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Effective Field Theory Analysis of New Physics in $e^+e^- \to W^+W^-$ at a Linear Collider

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Abstract

We analyze new physics contributions to $e^+e^- \to W^+W^-$ at the TeV energy scale, employing an effective field theory framework. A complete basis of next-to-leading order operators in the standard model effective Lagrangian is used, both for the nonlinear and the linear realization of the electroweak sector. The elimination of redundant operators via equations-of-motion constraints is discussed in detail. Polarized cross sections for $e^+e^- \to W^+W^-$ (on-shell) are computed and the corrections to the standard model results are given in an expansion for large s/M_W^2. The dominant relative corrections grow with s and can be fully expressed in terms of modified gauge-fermion couplings. These corrections are interpreted in the context of the Goldstone boson equivalence theorem. Explicit new physics models are considered to illustrate the generation and the potential size of the coefficients in the effective Lagrangian. Brief comments are made on the production of W^+W^- pairs at the LHC.

1 Introduction

During the last decades there has been an intense scrutiny of the standard model in the search for traces of new physics effects. However, up to the energy scales probed until now and except for some occasional tensions, the standard model has proven to be an extremely successful theory. Moreover, the latest results from the LHC not only seem to confirm the Higgs-like nature of the newly-found scalar [1,2] but continuously increase the gap between the standard model particles and the scale of new physics Λ.

In the absence of new heavy particles in direct searches we should expect new physics to be first seen as virtual effects. These can generically be encoded as anomalous couplings for the different sectors of the theory: gauge-fermion interactions, gauge boson interactions (oblique, triple-gauge and quartic) and scalar interactions. Given the large energy gap between the electroweak and the new physics scale, an effective field theory (EFT) treatment becomes the best strategy to parametrize the new physics effects in a model-independent way. The main virtue of the EFT treatment is that the standard model symmetries are automatically implemented in the anomalous couplings. The resulting constraints from $SU(2)_L \times U(1)_Y$ symmetry make it transparent that (i) the number of independent parameters is typically smaller than the number of couplings; (ii) arbitrarily setting some of the couplings to zero in experimental analysis is in general inconsistent with the electroweak symmetry; (iii) the naive scaling with energy of the form factors is ameliorated by $(SU(2)_L \times U(1)_Y)$-induced cancellations. Therefore, adopting an EFT becomes not a matter of choice, but the only way to ensure consistency at the field theoretical level. The advantages of the EFT approach have recently been re-emphasized in [3,4].

Global fits to the electroweak data using an EFT framework have been performed by several groups in the past. Unfortunately, the global analysis contains too many parameters and cross-correlations are too strong to obtain an informative fit [5,6]. As an alternative, the number of coefficients is commonly limited to a reduced set, inspired by the results of different models, and fits have been performed on this basis. A prototype of this approach is the well-known S, T, U parameter analysis of [7] .

In this paper we will explicitly show that the above shortcoming of the global fit to EFT couplings can be ameliorated by studying individual processes at high energies ($v \ll \sqrt{s} \ll \Lambda$). As an example, we will present a detailed EFT-based study of W^+W^- production at linear colliders. $e^+e^- \to W^+W^-$ has been the benchmark process in the study of charged triple-gauge corrections first at LEP [8] and subsequently for future linear collider facilities [9–19]. W^+W^- production at the LHC has been considered for instance in [17,20,21,22]. However, although there have been several studies in the literature emphasizing the need for an EFT approach to triple-gauge couplings in $e^+e^- \to W^+W^-$ [3,4], [23–27], a complete analysis is still missing.

The present analysis will be performed in the full nonlinear EFT basis recently studied in [28]. Our final results will provide expressions for the (initial and final state) polarized cross-sections in the large-s expansion, which is an excellent approximation for the projected energies at future linear colliders. The leading corrections will grow with

s relative to the standard model results, reflecting the fact that the nonlinear effective theory violates unitarity in the UV. The s/v^2 enhancement at energies where the EFT is still valid, improves the visibility of small new physics coefficients. Actually, the new physics effects at, say, $s = 800$ GeV could be typically as large as 20%.

One of the interesting properties of $e^+e^- \to W^+W^-$ is that, up to tiny mass corrections, it is independent of couplings to a physical Higgs sector. We will show that this is indeed the case by comparing our results with the linear EFT basis of [29,30]. Besides the results for the cross-sections, our main findings can be summarized as follows:

- Despite the sizable number of operators contributing to the process at next-to-leading order (NLO), the final result for new physics effects in the large-s limit can be encoded in terms of just three parameters. These can be expressed as the corrections to the left and right-handed gauge-fermion vertices.

- Three of the gauge-fermion operators and the three leading (C, P and CP-conserving) triple-gauge operators are related by field redefinitions. Therefore, in the case of $e^+e^- \to W^+W^-$, omitting the gauge-fermion operators is not an approximation but an exact field-theoretical result: they can be traded for triple-gauge operators and vice versa, depending on the chosen operator basis. We stress that this is because only three independent gauge-fermion couplings enter the process $e^+e^- \to W^+W^-$. In general, there are many more gauge-fermion operators and it is not possible to eliminate all of them.

The last point above implies that statements about gauge-fermion or triple-gauge operators *per se* are basis-dependent and therefore ill-defined. For instance, in the basis where gauge-fermion operators are kept, the electroweak fit [5] does not support the common claim that they are tightly constrained. Furthermore, our analysis contradicts the statement that W^+W^- production directly tests triple-gauge corrections. Rather, what one finds is that at large-s one can put bounds on gauge-fermion couplings or *equivalently* on triple-gauge couplings, since they are not independent.

The existence of field-theoretical relations binding gauge-fermion, triple-gauge and oblique operators raises the question of which basis should be preferred for experimental analyses of electroweak physics. In the particular case of $e^+e^- \to W^+W^-$ the possibility of eliminating the gauge-fermion operators altogether might suggest itself. However, in view of the general electroweak fit, it seems more natural to eliminate triple-gauge operators and keep the full set of gauge-fermion operators. As we will show, the emerging picture in this basis turns out to be rather simple: only a single triple-gauge operator appears ($\mathcal{O}_{XU3}$ in (8) below), which is both parity and isospin-breaking, and therefore expected to be numerically small.[1] Additionally, in the large-s limit oblique corrections and the surviving triple-gauge operator can be shown to be generically subleading, such that the leading large-s contribution naturally singles out gauge-fermion operators.

[1] CP-violating triple-gauge operators are also present but do not interfere with the standard model in the cross sections.

These rather simple and counterintuitive results follow from carefully eliminating redundant operators and therefore stress the importance of working with a complete and minimal basis in EFT-based analyses. Comments on how this picture would generalize to hadron colliders will be made but details will be left to future work.

This paper is organized as follows: in section 2 we will briefly review the EFT of the standard model at NLO and fix our notation and conventions. In section 3 we will apply the EFT formalism to $e^+e^- \to W^+W^-$, discussing in detail direct contributions and parameter redefinitions. In section 4 we collect the results for the differential cross sections for the different initial and final state polarizations. The issue of redundant operators and choice of basis is addressed in section 5. A complementary view of the large-s limit from the perspective of the equivalence theorem is given in section 6. In section 7 we discuss the case of a linearly-realized EFT. In order to get an estimate of the expected effects at linear colliders, in section 8 the size of EFT couplings is estimated from different benchmark UV completions. In section 9 we briefly comment on W^+W^- production at the LHC. Conclusions are given in section 10, while technical details are relegated to an Appendix.

