Theory of Radiative Rare B-Decays

Ahmed Ali
DESY, Germany

LHCb-Workshop, Valencia
Valencia, February 19, 2014

Interest in Rare B Decays

- Rare B Decays $(b \to (s,d)\gamma, b \to (s,d)\ell^+\ell^-, ...)$ are Flavour-Changing-Neutral-Current (FCNC) processes $(|\Delta B| = 1, |\Delta Q| = 0)$; not allowed at the Tree level in the SM

- These decays are governed by the GIM mechanism, which imparts them sensitivity to higher scales in the SM (m_t, m_W) and the CKM matrix elements, in particular, V_{td}, V_{ts} and V_{tb}

- In principle sensitive to physics beyond the SM (BSM), such as supersymmetry. Precise experiments and theory are needed to establish or definitively rule out BSM effects

- Hence, Rare B-decays have enjoyed great attention in the past experimental programme in flavour physics (CLEO, BABAR, BELLE, CDF, D0) and rightly continue to do so in the current and planned experiments at the LHC and the Super-B factory at KEK

The Cabibbo-Kobayashi-Maskawa Matrix

$$V_{\rm CKM} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

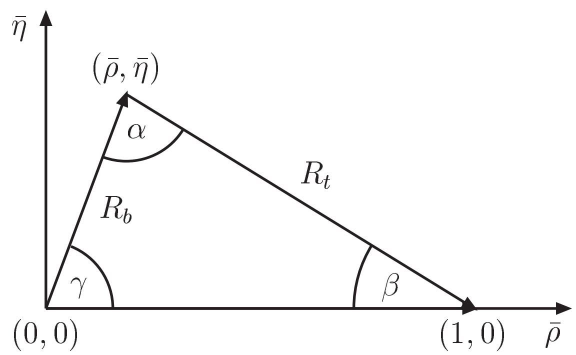
- Customary to use the handy Wolfenstein parametrization

$$V_{\rm CKM} \simeq \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda(1+iA^2\lambda^4\eta) & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2(1+i\lambda^2\eta) & 1 \end{pmatrix}$$

- Four parameters: $A, \ \lambda, \ \ \rho, \ \ \eta$
- Perturbatively improved version of this parametrization

$$\bar{\rho} = \rho(1-\lambda^2/2), \ \ \bar{\eta} = \eta(1-\lambda^2/2)$$

- The CKM-Unitarity triangle $[\phi_1 = \beta; \ \ \phi_2 = \alpha; \ \ \phi_3 = \gamma]$



The Standard Candle: $B \to X_s\gamma$

- First measurements by CLEO (1995); well measured at the B-factories by Belle and BaBar; more precise measurements anticipated at SuperB-factories

Theoretical Interest:

- A monumental theoretical effort has gone in improving the perturbative precision; $B \to X_s\gamma$ in NNLO completed in 2006

  - First estimate of $\mathcal{B}(B \to X_s\gamma)$: M. Misiak et al., Phys. Rev. Lett. 98:022002 (2007)

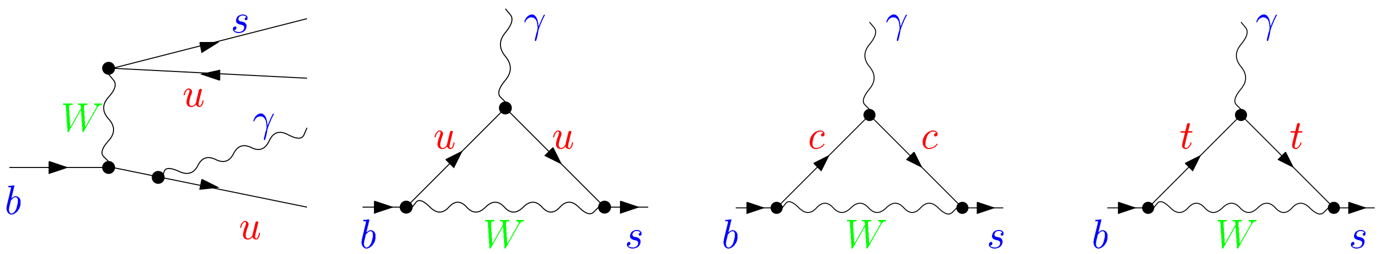
  - Analysis of $\mathcal{B}(B \to X_s\gamma)$ at NNLO with a cut on the Photon energy, T. Becher and M. Neubert, Phys. Rev. Lett. 98:022003 (2007)

- Non-perturbative effects under control thanks to HQET

- Sensitivity to new physics; hence constrains parameters of the BSM models such as the 2HDMs and Supersymmetry

- A crucial input in a large number of precision tests of the SM in $b \to s$ processes, such as $B \to X_s\ell^+\ell^-$

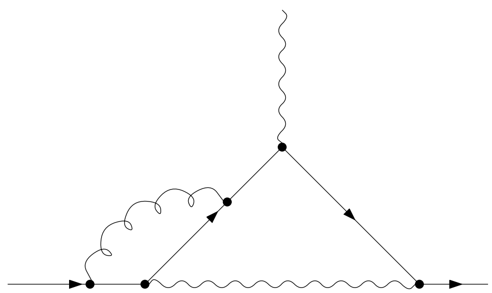
Examples of the leading electroweak diagrams for $B \to X_s \gamma$



$$\left|\frac{V_{ub}V_{us}}{V_{cb}}\right| \simeq \left|\frac{V_{ub}V_{us}}{V_{ts}}\right| \simeq 2\%$$

$\sim +200\%$ $\sim -100\%$

In the amplitude, after including LO QCD effects.



- QCD logarithms $\alpha_s \ln \frac{M_W^2}{m_b^2}$ enhance $\mathrm{BR}(B \to X_s\gamma)$ more than twice
- Effective field theory (obtained by integrating out heavy fields) used for resummation of such large logarithms

The effective Lagrangian for $B \to X_s \gamma$ and $B \to X_s \ell^+ \ell^-$

$$\mathcal{L} = \mathcal{L}_{QCD\times QED}(q,l) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i$$

$$(q = u, d, s, c, b, \; l = e, \mu, \tau)$$

$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b), & i = 1, 2, \quad |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma_i' q), & i = 3, 4, 5, 6, \quad |C_i(m_b)| < 0.07 \\ \frac{em_b}{16\pi^2}\bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, \quad C_7(m_b) \sim -0.3 \\ \frac{gm_b}{16\pi^2}\bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}, & i = 8, \quad C_8(m_b) \sim -0.15 \\ \frac{e^2}{16\pi^2}(\bar{s}_L \gamma_\mu b_L)(\bar{l}\gamma^\mu \gamma_5 l), & i = 9, 10 \quad |C_i(m_b)| \sim 4 \end{cases}$$

Three steps of the calculation:

Matching: Evaluating $C_i(\mu_0)$ at $\mu_0 \sim M_W$ by requiring equality of the SM and the effective theory Green functions

Mixing: Deriving the effective theory RGE and evolving $C_i(\mu)$ from μ_0 to $\mu_b \sim m_b$

Matrix elements: Evaluating the on-shell amplitudes at $\mu_b \sim m_b$

Structure of the SM calculations for $\bar{B} \to X_s \gamma$ & $\bar{B} \to X_s \ell^+ \ell^-$

$$\mathcal{H}_{\text{eff}} \sim \sum_{i=1}^{10} C_i(\mu) O_i$$

• $\mathcal{H}_{\text{eff}}$ independent of the scale μ, while $C_i(\mu)$ and $O_i(\mu)$ depend on μ
$\Longrightarrow$ Renormalization Group Equation (RGE) for $C_i(\mu)$:

$$\mu \frac{d}{d\mu} C_i(\mu) = \gamma_{ij}^{\mathrm{T}} C_j(\mu)$$

• γ_{ij}: anomalous dimension matrix

• Matching usually done at high scale $(\mu_0 \sim M_W, m_t)$

• Full theory and the matrix elements of the effective operators have the same large logarithms

$\mu_0 \sim O(M_W)$
$\downarrow$ RGE
$\mu_b \sim O(m_b)$: matrix elements of the operators at this scale don't have large logs; they are contained in the $C_i(\mu_b)$

• Evaluation of the on-shell amplitudes at $\mu_b \sim m_b$

Status of the SM calculations for $\bar{B} \to X_s \gamma$

Matching $(\mu_0 \sim M_W, m_t)$:

$$C_i(\mu_0) = C_i^{(0)}(\mu_0) + \frac{\alpha_s(\mu_0)}{4\pi} C_i^{(1)}(\mu_0) + \left(\frac{\alpha_s(\mu_0)}{4\pi}\right)^2 C_i^{(2)}(\mu_0)$$

