m}}(v) \otimes \bar{\jmath}^{(0)}_{\bar{\mathbf{m}}}(\bar{v}) \right\rangle$$

$$\int_{\mathbb{C}_\epsilon} \frac{d^2z}{\pi} \left\langle V^{(0)} \otimes \jmath^{(1)}_{\mathbf{m}}(u) \otimes \bar{\jmath}^{(0)}_{\bar{\mathbf{m}}}(\bar{u}) \otimes V^{(0)} \otimes \jmath^{(0)}_{\mathbf{m}}(v) \otimes \bar{\jmath}^{(0)}_{\bar{\mathbf{m}}}(\bar{v}) \Omega_3(z,\bar{z}) \right\rangle$$

$$\int_{\mathbb{C}_\epsilon} \frac{d^2z}{\pi} \frac{d^2w}{\pi} \left\langle V^{(0)} \otimes \jmath^{(0)}_{\mathbf{m}}(u) \otimes \bar{\jmath}^{(0)}_{\bar{\mathbf{m}}}(\bar{u}) \otimes V^{(0)} \otimes \jmath^{(0)}_{\mathbf{m}}(v) \otimes \bar{\jmath}^{(0)}_{\bar{\mathbf{m}}}(\bar{v}) \Omega_3(z,\bar{z}) \Omega_3(w,\bar{w}) \right\rangle$$

$$\int_{\mathbb{C}_\epsilon} \frac{d^2z}{\pi} \left\langle V^{(1)} \otimes \jmath^{(0)}_{\mathbf{m}}(u) \otimes \bar{\jmath}^{(0)}_{\bar{\mathbf{m}}}(\bar{u}) \otimes V^{(0)} \otimes \jmath^{(0)}_{\mathbf{m}}(v) \otimes \bar{\jmath}^{(0)}_{\bar{\mathbf{m}}}(\bar{v}) \Omega_3(z,\bar{z}) \right\rangle$$

$$\left\langle V^{(1)} \otimes \jmath^{(1)}_{\mathbf{m}}(u) \otimes \bar{\jmath}^{(0)}_{\bar{\mathbf{m}}}(\bar{u}) \otimes V^{(0)} \otimes \jmath^{(0)}_{\mathbf{m}}(v) \otimes \bar{\jmath}^{(0)}_{\bar{\mathbf{m}}}(\bar{v}) \right\rangle$$

Conclusions

We have studied some features of symmetric and semisymmetric coset NLSM

$\mathbb{Z}_2$:

- We have studied a proposed duality that would allow us to study sigma models on backgrounds with strong curvature.
- We have seen some examples supporting this duality, comparing directly the spectra for the first few levels, for some 1/2 BPS operators.
- There is the issue of stability issue that needs to be fixed in order to be able to compare the spectra of the two theories.

$\mathbb{Z}_4$:

- We are working to find the one loop anomalous dimesion for a conformal theory.
- Spectrum, instabilities, dualities...

Thank you

GATIS

Gauge Theories as Integrable Systems