2 Electroweak chiral Lagrangian at NLO

The starting point of our analysis is the well-known leading order chiral Lagrangian of the electroweak standard model. To define our notation we quote here the terms of the leptonic sector relevant for $e^+e^- \to W^+W^-$. They read

$$\mathcal{L}_{\rm LO} = -\frac{1}{2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{v^2}{4}\,\langle D_\mu U^\dagger D^\mu U\rangle + \bar{l}_L i\not{D} l_L + \bar{e}_R i\not{D} e_R \tag{1}$$

Here and in the following the trace of a matrix M is written as $\langle M\rangle$. The doublet of left-handed leptons is denoted by $l_L = (\nu_L, e_L)^T$, the right-handed electron by e_R, and we focus our attention on the first-generation fermions. The covariant derivatives of the fermions are

$$D_\mu l_L = \partial_\mu l_L + igW_\mu l_L - \frac{i}{2}g' B_\mu l_L, \qquad D_\mu e_R = \partial_\mu e_R - ig' B_\mu e_R \tag{2}$$

The electron mass is negligible and the associated Yukawa terms have been omitted from (1). Couplings to a physical Higgs field do not play a role in $e^+e^- \to W^+W^-$ and are likewise omitted from the Lagrangian. The Goldstone bosons of electroweak symmetry breaking are represented by the matrix field

$$U = \exp(2i\Phi/v), \qquad \Phi = \varphi^a T^a = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\varphi^0}{\sqrt{2}} & \varphi^+ \\ \varphi^- & -\frac{\varphi^0}{\sqrt{2}} \end{pmatrix} \tag{3}$$

with $T^a = T_a$ the generators of $SU(2)$. The U-field transforms as

$$U \to g_L U g_R^\dagger, \qquad g_{L,R} \in SU(2)_{L,R} \tag{4}$$

where g_L and the $U(1)_Y$ subgroup of g_R are gauged, so that the covariant derivative of U is given by

$$D_\mu U = \partial_\mu U + igW_\mu U - ig' B_\mu U T_3 \tag{5}$$

The effective Lagrangian (1) describes physics at the electroweak scale $v = 246\,\mathrm{GeV}$, assumed to be small in comparison with a new physics scale Λ. This Lagrangian is non-renormalizable in general, except when a Higgs field h is introduced with specific couplings, in which case the theory reduces to the conventional standard model (see e.g. [31] for a review). In the general case, additional terms will arise beyond the lowest order from the dynamics of electroweak symmetry breaking at the TeV scale. These subleading terms were first considered in [32–37]. A complete list of all NLO operators in this framework based on a systematic power counting has recently been given in [28]. Using the notation of this paper, the NLO operators relevant for $e^+e^- \to W^+W^-$ can be written as

$$\mathcal{L}_{\mathrm{NLO}} = \beta_1 \mathcal{O}_{\beta_1} + \sum_{i=1}^{6} C_{Xi}\mathcal{O}_{XUi} + \sum_{i=7}^{10} C_{Vi}\mathcal{O}_{\psi Vi} + C^*_{V9}\mathcal{O}^\dagger_{\psi V9} + \frac{C_{4f}}{\Lambda^2}\mathcal{O}_{4f} + \sum_{i=1}^{2}\frac{C_{Wi}}{\Lambda^2}\mathcal{O}_{Wi} \tag{6}$$

with operators $\mathcal{O}_k$ specified in (7) – (13). The complete basis of NLO operators [28] also contains the terms $\bar{e}_L e_R W^+_\mu W^{-\mu}$ and $\bar{e}_L \sigma^{\mu\nu} e_R W^+_\mu W^-_\nu$, which could in principle contribute to $e^+e^- \to W^+W^-$. Due to the chirality flip in the electron current the coefficients of these operators can be expected to be proportional to the Yukawa coupling of the electron and thus very much suppressed. In addition, the chirality-changing currents do not interfere with the vectorial currents of the leading-order amplitude. Those operators therefore give no first-order correction to the $e^+e^- \to W^+W^-$ cross sections and we have omitted them from (6). We have included the 4-fermion operator $\mathcal{O}_{4f}$, which contributes only indirectly through the renormalization of the Fermi constant G_F. Other 4-fermion operators from [28] do not give rise to first-order corrections to $e^+e^- \to W^+W^-$ cross sections and have been neglected.

The operators $\mathcal{O}_{Wi}$ (see (13) below) are strictly speaking terms that appear only at next-to-next-to-leading order (NNLO) in the effective Lagrangian. Their coefficients are generally loop-induced [38] and count as $C_{Wi} \sim 1/(16\pi^2) \sim v^2/\Lambda^2$, which multiplies the explicit prefactor $1/\Lambda^2$ in the last term of (6). We have included them here in order to facilitate the transition to the basis of operators within the framework of a linearly transforming Higgs field, to be considered in section 7. In this case they belong to the full list of operators of dimension 6, and we include them for completeness in our analysis. In the present context, and working consistently to NLO, the coefficients C_{Wi} may be put to zero.

All operators in the Lagrangian (6) are hermitian and have real coefficients, except $\mathcal{O}_{\psi V9}$. They have already been known from the work of [33,34,36]. However, the basis of operators used there contains redundant terms, which can be eliminated using the equations of motion [28,39,40].

The operators in (6) have the following explicit form, where the second expression in each case refers to unitary gauge with $U = 1$:

$$\mathcal{O}_{\beta_1} = v^2 \langle U^\dagger D_\mu U T_3 \rangle^2 = -M_Z^2 Z_\mu Z^\mu \tag{7}$$

$$\begin{aligned}
\mathcal{O}_{XU1} &= g' g \; B_{\mu\nu} \; \langle U^\dagger W^{\mu\nu} U T_3 \rangle = \frac{g' g}{2} \; B^{\mu\nu} W^3_{\mu\nu} \\
\mathcal{O}_{XU2} &= g^2 \; \langle U^\dagger W_{\mu\nu} U T_3 \rangle \; \langle U^\dagger W^{\mu\nu} U T_3 \rangle = \frac{g^2}{4} W^3_{\mu\nu} W^{3\mu\nu} \\
\mathcal{O}_{XU3} &= g \; \varepsilon^{\mu\nu\lambda\rho} \; \langle U^\dagger W_{\mu\nu} D_\lambda U \rangle \; \langle U^\dagger D_\rho U T_3 \rangle \\
&= \frac{g}{4} \varepsilon^{\mu\nu\lambda\rho} \left[g W^a_{\mu\nu} W^a_\lambda - g' W^3_{\mu\nu} B_\lambda \right] \left[g' B_\rho - g W^3_\rho \right] \\
\mathcal{O}_{XU4} &= g' g \; \varepsilon^{\mu\nu\lambda\rho} \; B_{\mu\nu} \; \langle U^\dagger W_{\lambda\rho} U T_3 \rangle = \frac{g' g}{2} \varepsilon^{\mu\nu\lambda\rho} B_{\mu\nu} W^3_{\lambda\rho} \\
\mathcal{O}_{XU5} &= g^2 \; \varepsilon^{\mu\nu\lambda\rho} \; \langle U^\dagger W_{\mu\nu} U T_3 \rangle \; \langle U^\dagger W_{\lambda\rho} U T_3 \rangle = \frac{g^2}{4} \varepsilon^{\mu\nu\lambda\rho} W^3_{\mu\nu} W^3_{\lambda\rho} \\
\mathcal{O}_{XU6} &= g \; \langle U^\dagger W_{\mu\nu} D^\mu U \rangle \; \langle U^\dagger D^\nu U T_3 \rangle \\
&= \frac{g}{4} \left[g W^a_{\mu\nu} W^{a\mu} - g' W^3_{\mu\nu} B^\mu \right] \left[g' B^\nu - g W^{3\nu} \right]
\end{aligned} \tag{8}$$