$i = 1, ..., 6$: tree — 1-loop — 2-loop [Bobeth, Misiak, Urban, NPB 574 (2000) 291]

$i = 7, 8$: 1-loop — 2-loop — 3-loop [Steinhauser, Misiak, hep-ph/0401041]

The 3-loop matching has less than 2% effect on BR$(\bar{B} \to X_s \gamma)$

Mixing:

$$\hat{\gamma} = \frac{\alpha_s}{4\pi} \begin{pmatrix} 1\text{L} & 2\text{L} \\ 0 & 1\text{L} \end{pmatrix} + \left(\frac{\alpha_s}{4\pi}\right)^2 \begin{pmatrix} 2\text{L} & 3\text{L} \\ 0 & 2\text{L} \end{pmatrix} + \left(\frac{\alpha_s}{4\pi}\right)^3 \begin{pmatrix} 3\text{L} & 4\text{L} \\ 0 & 3\text{L} \end{pmatrix}$$

Haisch, Gorbahn, Gambino, Schröder, Czakon

Matrix elements $(\mu_b \sim m_b)$:

$$\langle O_i \rangle(\mu_b) = \langle O_i \rangle^{(0)}(\mu_b) + \frac{\alpha_s(\mu_b)}{4\pi} \langle O_i \rangle^{(1)}(\mu_b) + \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^2 \langle O_i \rangle^{(2)}(\mu_b)$$

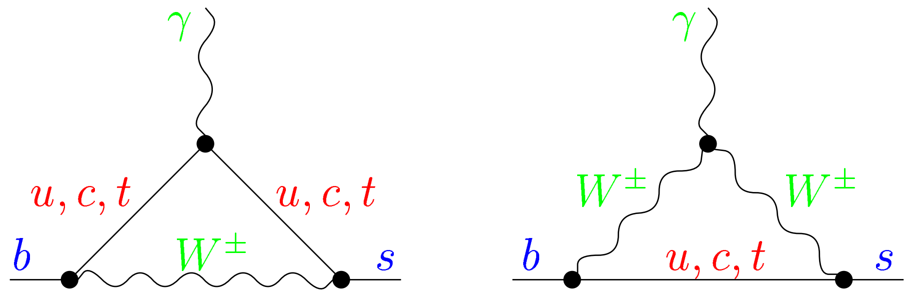
$i = 1, ..., 6$: 1-loop — 2-loop — 3-loop [Bieri, Greub, Steinhauser, hep-ph/0302051]

$\mathcal{O}(\alpha_s^2 n_f)$, Steinhauser, Misiak

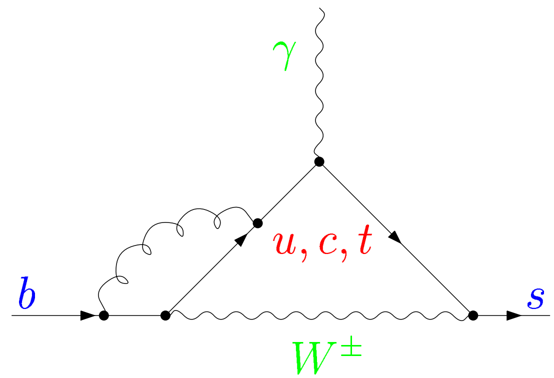
$i = 7, 8$: tree — 1-loop — 2-loop [Greub, Hurth, Asatrian]

Examples of SM diagrams for the matching of $C_7(\mu_0)$:

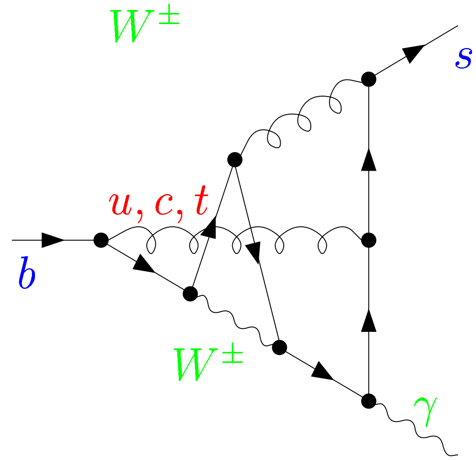
LO:
[Inami, Lim, 1981]



NLO:
[Adel, Yao, 1993]



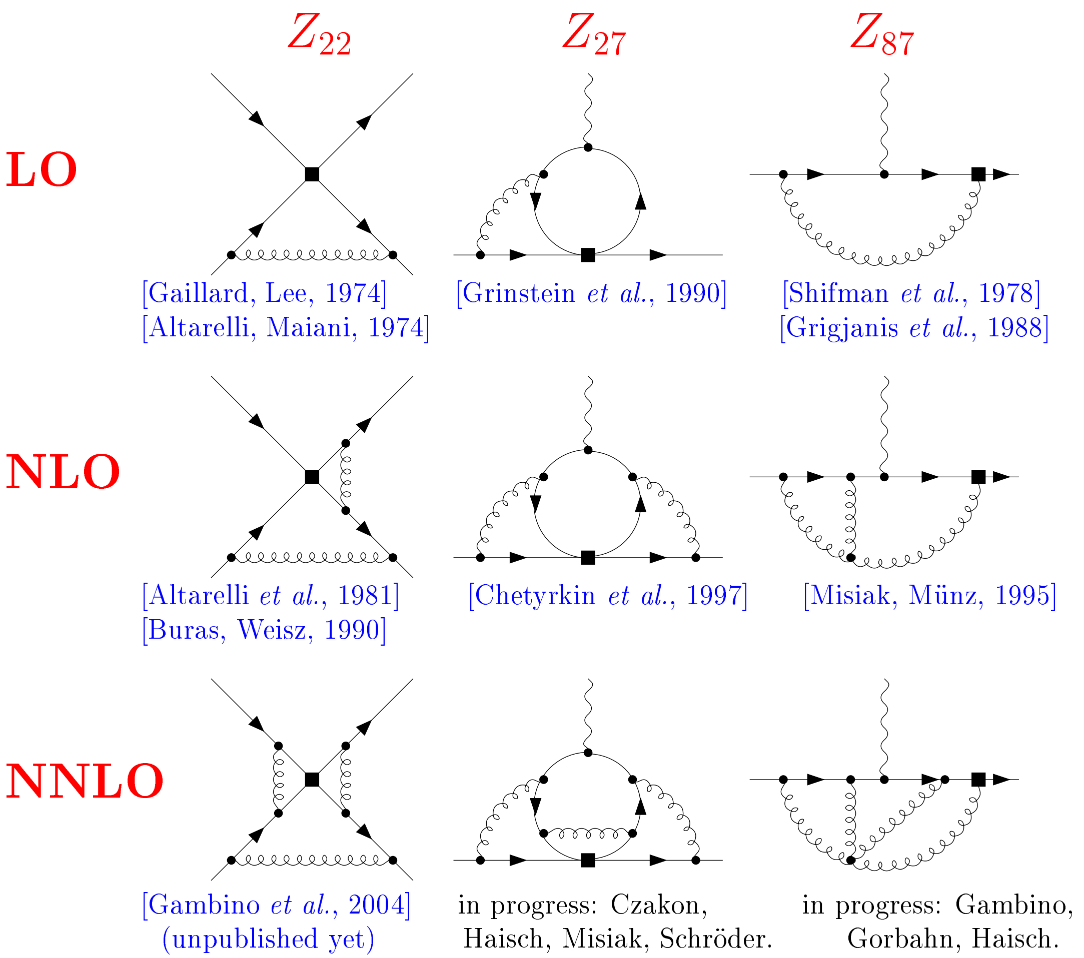
NNLO:
[Steinhauser, Misiak, 2004]



Resummation of large logarithms $\left(\alpha_s \ln \frac{M_W^2}{m_b^2}\right)^n$ in $b \to s\gamma$ amplitude

RGE for the Wilson coefficients $\mu \frac{d}{d\mu} C_j(\mu) = C_i(\mu)\gamma_{ij}(\mu)$

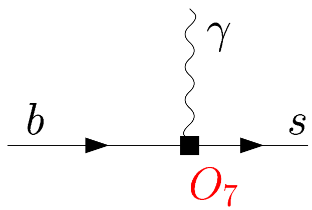
- Renormalization constants $\Longrightarrow$ γ_{ij}: $C_j(\mu)$ known in the meanwhile to NLL accuracy