$$\begin{aligned}
\mathcal{O}_{\psi V7} &= \bar{l}_L \gamma^\mu l_L \; \langle U^\dagger i D_\mu U T_3 \rangle = -\frac{\sqrt{g^2 + g'^2}}{2} \bar{l}_L \gamma^\mu l_L Z_\mu \\
\mathcal{O}_{\psi V8} &= \bar{l}_L \gamma^\mu U T_3 U^\dagger l_L \; \langle U^\dagger i D_\mu U T_3 \rangle = -\frac{\sqrt{g^2 + g'^2}}{2} \bar{l}_L \gamma^\mu T_3 l_L Z_\mu \\
\mathcal{O}_{\psi V9} &= \bar{l}_L \gamma^\mu U P_{12} U^\dagger l_L \; \langle U^\dagger i D_\mu U P_{21} \rangle = -\frac{g}{\sqrt{2}} \bar{\nu}_L \gamma^\mu e_L W^+_\mu \\
\mathcal{O}_{\psi V10} &= \bar{e}_R \gamma^\mu e_R \; \langle U^\dagger i D_\mu U T_3 \rangle = -\frac{\sqrt{g^2 + g'^2}}{2} \bar{e}_R \gamma^\mu e_R Z_\mu
\end{aligned} \tag{9}$$

$$\mathcal{O}_{4f} = \frac{1}{2} (\mathcal{O}_{LL5} - 4 \mathcal{O}_{LL15}) = \bar{e}_L \gamma^\mu \mu_L \, \bar{\nu}_{\mu L} \gamma_\mu \nu_{eL} + h.c. \tag{10}$$

where the appropriate flavour structure is understood for $\mathcal{O}_{LL5}$, $\mathcal{O}_{LL15}$ from [28].

In (9) we have used the definitions $P_{12} \equiv T_1 + i T_2$, $P_{21} \equiv T_1 - i T_2$. It is convenient to work with the following linear combinations of operators $\mathcal{O}_{\psi V7,8}$

$$\mathcal{O}_{\psi V\pm} \equiv \frac{1}{2} \mathcal{O}_{\psi V7} \pm \mathcal{O}_{\psi V8} \tag{11}$$

whose coefficients become

$$C_{V\pm} \equiv C_{V7} \pm \frac{1}{2} C_{V8} \tag{12}$$

Only one of these coefficients, C_{V-}, appears in the amplitudes for $e^+e^- \to W^+W^-$ within our approximations. This is most clearly seen in unitary gauge, where $\mathcal{O}_{\psi V-}$ couples the Z to electrons and $\mathcal{O}_{\psi V+}$ to neutrinos.

Finally, the NNLO terms $\mathcal{O}_{Wi}$ are

$$\begin{aligned}
\mathcal{O}_{W1} &= g^3 \varepsilon^{abc} W_\mu^{a\nu} W_\nu^{b\lambda} W_\lambda^{c\mu} \\
\mathcal{O}_{W2} &= g^3 \varepsilon^{abc} \tilde{W}_\mu^{a\nu} W_\nu^{b\lambda} W_\lambda^{c\mu}
\end{aligned} \tag{13}$$

with

$$\tilde{W}^a_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} W^{a,\rho\sigma}, \qquad \varepsilon^{0123} = -1 \tag{14}$$

3 Anomalous couplings

The NLO terms in the effective Lagrangian modify the lowest order vertices of the standard model. Their effect can be cast in the form of anomalous couplings.

For the triple-gauge vertex (TGV), coupling a virtual, neutral vector boson V to a W^+W^- pair in the final state, the Feynman rule can be written as

$$V^\rho(k) \to W^{-\mu}(p) W^{+\nu}(q) : \qquad -i \left\{ \begin{matrix} g c_Z \\ g s_Z \end{matrix} \right\} \Gamma_V^{\mu\nu\rho}(p, q; k), \quad V = \left\{ \begin{matrix} Z \\ A \end{matrix} \right. \tag{15}$$

where [36,41]

$$\begin{aligned}
\Gamma_V^{\mu\nu\rho}(p, q; k) = {} & g_1^V (p - q)^\rho g^{\mu\nu} + (g_1^V + \kappa_V)(k^\mu g^{\nu\rho} - k^\nu g^{\mu\rho}) \\
& + i g_4^V (k^\mu g^{\nu\rho} + k^\nu g^{\mu\rho}) - i g_5^V \varepsilon^{\mu\nu\lambda\rho} (p - q)_\lambda + \tilde{\kappa}_V \varepsilon^{\mu\nu\lambda\rho} k_\lambda \\
& - \frac{\lambda_V}{\Lambda^2} (p - q)^\rho k^\mu k^\nu - \frac{\tilde{\lambda}_V}{\Lambda^2} (p - q)^\rho \varepsilon^{\mu\nu\sigma\tau} p_\sigma q_\tau
\end{aligned} \tag{16}$$

Here s_Z, c_Z are, respectively, sine and cosine of the weak mixing angle in the Z-standard definition ($\alpha = \alpha(M_Z)$)

$$s_Z^2 c_Z^2 \equiv \frac{\pi \alpha}{\sqrt{2} G_F M_Z^2} \tag{17}$$

and g is the $SU(2)_L$ gauge coupling, where $g s_Z = e = \sqrt{4\pi\alpha}$. The anomalous-coupling parameters in (16) encode deviations from the standard model, in which $g_1^V = \kappa_V = 1$ and $g_{4,5}^V = \tilde{\kappa}_V = \lambda_V = \tilde{\lambda}_V = 0$.

Similarly, the gauge-fermion interactions can be parametrized through the Feynman rules

$$\begin{array}{rccc}
\bar{\nu} e_{L,R} W : & -\frac{ig}{\sqrt{2}} \kappa_c \gamma^\mu P_L & & 0 \\
\bar{e} e_{L,R} Z : & \frac{ig}{2c_Z} (\kappa_1 - 2 s_Z^2 \kappa_2) \gamma^\mu P_L & & \frac{ig}{2c_Z} (-2 s_Z^2 \kappa_2) \gamma^\mu P_R
\end{array} \tag{18}$$

for left- and right-handed electrons, respectively, with the corresponding projectors $P_{L,R} = (1 \mp \gamma_5)/2$. The couplings to the photon ($\bar{e}e_{L,R}A$) are not modified by anomalous couplings because of electromagnetic gauge invariance. The κ_i in (18) parametrize deviations from the standard model, in which $\kappa_c = \kappa_1 = \kappa_2 = 1$.

Working in the framework of an effective theory, the anomalous couplings should be expressed in terms of the operator coefficients in the effective Lagrangian (6). The operators $\mathcal{O}_{\beta_1}$, $\mathcal{O}_{XU1}$ and $\mathcal{O}_{XU2}$ contain terms bilinear in the gauge fields Z and A, which can be absorbed into the canonical kinetic terms through the renormalizations [42] (see also [36])

$$Z_0 = (1+\delta_Z)Z, \qquad A_0 = (1+\delta_A)A + \delta_{AZ}Z, \qquad M_{Z0} = (1-\delta_{M_Z})M_Z \tag{19}$$

Here the subscript 0 denotes fields and parameters in the absence of any NLO terms in the Lagrangian. We also have

$$g_0 s_0 = e_0, \qquad e_0 = (1-\delta_A)e, \qquad G_{F0} = (1-2\delta_G)G_F \tag{20}$$

and, from (17),

$$s_0 c_0 = s_Z c_Z (1 - \delta_A + \delta_{M_Z} + \delta_G) \tag{21}$$

Corrections to the Fermi constant come from $\mathcal{O}_{V9}$ and $\mathcal{O}_{4f}$. They lead to

$$\delta_G = \frac{1}{2}\mathrm{Re}\,(C^e_{V9} + C^\mu_{V9}) - \frac{v^2}{4\Lambda^2}C_{4f} = C_{V9} - \frac{v^2}{4\Lambda^2}C_{4f} \tag{22}$$

The first expression allows for general, flavour non-universal and complex coefficients of $\mathcal{O}_{V9}$. In the opposite case, δ_G simplifies to the second expression in (22).

Within the basis of operators in (7) – (9) the anomalous couplings can finally be expressed as

$$g_1^Z = 1 + \left[\frac{\beta_1 - \delta_G + C_{X1}e^2/c_Z^2}{c_Z^2 - s_Z^2}\right] + 3\frac{e^2}{s_Z^2}\frac{k^2}{\Lambda^2}C_{W1}, \qquad g_1^A = 1 + 3\frac{e^2}{s_Z^2}\frac{k^2}{\Lambda^2}C_{W1} \tag{23}$$

$$\kappa_Z = 1 + \left[\frac{\beta_1 - \delta_G + C_{X1}e^2/c_Z^2}{c_Z^2 - s_Z^2}\right] + \frac{e^2}{c_Z^2}C_{X1} - \frac{e^2}{s_Z^2}C_{X2} + 3\frac{e^2}{s_Z^2}\frac{2M_W^2 - k^2}{\Lambda^2}C_{W1} \tag{24}$$

$$\kappa_A = 1 - \frac{e^2}{s_Z^2}(C_{X1} + C_{X2}) + 3\frac{e^2}{s_Z^2}\frac{2M_W^2 - k^2}{\Lambda^2}C_{W1} \tag{25}$$

$$g_4^Z = \frac{e^2}{4s_Z^2 c_Z^2}C_{X6}, \qquad g_4^A = 0 \tag{26}$$

$$g_5^Z = -\frac{e^2}{2s_Z^2 c_Z^2}C_{X3}, \qquad g_5^A = 0 \tag{27}$$

$$\tilde{\kappa}_Z = 2\left(\frac{e^2}{c_Z^2}C_{X4} - \frac{e^2}{s_Z^2}C_{X5}\right) - 6\frac{e^2}{s_Z^2}\frac{M_W^2}{\Lambda^2}C_{W2} \tag{28}$$

$$\tilde{\kappa}_A = -2\frac{e^2}{s_Z^2}(C_{X4}+C_{X5}) - 6\frac{e^2}{s_Z^2}\frac{M_W^2}{\Lambda^2}C_{W2} \tag{29}$$

$$\lambda_{Z,A} = 6\frac{e^2}{s_Z^2}C_{W1}, \qquad \tilde{\lambda}_{Z,A} = 6\frac{e^2}{s_Z^2}C_{W2} \tag{30}$$

$$\kappa_c = 1 + \left[\frac{C_{X1}e^2 + c_Z^2(\beta_1-\delta_G)}{c_Z^2 - s_Z^2} - \frac{C_{X2}e^2}{2s_Z^2}\right] + C_{V9} \tag{31}$$

$$\kappa_1 = 1 + [\beta_1 - \delta_G] - C_{V-} + C_{V10} \tag{32}$$

$$\kappa_2 = 1 + \left[\frac{\delta_G - \beta_1 - C_{X1}e^2/s_Z^2}{c_Z^2 - s_Z^2}\right] + \frac{1}{2s_Z^2}C_{V10} \tag{33}$$

The terms in (23) through (33) that arise from renormalizing A, Z, e, s_Z, c_Z are indicated by square brackets. The remaining corrections represent the direct effect of the NLO operators on the interaction vertices. Note that the coefficients β_1 and δ_G always appear in the combination $\beta_1 - \delta_G$.

4 Cross sections

In the following we present cross-section formulas for $e^+e^- \to W^+W^-$, focussing on the new-physics corrections from the NLO Lagrangian (6). The amplitude is determined by the s-channel (Z, γ) and t-channel (ν) exchange diagrams. Of particular interest for a future linear collider will be the limit of large centre-of-mass energy $\sqrt{s}$, defined as $v^2 \ll s \ll \Lambda^2$ [43]. In this window $\sqrt{s}$ is considered to be much larger than the electroweak scale v, and also $M_{W,Z}$, but still smaller than the new-physics scale Λ that determines the range of validity of the effective theory. With the inequality $M_{W,Z}^2 \ll s$, the corrections to the cross sections can be expanded in inverse powers of s. Relative to the standard model, the potentially leading corrections grow as $\mathcal{O}(s)$, subleading terms are of $\mathcal{O}(1)$, whereas all further terms, suppressed as $\mathcal{O}(v^2/s)$ or higher, can be expected to be irrelevant in practice.

We provide results for cross sections with different polarizations of the initial and final state particles [44]. The case of left-handed and right-handed e^- will be denoted by LH and RH, respectively. For the W^+W^- bosons we consider either longitudinal (L) or transverse polarization (T) for each, which leads to the cases LL, TT and LT. For LT the cross sections are the same whether W^+ or W^- is longitudinally polarized ($LT = TL$). The polarized cross sections are quoted relative to their standard model expressions, where only the $\mathcal{O}(s)$ enhanced terms are given here. The corrections of $\mathcal{O}(1)$, $f_{LL}^{LH}, \ldots$, can be found in the appendix.