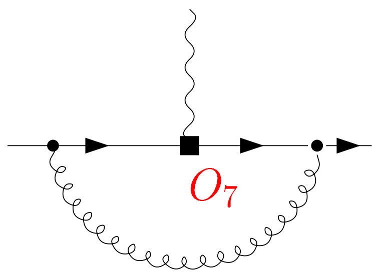
The $b \to s\gamma$ matrix elements

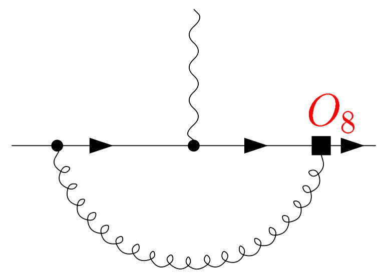
Perturbative on-shell amplitudes

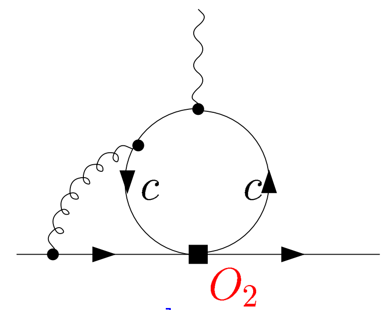
LO



NLO



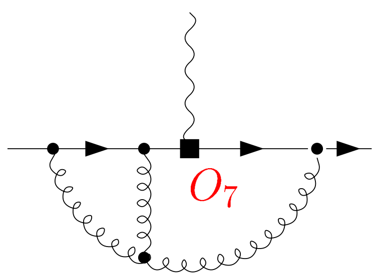


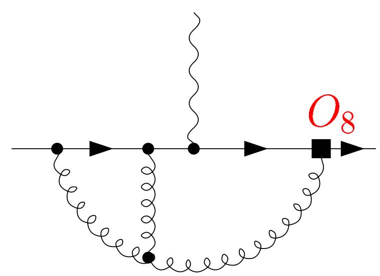


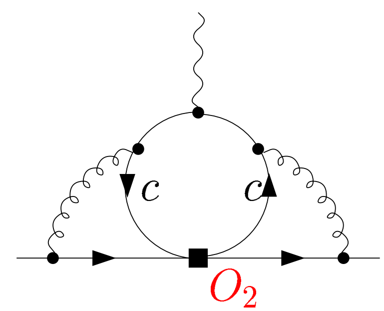
[Ali, Greub, 1991]

[Greub, Hurth, Wyler, 1996]

NNLO







in progress: Asatrian, Greub, Hurth

[Bieri *et al*, 2003] ($\mathcal{O}(\alpha_s^2 n_f)$)
in progress: Steinhauser, Misiak
(extrapolation in m_c)

Wilson Coefficients in the SM

Wilson Coefficients of Four-Quark Operators

	$C_1(\mu_b)$	$C_2(\mu_b)$	$C_3(\mu_b)$	$C_4(\mu_b)$	$C_5(\mu_b)$	$C_6(\mu_b)$
LL	-0.257	1.112	0.012	-0.026	0.008	-0.033
NLL	-0.151	1.059	0.012	-0.034	0.010	-0.040

Wilson Coefficients of Other Operators

	$C_7^{\text{eff}}(\mu_b)$	$C_8^{\text{eff}}(\mu_b)$	$C_9(\mu_b)$	$C_{10}(\mu_b)$
LL	-0.314	-0.149	2.007	0
NLL	-0.308	-0.169	4.154	-4.261
NNLL	-0.290		4.214	-4.312

- Obtained for the following input:

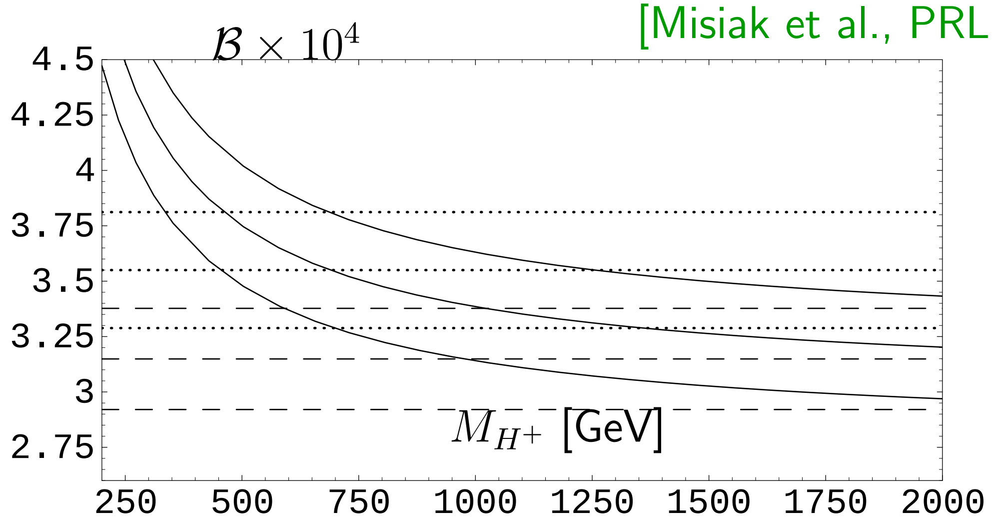
$$\mu_b = 4.6 \text{ GeV} \qquad \bar{m}_t(\bar{m}_t) = 167 \text{ GeV}$$

$$M_W = 80.4 \text{ GeV} \qquad \sin^2\theta_W = 0.23$$

- Three-loop running is used for α_s coupling with $\Lambda_{\overline{\text{MS}}}^{(5)} = 220$ MeV

$\mathcal{B}(\bar{B} \to X_s \gamma)$: Experiment vs. SM & 2HDM

[Misiak et al., PRL 98:022002 (2007)]



[$\cdots$ (exp); $- - - -$ (SM); solid (2HDM)]

- Expt. [Summer 2013]: ($E_\gamma > 1.6$ GeV): $\mathcal{B}(\bar{B} \to X_s \gamma) = (3.43 \pm 0.21) \times 10^{-4}$
- NNLO SM: $\mathcal{B}(\bar{B} \to X_s \gamma) = (3.15 \pm 0.23) \times 10^{-4}$
- Ratio=Expt./SM $= 1.09 \pm 0.10$, Limits most NP models
- In 2HDM, $\mathcal{B}(B \to X_s \gamma)$ bounds M_{H^+} and $\tan \beta$

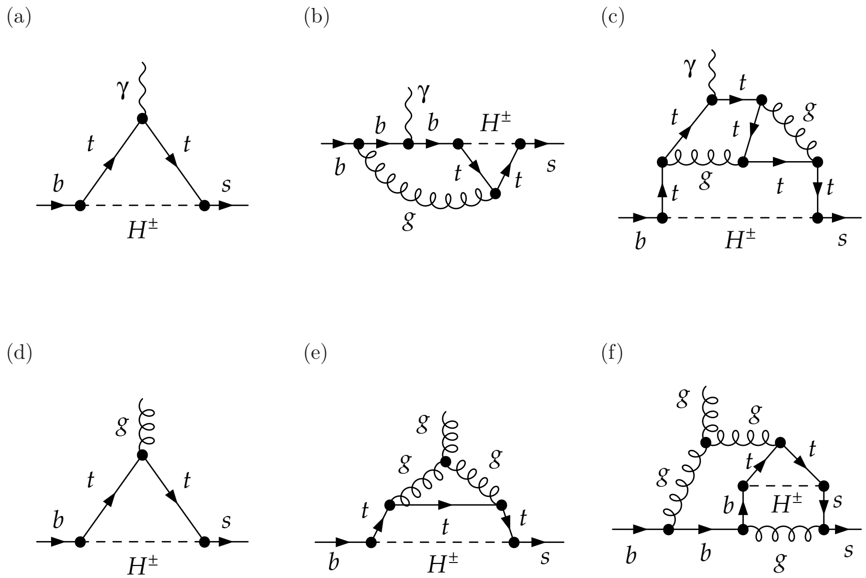
$B \to X_s\gamma$ in 2HDM

- NNLO in 2HDM calculated recently [Hermann, Misiak, Steinhauser; arxiv:1208.2788]

$$\mathcal{L}_{H^+} = (2\sqrt{2}G_F)^{1/2}\Sigma_{i,j=1}^{3}\bar{u}_i(A_u m_{u_i} V_{ij} P_L - A_d m_{d_j} V_{ij} P_R) d_j H^* + h.c.$$

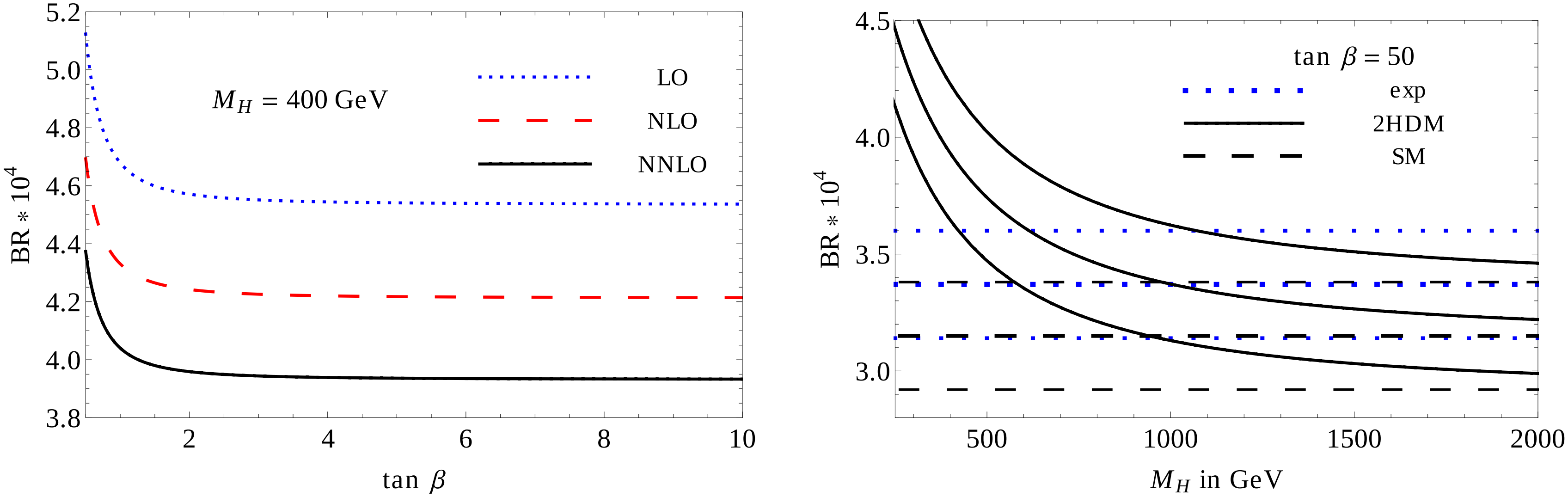
with $P_{L/R} = (1 \mp \gamma_5)/2$

- 2HDM contributions to the Wilson coefficients are proportional to $A_i A_j^*$
  - 2HDM of type-I: $A_u = A_d = \frac{1}{\tan\beta}$
  - 2HDM of type-II: $A_u = -1/A_d = \frac{1}{\tan\beta}$



$\bar{B} \to X_s \gamma$ in Type-II 2HDM

[Hermann, Misiak, Steinhauser; arxiv:1208.2788]



- $M_{H^+} > 380$ GeV (at 95% C.L.)
- $M_{H^+} > 289$ GeV (at 99% C.L.)

Experimental data

Experimental Data on $B \to V\gamma$ Decays

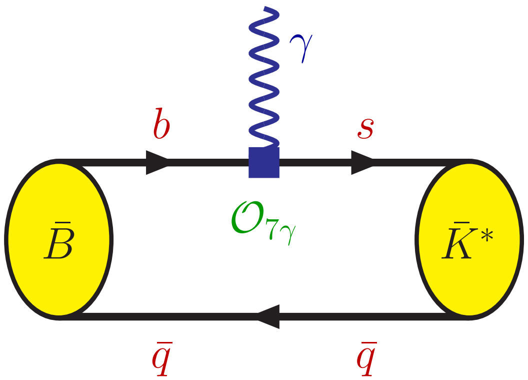
Branching ratios (in units of 10^{-6}) [HFAG, Summer 2013]

Mode	BABAR	BELLE	CLEO	Average [HFAG]
$B \to X_s\gamma$	$332 \pm 16 \pm 30$	$350 \pm 15 \pm 41$	$328 \pm 44 \pm 28$	343 ± 21 ‡
$B^+ \to K^*(892)^+\gamma$	$42.1 \pm 1.4 \pm 1.6$	$42.5 \pm 3.1 \pm 2.4$	$37.6^{+8.9}_{-8.3} \pm 2.8$	42.1 ± 1.8
$B^0 \to K^*(892)^0\gamma$	$44.7 \pm 1.0 \pm 1.6$	$40.1 \pm 2.1 \pm 1.7$	$45.5^{+7.2}_{-6.8} \pm 3.4$	43.3 ± 1.5
$B^+ \to K_1(1270)^+\gamma$		$43 \pm 9 \pm 9$		43 ± 12
$B^+ \to K_2^*(1430)^+\gamma$	$14.5 \pm 4.0 \pm 1.5$			14.5 ± 4.3
$B^0 \to K_2^*(1430)^0\gamma$	$12.2 \pm 2.5 \pm 1.0$	$13.0 \pm 5.0 \pm 1.0$		12.4 ± 2.4
$B^+ \to \rho^+\gamma$	$1.20^{+0.42}_{-0.37} \pm 0.20$	$0.87^{+0.29+0.09}_{-0.27-0.11}$	< 13.0	$0.98^{+0.25}_{-0.24}$
$B^0 \to \rho^0\gamma$	$0.97^{+0.24}_{-0.22} \pm 0.06$	$0.78 \pm 0.17 \pm 0.09$	< 17.0	0.86 ± 0.14
$B^0 \to \omega\gamma$	$0.50^{+0.27}_{-0.23} \pm 0.09$	$0.40^{+0.19}_{-0.17} \pm 0.11$	< 9.2	$0.44^{+0.18}_{-0.16}$
$B \to (\rho, \omega)\gamma$	$1.63 \pm 0.29 \pm 0.16$	$1.14 \pm 0.20 \pm 0.11$	< 14.0	1.30 ± 0.18
$B^0 \to \phi\gamma$	< 0.85		< 3.3	< 0.85
$B^0 \to J/\psi\,\gamma$	< 1.6			< 1.6

‡ Calculated for the photon energy range $E_\gamma > 1.6$ GeV

$B \to K^*\gamma$ Branching Fraction in LO

- In LO, only the electromagnetic penguin operator $\mathcal{O}_{7\gamma}$ contributes to the $B \to K^*\gamma$ amplitude; involves the form factor $T_1^{(K^*)}(0)$



$$\mathcal{M}^{\rm LO} = -\frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* C_7^{(0)\rm eff} \frac{e\bar{m}_b}{4\pi^2} T_1^{(K^*)}(0) \left[(Pq)(e^*\varepsilon^*) - (e^*P)(\varepsilon^*q) + i\,{\rm eps}(e^*, \varepsilon^*, P, q)\right]$$

Here, $P^\mu = p_B^\mu + p_K^\mu$; $q^\mu = p_B^\mu - p_K^\mu$ is the photon four-momentum; e^μ is its polarization vector; ε^μ is the K^*-meson polarization vector

- Branching ratio:

$$\mathcal{B}^{\rm LO}(B \to K^*\gamma) = \tau_B \frac{G_F^2 |V_{tb}V_{ts}^*|^2 \alpha M^3}{32\pi^4} \bar{m}_b^2(\mu_b) \, |C_7^{(0)\rm eff}(\mu_b)|^2 \, |T_1^{(K^*)}(0)|^2$$

$B \to K^* \gamma$ decay rates in NLO

- Perturbative improvements undertaken in three approaches (QCD-F; PQCD; SCET)
- Will concentrate on the QCD-F and SCET approaches

Factorization Ansatz (QCDF):

[Beneke, Buchalla, Neubert, Sachrajda; Beneke & Feldmann]