The LL cross sections read:

$$\frac{d\sigma_{LL}^{LH}}{d\cos\theta} = \frac{d\sigma_{LL,SM}^{LH}}{d\cos\theta}\left[1 + s\frac{4\,\mathrm{Re}\,C_{V9}^e}{M_Z^2} + s\frac{2C_{V-}}{M_Z^2} + f_{LL}^{LH} + \mathcal{O}\left(s^{-1}\right)\right] \tag{34}$$

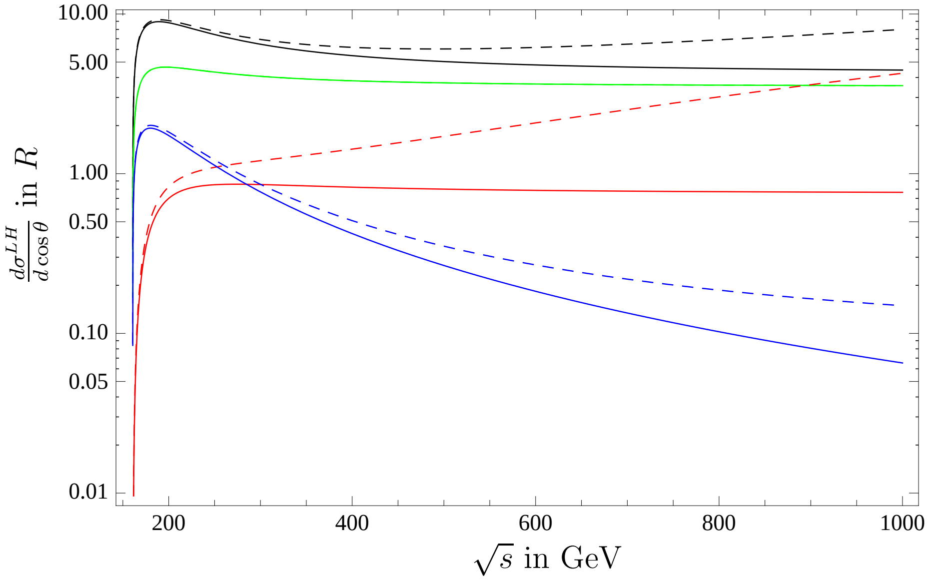

Figure 1: Energy dependence of scattering cross sections for left-handed electrons at $\cos\theta = 0$ in units of $R = 4\pi\alpha^2/3s$. The solid curves are, from top to bottom (at $\sqrt{s} = 600\,\text{GeV}$), the leading-order standard model results for unpolarized W^+W^-, and for W polarizations TT, LL and LT. The dashed curves are the corresponding results including leading new physics corrections. Note that these corrections are absent in the TT case.

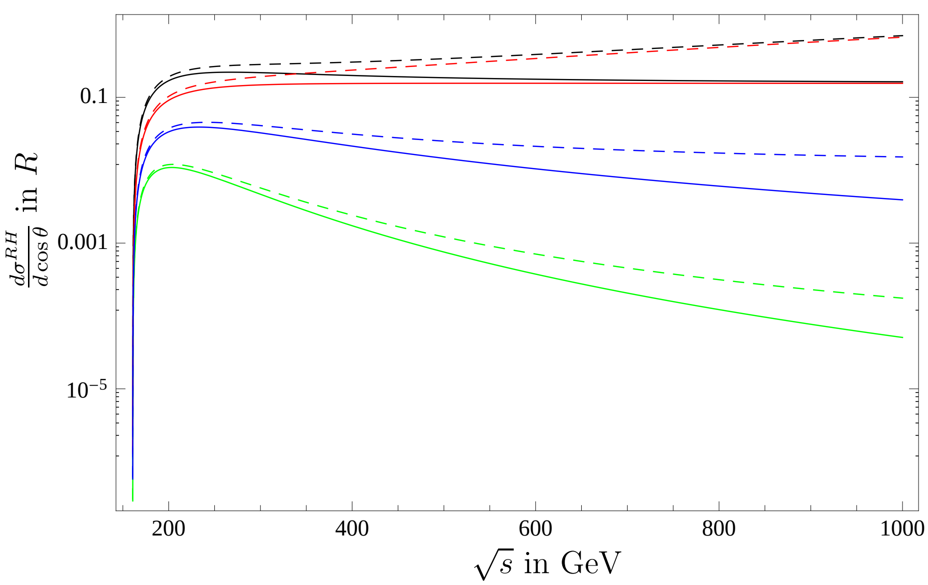

Figure 2: Energy dependence of scattering cross sections for right-handed electrons at $\cos\theta = 0$ in units of R. The solid curves are, from top to bottom (at $\sqrt{s} = 600\,\text{GeV}$), the leading-order standard model results for unpolarized W^+W^-, and for W polarizations LL, LT and TT. The dashed curves are the corresponding results including leading new physics corrections.

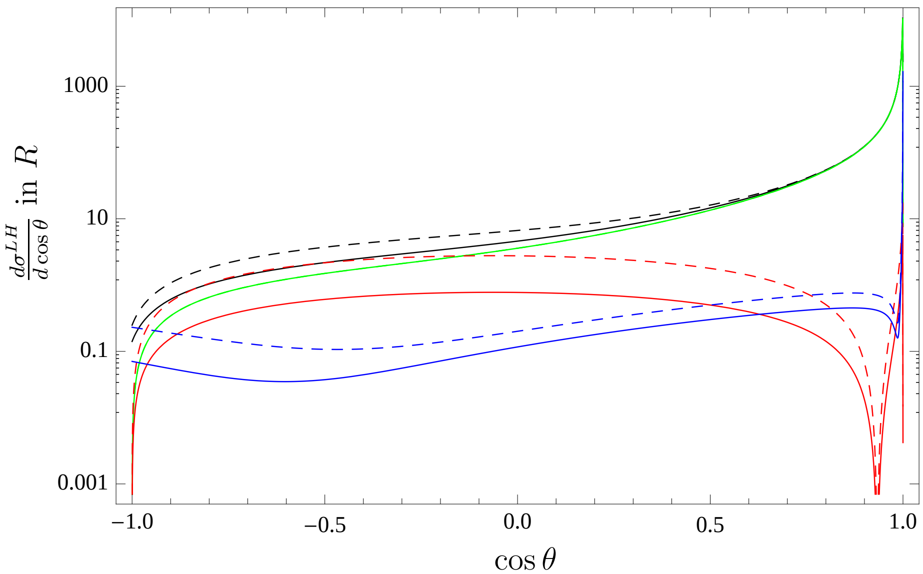

Figure 3: Angular dependence of scattering cross sections for left-handed electrons at $s = (750\,\mathrm{GeV})^2$ in units of R. The solid curves are, from top to bottom (at $\cos\theta = 0$), the leading-order standard model results for unpolarized W^+W^-, and for W polarizations TT, LL and LT. The dashed curves are the corresponding results including leading new physics corrections. Note that these corrections are absent in the TT case.