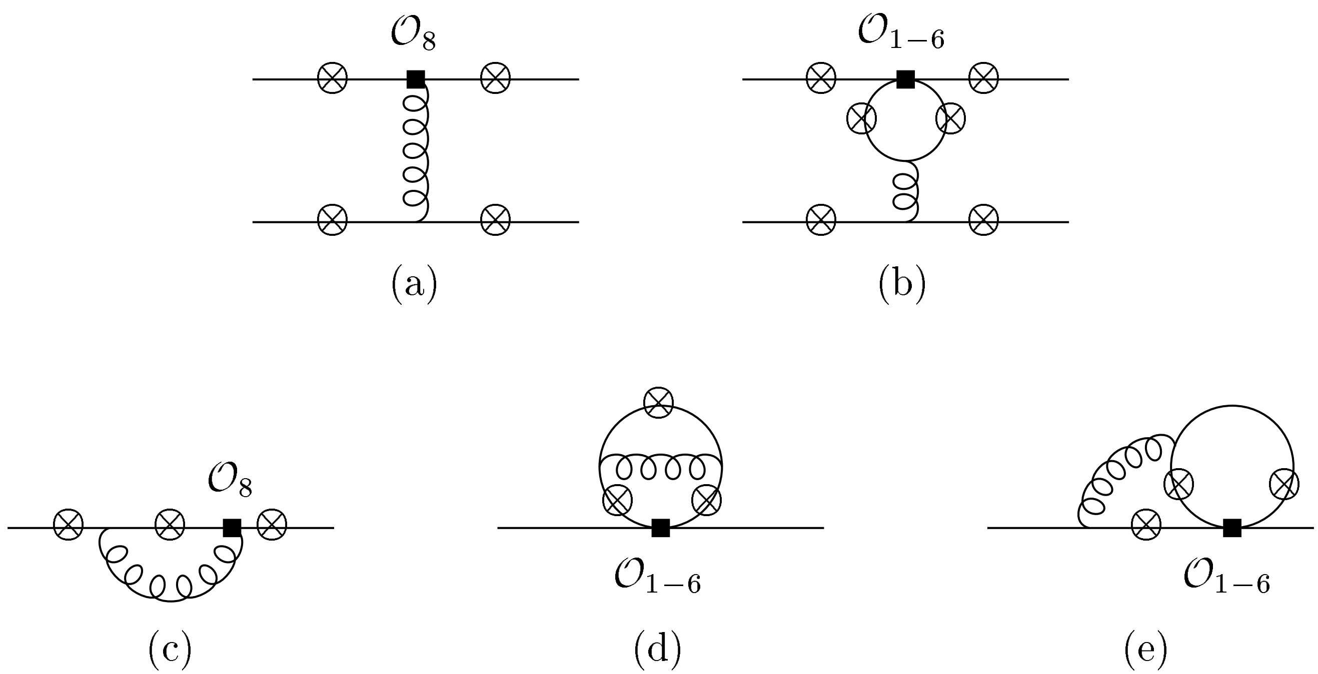
$$\langle V\gamma|Q_i|\bar{B}\rangle = t_i^I \zeta_{V_\perp} + t_i^{II} \otimes \phi_+^B \otimes \phi_\perp^V + \mathcal{O}(\frac{\Lambda_{\mathrm{QCD}}}{m_b})$$

- $\zeta_{V_\perp}$ (form factor) and $\phi^{B,V}$ (LCDAs) are non-perturbative functions
- t^I and t^{II} are perturbative hard-scattering kernels

$$t^I = \mathcal{O}(1) + \mathcal{O}(\alpha_s) + ..., \quad t^{II} = \mathcal{O}(\alpha_s) + ...$$

- The kernels t^I and t^{II} are known at $\mathcal{O}(\alpha_s)$ for some time; include Hard-scattering and Vertex corrections
  [Parkhomenko, AA; Bosch, Buchalla; Beneke, Feldmann, Seidel 2001]

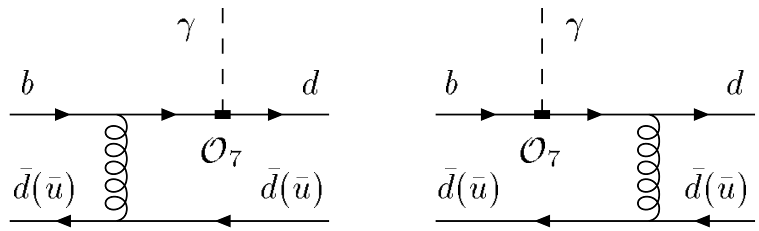
Nonfactorizable α_s Corrections



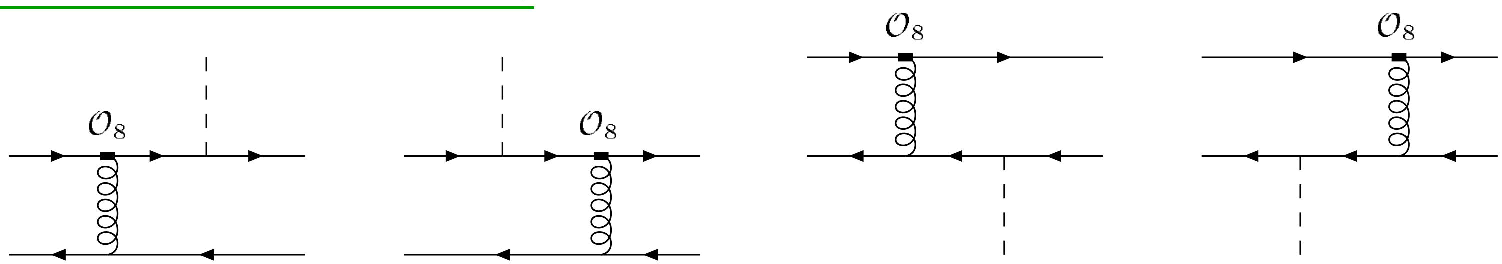
- First line: hard-spectator corrections
- Second line: $b \to s\gamma$ vertex corrections

Hard spectator contributions in $B \to (K^*, \rho)\,\gamma$

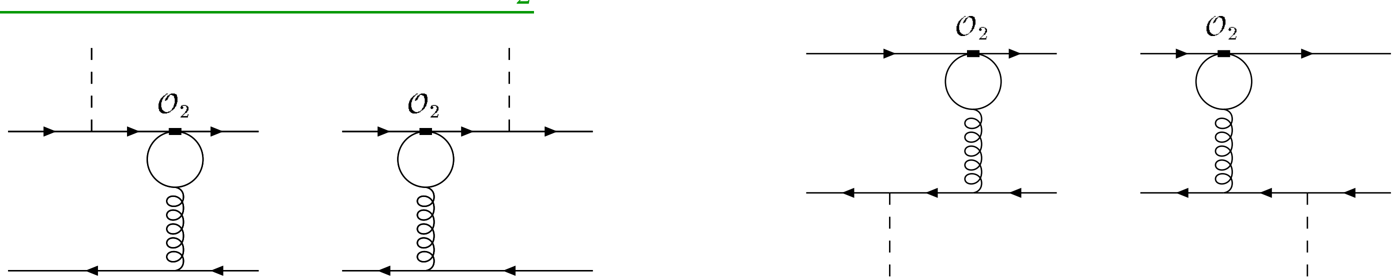
Spectator corrections due to $\mathcal{O}_7$



Spectator corrections due to $\mathcal{O}_8$



Spectator corrections due to $\mathcal{O}_2$



Comparison with data

$$\mathcal{B}_{\rm th}(B \to K^*\gamma) = \tau_B \frac{G_F^2 \alpha |V_{tb}V_{ts}^*|^2}{32\pi^4} m_{b,\rm pole}^2 M^3 \left[\xi_\perp^{(K^*)}\right]^2 \left(1 - \frac{m_{K^*}^2}{M^2}\right)^3 K_{\rm NLO} \left|C_7^{(0)\rm eff}\right|^2$$

$$K_{\rm NLO} = \frac{\left|C_7^{(0)\rm eff} + A^{(1)}(\mu)\right|^2}{\left|C_7^{(0)\rm eff}\right|^2} \quad \text{with} \quad 1.5 \leq K \leq 1.7$$

$$\mathcal{B}_{\rm th}(B^0 \to K^{*0}\gamma) \simeq (6.9 \pm 1.1) \times 10^{-5} \left(\frac{m_{b,\rm pole}}{4.65\ \text{GeV}}\right)^2 \left(\frac{\xi_\perp^{(K^*)}}{0.35}\right)^2$$

$$\mathcal{B}_{\rm th}(B^\pm \to K^{*\pm}\gamma) \simeq (7.4 \pm 1.2) \times 10^{-5} \left(\frac{m_{b,\rm pole}}{4.65\ \text{GeV}}\right)^2 \left(\frac{\xi_\perp^{(K^*)}}{0.35}\right)^2$$

- $T_1^{K^*}(0) = (1 + O(\alpha_s))\xi_\perp^{(K^*)}(0)$
  [Beneke, Feldmann]

Current Experimental Average

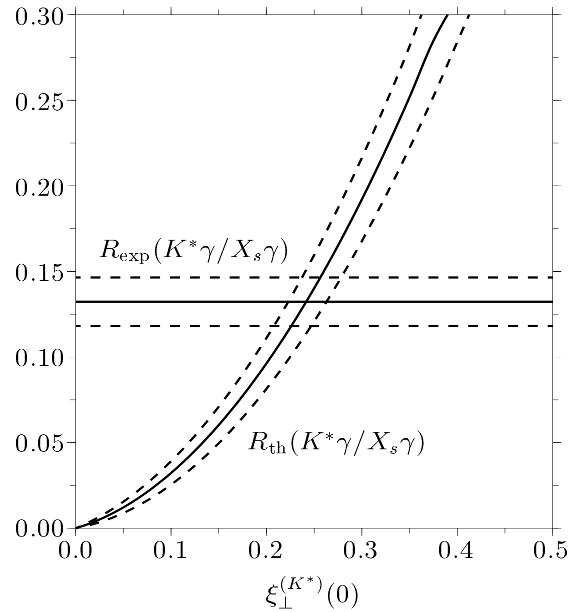
$\mathcal{B}(B^0 \to K^{*0}\gamma) = (4.33 \pm 0.15) \times 10^{-5}$