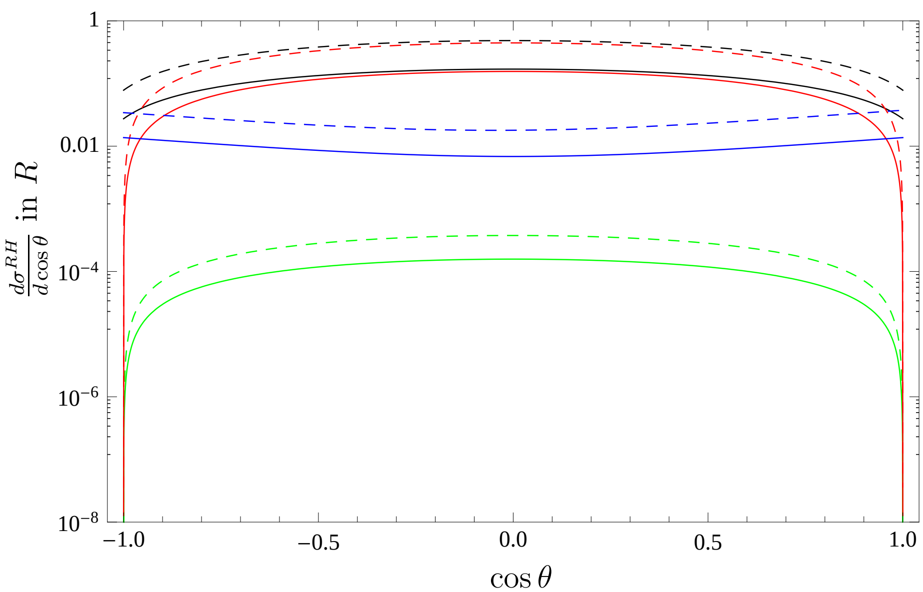

Figure 4: Angular dependence of scattering cross sections for right-handed electrons at $s = (750\,\mathrm{GeV})^2$ in units of R. The solid curves are, from top to bottom (at $\cos\theta = 0$), the leading-order standard model results for unpolarized W^+W^-, and for W polarizations LL, LT and TT. The dashed curves are the corresponding results including leading new physics corrections.

$$\frac{d\sigma_{LL}^{RH}}{d\cos\theta}=\frac{d\sigma_{LL,SM}^{RH}}{d\cos\theta}\left[1+s\frac{C_{V10}}{M_Z^2 s_Z^2}+f_{LL}^{RH}+\mathcal{O}\left(s^{-1}\right)\right] \tag{35}$$

The notation $\mathrm{Re}\, C_{V9}^e$ reflects the fact that, in general, the coefficient C_{V9} may be complex and flavour dependent. If these possibilities are neglected $\mathrm{Re}\, C_{V9}^e$ can be identified with C_{V9} (taken to be real), as it is frequently done throughout this paper.

The TT cross sections read:

$$\frac{d\sigma_{TT}^{LH}}{d\cos\theta}=\frac{d\sigma_{TT,SM}^{LH}}{d\cos\theta}\left[1+f_{TT}^{LH}+\mathcal{O}\left(s^{-1}\right)\right] \tag{36}$$

$$\frac{d\sigma_{TT}^{RH}}{d\cos\theta}=\frac{d\sigma_{TT,SM}^{RH}}{d\cos\theta}\left[1+s\frac{C_{V10}}{M_Z^2 s_Z^2}-s\frac{2e^2 C_{X1}}{M_Z^2 s_Z^2 c_Z^2}+s\frac{C_{W1}}{\Lambda^2}\frac{6e^2}{s_Z^2}+f_{TT}^{RH}\right] \tag{37}$$

In (37) the s-dependence of the square bracket is exact, $f_{TT}^{RH}=0$, and terms of $\mathcal{O}(s^{-1})$ or higher are absent in this case.

The LT cross sections are given by:

$$\begin{aligned}\frac{d\sigma_{LT}^{LH}}{d\cos\theta}=\frac{d\sigma_{LT,SM}^{LH}}{d\cos\theta}\Bigg[&1+s\frac{4\,\mathrm{Re}\, C_{V9}^e\xi}{M_Z^2\chi}+s\frac{2C_{V-}\xi}{M_Z^2\chi}-s\frac{e^2\xi C_{X1}}{M_Z^2 c_Z^2\chi}-s\frac{e^2\xi C_{X2}}{M_Z^2 s_Z^2\chi}\\&-s\frac{e^2\left(c_Z^2-s_Z^2\right)\left[\left(1+\cos\theta\right)c_Z^2+\cos\theta\right]C_{X3}}{M_Z^2 s_Z^2 c_Z^2\chi}-s\frac{C_{W1}}{\Lambda^2}\frac{6e^2 c_Z^2\xi}{s_Z^2\chi}\\&+f_{LT}^{LH}+\mathcal{O}\left(s^{-1}\right)\Bigg]\end{aligned} \tag{38}$$

$$\begin{aligned}\frac{d\sigma_{LT}^{RH}}{d\cos\theta}=\frac{d\sigma_{LT,SM}^{RH}}{d\cos\theta}\Bigg[&1-s\frac{e^2 C_{X3}\cos\theta}{M_Z^2 s_Z^2 c_Z^2(1+\cos^2\theta)}-s\frac{e^2 C_{X1}}{M_Z^2 c_Z^2 s_Z^2}+s\frac{C_{V10}}{M_Z^2 s_Z^2}\\&+f_{LT}^{RH}+\mathcal{O}\left(s^{-1}\right)\Bigg]\end{aligned} \tag{39}$$

with

$$\begin{aligned}\xi&=1+\left(2c_Z^2\left(1+\cos\theta\right)+\cos\theta\right)\cos\theta\\\chi&=1+\left(2c_Z^2\left(1+\cos\theta\right)+\cos\theta\right)^2\end{aligned} \tag{40}$$

Finally, we give the corresponding results also for the case of unpolarized W bosons (denoted by Σ):

$$\frac{d\sigma_{\Sigma}^{LH}}{d\cos\theta} = \frac{d\sigma_{\Sigma,SM}^{LH}}{d\cos\theta}\left[1 + s\frac{16\,\mathrm{Re}\,C_{V9}^{e}\sin^4\frac{\theta}{2}}{M_Z^2\eta} + s\frac{8C_{V-}\sin^4\frac{\theta}{2}}{M_Z^2\eta} + f_{\Sigma}^{LH} + \mathcal{O}\left(s^{-1}\right)\right] \tag{41}$$

$$\frac{d\sigma_{\Sigma}^{RH}}{d\cos\theta} = \frac{d\sigma_{\Sigma,SM}^{RH}}{d\cos\theta}\left[1 + s\frac{C_{V10}}{M_Z^2 s_Z^2} + f_{\Sigma}^{RH} + \mathcal{O}\left(s^{-1}\right)\right] \tag{42}$$

with

$$\eta = \left(1+\cos^2\theta\right)\left(1+8c_Z^4\right) - 2\cos\theta \tag{43}$$

It is useful to present the latter results for unpolarized W bosons also in a slightly more explicit and complementary form. In the high-energy limit ($s \gg M_W^2$) the differential cross sections for the scattering of polarized e^+e^- into unpolarized W^+W^- can be written as

$$\begin{aligned}\frac{d\sigma(e_L^- e_R^+ \to W^-W^+)}{d\cos\theta} &= \frac{\pi\alpha^2}{2s}\left[\frac{1-\cos^2\theta}{16c_Z^4 s_Z^4} + \frac{(1+\cos\theta)(1+\cos^2\theta)}{2s_Z^4(1-\cos\theta)}\right. \\ &\quad \left. - \frac{s(1-\cos^2\theta)}{8M_W^2 c_Z^2 s_Z^4}\left(\delta\kappa_1 - 2\delta\kappa_c + \delta\kappa_Z - 2s_Z^2(\delta\kappa_2 - \delta\kappa_A + \delta\kappa_Z)\right)\right]\end{aligned} \tag{44}$$

and

$$\frac{d\sigma(e_R^- e_L^+ \to W^-W^+)}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\frac{M_Z^4}{4M_W^4}(1-\cos^2\theta)\left[1 + \frac{2s}{M_Z^2}(\delta\kappa_2 - \delta\kappa_A + \delta\kappa_Z)\right] \tag{45}$$

Here only the leading terms in M_W^2/s have been kept, both for the standard model results and for the new physics corrections. The latter are expressed in terms of the anomalous contributions to the couplings, $\delta\kappa_i \equiv \kappa_i - \kappa_{i,SM}$ defined in (16) and (18). In terms of the Lagrangian coefficients one finds for the parameters that determine the leading corrections

$$\delta\kappa_1 - 2\delta\kappa_c + \delta\kappa_Z - 2s_Z^2(\delta\kappa_2 - \delta\kappa_A + \delta\kappa_Z) = -C_{V-} - 2\mathrm{Re}\ C_{V9}^{e} \tag{46}$$

$$\delta\kappa_2 - \delta\kappa_A + \delta\kappa_Z = \frac{C_{V10}}{2s_Z^2} \tag{47}$$

in agreement with (41) and (42).