$\mathcal{B}(B^\pm \to K^{*\pm}\gamma) = (4.21 \pm 0.18) \times 10^{-5}$

$R_{\rm exp}(K^*\gamma/X_s\gamma) = 0.124 \pm 0.012$

$\Longrightarrow \quad T_1^{K^*}(0) = 0.27 \pm 0.02$

$= 0.33 \pm 0.06$ (Lattice-QCD: Z. Liu et al., arxiv: 1101.2726)



Basics of SCET: $B \to K^*\gamma$ and $B_s \to \phi\gamma$ decays

- The connection between SCET and perturbative QCD is provided by the method of regions [Smirnov; Beneke, Smirnov]
- A number of different momentum regions appear in the analysis. To identify these, introduce two light-like vectors $n_\pm$

  $n^\mu = (1,0,0,1)$, $\bar{n}^\mu = (1,0,0,-1)$, satisfying $n^2 = \bar{n}^2 = 0$ and $n \cdot \bar{n} = 2$

- The outgoing K^* is assumed to be along the n_- direction, and define n_+ such that the velocity of the b quark is given by

$$v^\mu = n_-^\mu \frac{n_+ v}{2} + n_+^\mu \frac{n_- v}{2}$$

- To perform the expansion in $1/m_b$, we define the parameter $\Lambda^2 = (p_B - m_b v)^2$ and the dimensionless parameter $\lambda = \Lambda/m_b \ll 1$
- The regions are classified according to the scaling of their light-cone components with the expansion parameter λ

- Denoting the light-cone components of a generic four-vector p by $(n_+p, p_\perp, n_-p)$, the relevant momentum regions are

Perturbative

hard	$m_b(1,1,1)$
hard-collinear	$m_b(1,\sqrt{\lambda},\lambda)$

Non-perturbative

soft	$m_b(\lambda,\lambda,\lambda)$
collinear	$m_b(1,\lambda,\lambda^2)$
soft-collinear	$m_b(\lambda,\lambda^{3/2},\lambda^2)$

- Momentum scales: $m_b^2 \sim 5$ GeV^2 ("hard"); $m_b\Lambda_{\rm QCD} \sim 1.5$ GeV^2 ("hard-collinear"); $\Lambda_{\rm QCD}^2 \sim (0.5)$ GeV^2 ("soft, collinear)
- Factorize the two perturbative scales m_b^2 and $m_b\Lambda$ using a two-step matching procedure $\mathrm{QCD} \to \mathrm{SCET_I} \to \mathrm{SCET_{II}}$
- In the effective theory, contributions from the perturbative regions are encoded in Wilson coefficients of operators built from fields representing the regions of lower virtuality

Effective Fields of SCET

- Hard mode (h)
  $P \sim E(1,1,1)$, integrated out in QCD $\to$ SCET$_I$

- Hard-collinear mode (hc)
  $P \sim E(1,\sqrt{\lambda},\lambda)$, integrated out in SCET$_I$ $\to$ SCET$_{II}$,

$$\xi_{hc} \sim \sqrt{\lambda} \qquad A_{hc} \sim (1,\sqrt{\lambda},\lambda)$$

- Power counting
  $\xi_{hc} = \frac{\not{n}\not{\bar{n}}}{4}\psi_{hc} \Longrightarrow$

  $\int d^4x\, e^{ip\cdot x}\, \langle 0|\, T\,\{\xi_{hc}(x)\bar{\xi}_{hc}(0)\}\,|0\rangle = \frac{\not{n}\not{\bar{n}}}{4}(\frac{i\not{p}}{p^2})\frac{\not{\bar{n}}\not{n}}{4} = \frac{i\bar{n}\cdot p}{p^2}\frac{\not{n}}{2}$

  $d^4x \sim 1/d^4p \sim \lambda^{-2}$, $p^2 \sim \lambda$, $\bar{n}\cdot p \sim 1 \Longrightarrow \xi_{hc} \sim \sqrt{\lambda}$

- Collinear mode (c)
  $P \sim E(1,\lambda,\lambda^2)$, long-distance mode, $\quad \xi_c \sim \lambda \quad A_c \sim (1,\lambda,\lambda^2)$

- Soft mode (s)
  $P \sim E(\lambda,\lambda,\lambda)$, long-distance mode, $\quad q_s \sim \lambda^{3/2} \quad A_s \sim (\lambda,\lambda,\lambda)$

SCET approach to $B \to K^*\gamma$ decay

- The objects of interest are the hadronic matrix elements $\langle K^*\gamma|Q_i|B\rangle$
- First matching step: the hard scale m_b^2 is integrated out by matching the operators Q_i onto a set of operators in $\mathrm{SCET_I}$
- For $B \to V\gamma$, the matching takes the form

$$Q_i \to \Delta_i C^A J^A + \Delta_i C^{B1} \star J^{B1} + \Delta_i C^{B2} \star J^{B2}$$

- The momentum-space Wilson coefficients depend only on quantities at the hard scale m_b^2. The exact form of the operators $J^{(i)}$ is:

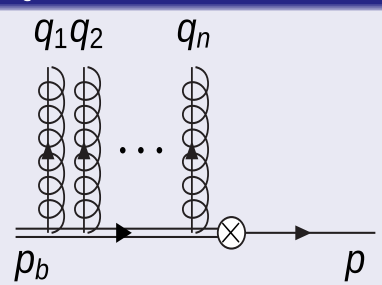
$$\begin{aligned} J^A &= (\bar{\xi} W_{hc})\, \not{\epsilon}_\perp (1-\gamma_5) h_v, \\ J^{B1} &= (\bar{\xi} W_{hc})\, \not{\epsilon}_\perp \not{\mathcal{A}}_{hc\perp} (1+\gamma_5) h_v, \\ J^{B2} &= (\bar{\xi} W_{hc})\, \not{\mathcal{A}}_{hc\perp} \not{\epsilon}_\perp (1+\gamma_5) h_v \end{aligned}$$

- The operators contain a hard-collinear quark field ξ, a composite object $\mathcal{A}_{hc}$, which in light-cone gauge is the hard-collinear gluon field, and W_{hc}, a Wilson line
- In SCET the b-quark field is treated as in HQET
- The B-type operators are power suppressed in $\mathrm{SCET_I}$, but contribute at the same order as the A-type operator upon the transition to $\mathrm{SCET_{II}}$

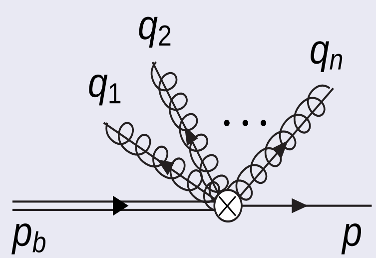
Defining $\chi_{\rm hc}$

Wilson lines

Full QCD



SCET$_I$



Hard-collinear Wilson line

$$W_{hc} = \mathrm{P}\exp\left(ig \int_{-\infty}^{y} ds\ \bar{n} \cdot A_{hc}(s\bar{n})\right)$$

$$\bar{q}\Gamma b \quad \Longrightarrow \quad (\bar{\xi}_{hc} W_{hc})\Gamma' h_v$$

- $\mathcal{X}_{hc} = W_{hc}^{\dagger} \xi_{hc}$: collinear gauge invariance

SCET factorization formula for $B \to K^* \gamma$

[Chay, Kim '03; Grinstein, Grossman, Ligeti '04; Becher, Hill, Neubert '05]

$$\langle V\gamma|Q_i|\bar{B}\rangle = \Delta_i C^A \zeta_{V_\perp} + (\Delta_i C^{B1} \otimes j_\perp) \otimes \phi_\perp^V \otimes \phi_+^B$$

- $\zeta_{V_\perp}, \phi_\perp^V, \phi_+^B$ are matrix elements of SCET operators
- Hard-scattering kernels t^I, t^{II} = SCET matching coefficients

$$t_i^I = \Delta_i C^A(m_b); \quad t_i^{II} = \Delta_i C^{B1}(m_b) \otimes j_\perp(\sqrt{m_b \Lambda}) \quad \textbf{(subfactorization)}$$

- Derivation of factorization in SCET
  1) QCD $\to$ SCET$_I$: Integrate out m_b; Defines vertex corrections $\Delta_i C^A = t_i^I$

$$Q_i \to \Delta_i C^A(m_b) J^A + \Delta_i C^{B1}(m_b) \otimes J^{B1} + \ldots$$

  2) SCET$_I$ $\to$ SCET$_{II}$: Integrate out $\sqrt{m_b \Lambda_{\rm QCD}}$; Defines spectator corrections

$$J^{B1} \to j_\perp(\sqrt{m_b \Lambda_{\rm QCD}}) \otimes O^{B1,{\rm SCET}_{II}}(\Lambda_{\rm QCD})$$

  3) Large logs in t_i^{II} resummed by solving RG equations

$$[\Delta_i C^{B1} \otimes j_\perp] \to [\Delta_i C^{B1}(\mu_h) \otimes U(\mu_h, \mu_{hc}) \otimes j_\perp(\mu_{hc})]$$

$B \to K^* \gamma$ in SCET at NNLO

[Pecjak, Greub, AA '07]

Vertex Corrections

$$\Delta_i C^A = \Delta_7 C^{A(0)} \left[\Delta_{i7} + \frac{\alpha_s(\mu)}{4\pi} \Delta_i C^{A(1)} + \frac{\alpha_s^2(\mu)}{(4\pi)^2} \Delta_i C^{A(2)} \right]$$

- Contributions from O_7 and O_8 exact to NNLO $O(\alpha_s^2)$
- Contribution from O_2 exact at NLO $O(\alpha_s)$ but only large-β_0 limit at $O(\alpha_s^2)$

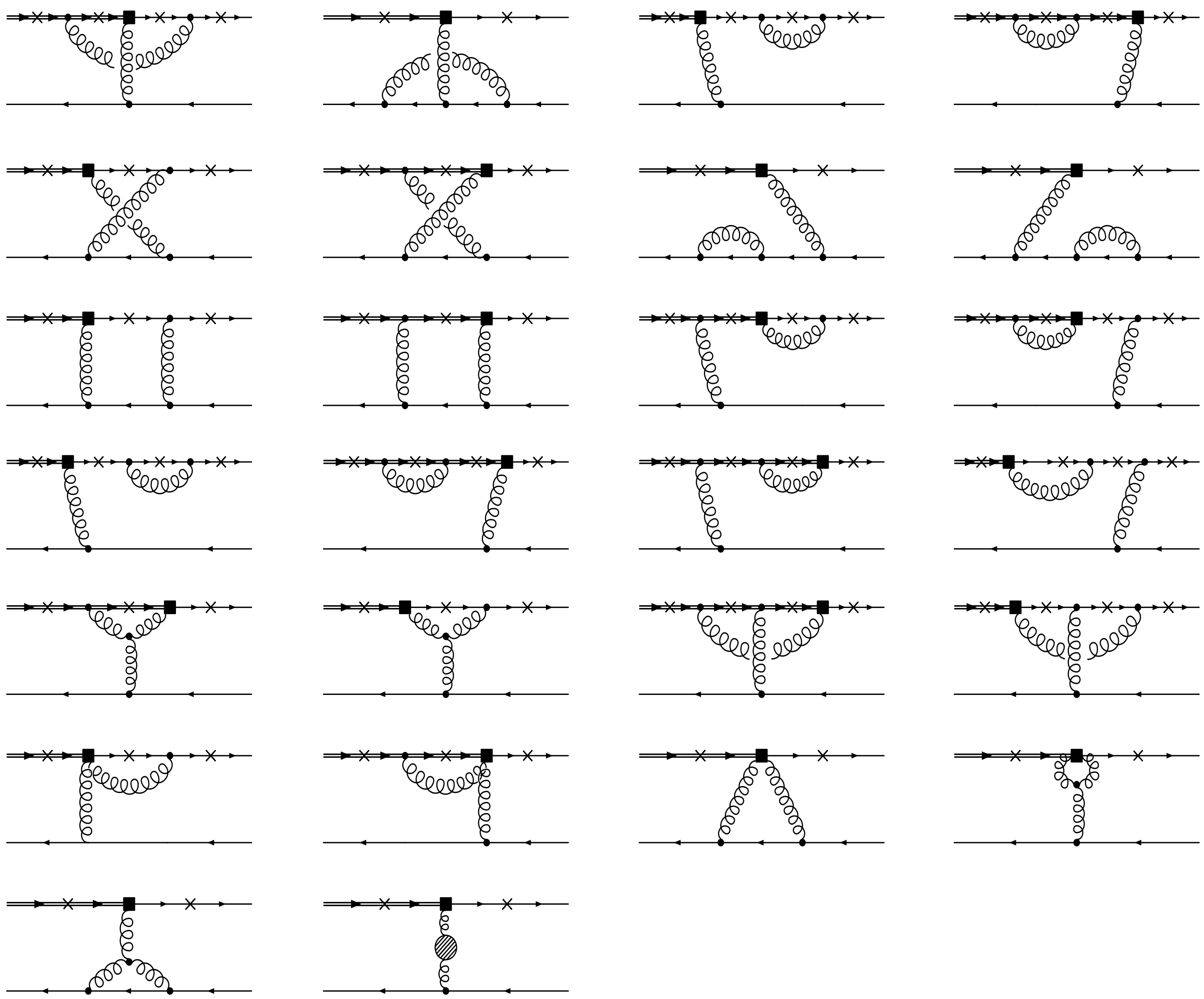
Spectator Corrections at $O(\alpha_s^2)$

$$t_i^{II(1)}(u,\omega) = \Delta_i C^{B1(1)} \otimes j_\perp^{(0)} + \Delta_i C^{B1(0)} \otimes j_\perp^{(1)}$$

- Status of $O(\alpha_s^2)$ Calculations
  - The one-loop jet-function $j_\perp^{(1)}$ known [Becher and Hill '04; Beneke and Yang '05]
  - The one-loop hard coefficient $\Delta_7 C^{B1(1)}$ known [Beneke, Kiyo, Yang '04; Becher and Hill '04]
  - The one-loop hard coefficient $\Delta_8 C^{B1(1)}$ known [Pecjak, Greub, AA '07]
  - $\Delta_i C^{B1(1)}$ $(i = 1, ..., 6)$ remain unknown (require two loops)

One-loop corrections to spectator scattering with O_8

Greub, Pecjak, AA (2008)



Estimates of BR($B \to K^* \gamma$) in SCET at NNLO

[Pecjak, Greub, AA; EPJ C55: 577 (2008)]

Estimates at NNLO in units of 10^{-5}

$$\mathcal{B}(B^+ \to K^{*+}\gamma) = 4.6 \pm 1.2[\zeta_{K^*}] \pm 0.4[m_c] \pm 0.2[\lambda_B] \pm 0.1[\mu]$$

[Expt. 4.2 ± 0.18 (HFAG 2012)];

$$\mathcal{B}(B^0 \to K^{*0}\gamma) = 4.3 \pm 1.1[\zeta_{K^*}] \pm 0.4[m_c] \pm 0.2[\lambda_B] \pm 0.1[\mu]$$

[Expt.: 4.33 ± 0.15 (HFAG 2012)];

$$\mathcal{B}(B_s \to \phi\gamma) = 4.3 \pm 1.1[\zeta_{\phi}] \pm 0.3[m_c] \pm 0.3[\lambda_B] \pm 0.1[\mu]$$

[Expt.: $5.7^{+1.8}_{-1.5} \pm 1.1$ (BELLE); 3.5 ± 0.4 (LHCb)]

Comparison with current experiments

- $\frac{\mathcal{B}(B^+ \to K^{*+}\gamma)_{\rm NNLO}}{\mathcal{B}(B^+ \to K^{*+}\gamma)_{\rm exp}} = 1.10 \pm 0.35[\text{theory}] \pm 0.07[\text{exp}]$
- $\frac{\mathcal{B}(B^0 \to K^{*0}\gamma)_{\rm NNLO}}{\mathcal{B}(B^+ \to K^{*0}\gamma)_{\rm exp}} = 1.00 \pm 0.32[\text{theory}] \pm 0.07[\text{exp}]$
- $\frac{\mathcal{B}(B_s \to \phi\gamma)_{\rm NNLO}}{\mathcal{B}(B_s \to \phi\gamma)_{\rm exp}} = 1.23 \pm 0.32[\text{theory}] \pm 0.11[\text{exp}]$
- Theory error is about 30%; dominantly from $\zeta_{V_\perp}$, m_c and λ_B; SM decay rates in good agreement with the data