The (full) energy dependence of the leading-order standard model cross sections is plotted in Figs. 1 and 2, their angular dependence in Figs. 3 and 4 (solid lines). For illustration, the typical size of potential, s-enhanced new physics corrections is also indicated (dashed lines). The following input parameters have been used:

$$M_W = 80.4\,\mathrm{GeV}, \qquad M_Z = 91.19\,\mathrm{GeV}, \qquad G_F = 1.166\cdot 10^{-5}\,\mathrm{GeV}^{-2}, \qquad \alpha = 1/129 \tag{48}$$

The sine of the weak mixing angle, in the definition used here, is then determined through (17) to be

$$s_Z^2 = 0.231 \tag{49}$$

In order to display the potential impact of new physics we include for each cross section the leading $\mathcal{O}(s)$ corrections, exemplarily setting the relevant coefficients to $C_{V-} = C_{V9} = C_{V10} = C_{X1} = C_{X2} = C_{X3} = 1/(16\pi^2)$. This value corresponds to the natural size expected from naive dimensional analysis. In the plots, all cross sections are normalized to the quantity

$$R = \frac{4\pi\alpha^2}{3s} \tag{50}$$

We add several comments on the results presented above.

- It is instructive to recall the large-s behaviour of the cross sections in the standard model. The dominant ones scale as $1/s$. They are:

$$\sigma_{LL}^{LH}, \qquad \sigma_{TT}^{LH}, \qquad \sigma_{LL}^{RH} \tag{51}$$

  The remaining cross sections are subleading at high energies and scale as

$$\sigma_{LT}^{LH} \sim \frac{1}{s^2}, \qquad \sigma_{LT}^{RH} \sim \frac{1}{s^2}, \qquad \sigma_{TT}^{RH} \sim \frac{1}{s^3} \tag{52}$$

- The leading sensitivity to new physics comes from the $\mathcal{O}(s)$ enhanced corrections to the dominant cross sections (51). It depends on the coefficients

$$\sigma_{LL}^{LH} : C_{V-,V9}, \qquad \sigma_{TT}^{LH} : 0, \qquad \sigma_{LL}^{RH} : C_{V10} \tag{53}$$

  The fact that σ_{TT}^{LH} receives no leading corrections is clearly visible from Figs. 1 and 3. This feature also implies (Fig. 1) that the large-s enhancement in the cross section for left-handed electrons into unpolarized W^+W^- is contributed entirely by the longitudinal W bosons, even though the transverse W bosons have a larger cross section.

- The CP odd operators $\mathcal{O}_{XU4}$, $\mathcal{O}_{XU5}$, $\mathcal{O}_{XU6}$ and $\mathcal{O}_{W2}$ do not contribute to the cross sections considered here.

- The triple-W operators in (13) arise only at NNLO ($\sim v^2/(16\pi^2\Lambda^2)$) in the effective Lagrangian. Accordingly, their coefficients give only subleading contributions to the cross sections. The coefficient C_{W1} (CP even operator) enters the correction terms f_{LT}^{RH}, f_{TT}^{LH}, f_{LT}^{LH}, f_{Σ}^{LH} as well as the $\mathcal{O}(s)$ corrections in σ_{TT}^{RH} and σ_{LT}^{LH}. In the former case C_{W1} is strongly suppressed by a factor M_Z^2/Λ^2. In the latter case the suppression is milder, by a factor s/Λ^2. However, this is compensated by the overall suppression of these cross sections at large s, $\sigma_{TT}^{RH} \sim 1/s^3$ and $\sigma_{LT}^{LH} \sim 1/s^2$. Therefore the effect of C_{W1} can be expected to be negligible in practice [36,38].

- For high-precision studies standard-model radiative corrections in $e^+e^- \to W^+W^-$, which are neglected here, have to be taken into account [45,46,47]. However, these corrections cannot affect the leading relative corrections from new physics enhanced by s/M_Z^2.

- The expression of anomalous couplings in terms of effective theory coefficients, (23) through (33), is fully general and can be used to compute further observables in $e^+e^- \to W^+W^-$ [48,49,50].

5 Redundant operators

In addition to the dimension-4 operators in (8), built from $B_{\mu\nu}$, $W_{\mu\nu}$ and U, further operators of similar type can be written down. Those may also be used in describing modified gauge-boson vertices, but they can always be eliminated by appropriate field redefinitions (or, equivalently, using equations of motion) in favour of the terms in (8) [28,39,40]. In this section we discuss how these redundant operators would enter the anomalous couplings. We also show explicitly how their effect can be absorbed into the coefficients of the operators already present in our basis. This exercise facilitates the transformation to a different set of independent operators that one might want to consider. It also provides a useful consistency check of the expressions in (23) – (33).

There are 6 redundant operators that have been considered in the literature, $\mathcal{O}_{XUi}$, $i = 7, \ldots, 12$, in the notation of [28]. The 3 CP-violating operators $i = 10$, 11, 12 are trivially related to $\mathcal{O}_{XUi}$, $i = 4$, 5, 6, in (8) and we will not discuss them further here. The first of the remaining operators is

$$\mathcal{O}_{XU7} = -2ig' B_{\mu\nu}\langle D^\mu U^\dagger D^\nu U T_3\rangle = -ig' g^2 B^{\mu\nu} W_\mu^+ W_\nu^- \tag{54}$$

It is related to the other operators, up to a total derivative, as

$$\mathcal{O}_{XU7} = \frac{g'^2}{2} B_{\mu\nu}B^{\mu\nu} + g'^2 \mathcal{O}_{\beta_1} - \mathcal{O}_{XU1} - g'^2 \mathcal{O}_{\psi V7} - 2g'^2 \mathcal{O}_{\psi V10} \tag{55}$$

In writing (55) we have omitted operators similar to $\mathcal{O}_{\psi Vi}$ that involve quark fields. The first term on the r.h.s. only renormalizes the B-field kinetic term and has no effect on the anomalous couplings (see the discussion in section 7 below). Adding a term $C_{X7}\mathcal{O}_{XU7}$ to the NLO Lagrangian results in the following shift in the anomalous couplings

$$\Delta\kappa_Z = -\frac{e^2}{c_Z^2} C_{X7}, \qquad \Delta\kappa_A = \frac{e^2}{s_Z^2} C_{X7} \tag{56}$$

All other couplings in (23) – (33) remain unchanged. According to (55), an inclusion of $C_{X7}\mathcal{O}_{XU7}$ in the Lagrangian is equivalent to shifting the other coefficients by

$$(\Delta\beta_1, \Delta C_{X1}, \Delta C_{V7}, \Delta C_{V10}) = C_{X7}\,(g'^2, -1, -g'^2, -2g'^2) \tag{57}$$

This reflects the redundancy of $\mathcal{O}_{XU7}$ and can be checked explicitly with (23) – (33).