Photon Polarization in Radiative B-Decays

- SM predicts that photons are mainly left-handed in $b \to q\gamma$ ($q = s, d$) decay
- Define the ratio (ϕ: weak phase, δ: strong phase)

$$r_q\, e^{i(\phi_q+\delta_q)} \equiv \frac{A_R}{A_L} \equiv \frac{A(\bar{B}^0 \to f_q \gamma_R)}{A(\bar{B}^0 \to f_q \gamma_L)}$$

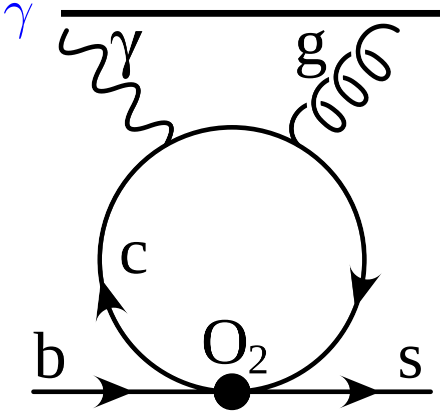
- If one assumes that the dipole operator $(\mathcal{O}_7)$ is dominant, then $r_q = m_q/m_b \ll 1$ in the SM [Atwood et al. (1997)]
- However, in the SM, the dominant contribution to r_q in inclusive decays $b \to (s,d)\gamma$ arises from QCD corrections, dominated by the four-quark operator $(\mathcal{O}_2)$
- In exclusive decays, such as $B \to K^*\gamma, B \to \rho\gamma$ and $B_s^0 \to \phi\gamma$, r_q arises from power correction of $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$ [Grinstein et al. (2005)]
- Time dependent CP asymmetry in $B \to f\gamma$, with f a CP eigenstate, is sensitive to r

$$\frac{\Gamma[\overline{B^0}(t) \to f\gamma] - \Gamma[B^0(t) \to f\gamma]}{\Gamma[\overline{B^0} \to f\gamma] + \Gamma[B^0(t) \to f\gamma]} = S_{f\gamma} \sin(\Delta m\, t) - C_{f\gamma} \cos(\Delta m\, t)$$

- In the SM, ϕ_s and $C_{f_s\gamma}$ are suppressed by $|(V_{ub}V_{us})/(V_{tb}V_{ts})|$

$$S_{f_s\gamma} = -2\, r_s \cos\delta_s \sin 2\beta$$

Photon Polarization in inclusive decay $\bar{B} \to X_s \gamma$



- Leading contribution to the inclusive decay $\bar{B} \to X_s \gamma_R$ is of $O(\alpha_s)$, arising dominantly from the bremsstrahlung contribution to the matrix element of the operator $(\mathcal{O}_2)$

- This diagram yields equal rates for $X_s\gamma_L$ and $X_s\gamma_R$ and the quantity r_s can be obtained by integrating the double differential rate $d\Gamma_{22}^{\rm brems}/dE_\gamma E_g$ [Greub, AA (91)]

$$\frac{\Gamma_{22}^{(\rm brem)}}{\Gamma_0} \simeq 0.025\,, \qquad \Gamma_0 = \frac{G_F^2 |V_{tb}V_{ts}^*|^2 \alpha_{\rm em}\, C_7^2\, m_b^5}{32\pi^4}$$

- At lowest order in g_s: $\langle r_s \rangle\big|_{x>0.75} = \sqrt{\Gamma_{22}^{(\rm brem)}/(2\Gamma_0)} \simeq 0.11$
- Need to know the strong phase $\cos\delta_s$ to determine $S_{X_s\gamma}$. An estimate obtained from the absorptive and dispersive parts of the inclusive result yields a large strong phase $\cos^2\delta_s \simeq 0.3$ [Grinstein et al. (2005)]
- Current Measurements based on the $K_s\pi^0\gamma$ final state with $1.1~{\rm GeV} \leq m(K_s\pi^0) \leq 1.8$ GeV yield [BABAR & Belle, (HFAG 2012)]

$$S_{X_s\gamma} = -0.15 \pm 0.20 : C_{X_s\gamma} = -0.07 \pm 0.12$$

Photon Polarization in Exlusive decays $B(B_s) \to K^*(\phi)\gamma$

- The first step is to match the effective Hamiltonian in QCD for $b \to s\gamma$ to the effective Hamiltonian in SCET$_I$

$$H_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{8} C_i(\mu)\, O_i(\mu)$$

$\Longrightarrow$

$$\begin{aligned} H_{\rm eff}({\rm SCET}) &= \frac{G_F V_{tb} V_{ts}^*\, e}{\sqrt{2}\,\pi^2} E_\gamma \Big[c(\omega)\, \bar{s}_{n,\omega} \not{A}^\perp m_b P_L\, b_v \\ &+ b_{1L}(\omega_i)\, O^{(1L)}(\omega_i) + b_{1R}(\omega_i)\, O^{(1R)}(\omega_i) + \mathcal{O}(\lambda^2) \Big], \end{aligned}$$

- The photon momentum is $q_\mu = E_\gamma \bar{n}_\mu$, the collinear s quark moves along n_μ, and $\mathcal{A}_\mu^\perp$ denotes the transverse photon field.
- The operator in the first line in $H_{\rm eff}({\rm SCET})$ occurs at leading order in $\lambda = \sqrt{\Lambda/m_b}$, with its Wilson coefficient $c(\omega) = C_7 + \mathcal{O}[\alpha_s(m_b)]$
- The operators in the second line are the only SCET$_{\rm I}$ operators suppressed by λ that couple to a transverse photon and are allowed by power counting and s chirality

$$\begin{aligned} O^{(1L)}(\omega_1,\omega_2) &= \bar{s}_{n,\omega_1} \not{A}^\perp \left[\frac{1}{\bar{n}\cdot\mathcal{P}} ig \not{B}_n^\perp \right]_{\omega_2} P_R\, b_v \\ O^{(1R)}(\omega_1,\omega_2) &= \bar{s}_{n,\omega_1} \left[\frac{1}{\bar{n}\cdot\mathcal{P}} ig \not{B}_n^\perp \right]_{\omega_2} \not{A}^\perp P_R\, b_v \end{aligned}$$

where $ig\mathcal{B}_n^\nu \equiv [\bar{n}\cdot iD_c\,, iD_{c\perp}^\nu]$ is the collinear gluon field.

- The operators $O^{(1L)}$ and $O^{(1R)}$ couple only to $\gamma_{L,R}$, respectively. Their Wilson coefficients are

$$\begin{aligned} b_{1L}(\omega_1,\omega_2) &= C_7 + C_2/3 + \mathcal{O}[C_{3-6},\alpha_s(m_b)] \\ b_{1R}(\omega_1,\omega_2) &= -C_2/3 + \mathcal{O}[C_{3-6},\alpha_s(m_b)] \end{aligned}$$

- The complete calculations for r_{K^*}, r_ρ and r_ϕ in SCET are both technical, and not worked out completely. Following estimates are based on dimensional arguments [Grisntein et al. PR D71, 011504 (2005)]

$$\mathbf{r}_\phi \sim r_{K^*} \simeq \frac{1}{3}\frac{C_2}{C_7}\frac{\Lambda_{\rm QCD}}{m_b} \sim 0.1$$

$$\mathbf{r}_\rho \sim r_{K^*}\left[1 + \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}\left(C_{\rm loop}\,\frac{m_c^2}{m_b^2} + C_{\rm WA}\,4\pi\,\frac{\Lambda_{\rm QCD}}{m_b}\right)\right]$$

- $C_{\rm loop}$ term is due to the non-cancellation of the u and c-loops, $C_{\rm WA}$ term arises from the weak annihilation. Expect r_ρ similar to r_{K^*}
- Current Measurements [HFAG 2012]

$$S_{K^*\gamma} = -0.16 \pm 0.22; \quad C_{K^*\gamma} = -0.04 \pm 0.14$$

Summary

- Thanks to dedicated experiments and progress in theoretical techniques (Pert. QCD, Lattice-QCD, QCD Sum Rules, Heavy quark Expansion, SCET), rare B-decays are under quantitative control, but the precision varies between (10 - 30)%, dominated by the imprecise knowledge of form factors in exclusive decays

- Rare B-Decays provide invaluable constraints on Beyond-the-SM Physics; we discussed briefly 2HDM in $B \to X_s\gamma$ and right-handed polarizations in inclusive and exclusive radiative decays

- In particular, the amplitude ratios $r_{X_s\gamma}$, r_{K^*}, r_ρ and r_ϕ have to be measured significantly above 0.1 to establish right-handed currents, which will be discussed by the next speaker

- We look forward to new data from the ongoing and planned experiments at the LHC and the Super-B factories