Similar considerations apply to the operator

$$\mathcal{O}_{XU8} = -2ig\langle W_{\mu\nu} D^\mu U D^\nu U^\dagger \rangle \tag{58}$$

which is related to the other operators as

$$\mathcal{O}_{XU8} = g^2 \langle W_{\mu\nu} W^{\mu\nu} \rangle - \frac{g^2}{2} v^2 \langle D_\mu U^\dagger D^\mu U \rangle - \mathcal{O}_{XU1} - 2g^2 \mathcal{O}_{\psi V8} - g^2 (\mathcal{O}_{\psi V9} + \mathcal{O}^\dagger_{\psi V9}) \tag{59}$$

up to total derivatives and contributions with quarks. The first two terms can be absorbed into the leading-order Lagrangian and have no effect on the anomalous couplings. A term $C_{X8}\mathcal{O}_{XU8}$ in the Lagrangian would shift the couplings by

$$\Delta\kappa_Z = \Delta\kappa_A = g^2 C_{X8}\,, \qquad \Delta g_1^Z = \frac{g^2}{c_Z^2} C_{X8} \tag{60}$$

with the remaining couplings in (23) – (33) unchanged. According to (59), an inclusion of $C_{X8}\mathcal{O}_{XU8}$ in the Lagrangian is equivalent to shifting the other coefficients by

$$(\Delta C_{X1}, \Delta C_{V8}, \Delta C_{V9}, \Delta\delta_G) = -C_{X8}\,(1, 2g^2, g^2, g^2) \tag{61}$$

as can be checked with (23) – (33).

Finally,

$$\mathcal{O}_{XU9} = -2ig\langle U^\dagger W_{\mu\nu} U T_3 \rangle \, \langle D^\mu U^\dagger D^\nu U T_3 \rangle \tag{62}$$

obeys the relation

$$\mathcal{O}_{XU9} = \frac{g^2}{4} \langle W_{\mu\nu} W^{\mu\nu} \rangle - \frac{g^2}{8} v^2 \langle D_\mu U^\dagger D^\mu U \rangle - \frac{g^2}{4} \mathcal{O}_{\beta_1} - \frac{1}{2} \mathcal{O}_{XU2} - \frac{g^2}{4} (\mathcal{O}_{\psi V9} + \mathcal{O}^\dagger_{\psi V9}) \tag{63}$$

The direct contribution from $C_{X9}\mathcal{O}_{XU9}$ reads

$$\Delta\kappa_Z = \Delta\kappa_A = \frac{g^2}{2} C_{X9} \tag{64}$$

which, using (63), is equivalent to shifting the other coefficients by

$$(\Delta\beta_1, \Delta C_{X2}, \Delta C_{V9}, \Delta\delta_G) = -\frac{C_{X9}}{4}\,(g^2, 2, g^2, g^2) \tag{65}$$

This is again consistent with (23) – (33).

We conclude this section with a discussion of an alternative operator basis, which includes the triple-gauge operators $\mathcal{O}_{XU7}$, $\mathcal{O}_{XU8}$ and $\mathcal{O}_{XU9}$, while eliminating three of the original operators in (7), (8) and (9). The choice of these three is in principle arbitrary. We emphasize, however, that it is not possible in general to eliminate all the gauge-fermion operators simultaneously since there are more than three (ten without counting flavour structure [28]). Because only three gauge-fermion operators ($\mathcal{O}_{\psi V-}$, $\mathcal{O}_{\psi V9} + h.c.$, $\mathcal{O}_{\psi V10}$) happen to contribute to $e^+e^- \to W^+W^-$, those may indeed be removed altogether from the basis in this case. Additional gauge-fermion terms will be required when other processes are considered, such as W^+W^- production from hadronic initial states (see section 9). Restricting our attention to $e^+e^- \to W^+W^-$ we may write

$$\begin{aligned}\mathcal{L}_{eff,NLO} &= \tilde{\beta}_1\mathcal{O}_{\beta_1} + \tilde{C}_{X1}\mathcal{O}_{XU1} + \tilde{C}_{X2}\mathcal{O}_{XU2} + \tilde{C}_{X7}\mathcal{O}_{XU7} + \tilde{C}_{X8}\mathcal{O}_{XU8} + \tilde{C}_{X9}\mathcal{O}_{XU9} + \ldots \\ &= \beta_1\mathcal{O}_{\beta_1} + C_{X1}\mathcal{O}_{XU1} + C_{X2}\mathcal{O}_{XU2} + C_{V-}\mathcal{O}_{\psi V-} + C_{V9}(\mathcal{O}_{\psi V9} + h.c.) + C_{V10}\mathcal{O}_{\psi V10} + \ldots\end{aligned} \tag{66}$$

where we disregard gauge-fermion operators other than $\mathcal{O}_{\psi V-}$, $\mathcal{O}_{\psi V9}$, $\mathcal{O}_{\psi V10}$. Further operators that are not affected by the change of basis are understood to be included but are not written explicitly. In terms of the coefficients, the transformation from one to the other basis in (66) is given by

$$\begin{aligned}&\beta_1 = \tilde{\beta}_1 + g'^2\tilde{C}_{X7} - \frac{g^2}{4}\tilde{C}_{X9}, \quad C_{X1} = \tilde{C}_{X1} - \tilde{C}_{X7} - \tilde{C}_{X8}, \quad C_{X2} = \tilde{C}_{X2} - \frac{1}{2}\tilde{C}_{X9} \\ &C_{V-} = -g'^2\tilde{C}_{X7} + g^2\tilde{C}_{X8}, \quad C_{V9} = -g^2\tilde{C}_{X8} - \frac{g^2}{4}\tilde{C}_{X9}, \quad C_{V10} = -2g'^2\tilde{C}_{X7}\end{aligned} \tag{67}$$

6 High-energy limit and the Goldstone boson equivalence theorem

The results of section 4 show that, despite the sizeable number of operators that parametrize new physics effects in $e^+e^- \to W^+W^-$, only 3 of them appear in the large-energy limit with a relative enhancement factor s/v^2, thus introducing potential violations of unitarity in the W^+W^- cross-section[2]. These unitarity violations are associated with the longitudinal modes of the W bosons as can be seen by inspection of our results or, more generally, by a straightforward application of the equivalence theorem [51,52]. A general discussion of the equivalence theorem in the context of chiral Lagrangians can be found in [53,54]. In this section we will rederive the large-s limit of the $e^+e^- \to W^+W^-$ cross-section in a more transparent way by working in the Landau gauge, where the Goldstone modes $\varphi^\pm$ appear explicitly.

The relevant topologies for $e^+e^- \to \varphi^+\varphi^-$ are collected in the second and third diagram of Fig. 5. The leftmost diagram is the standard model contribution. The $(\gamma, Z)\varphi^+\varphi^-$ vertices are obtained from the Goldstone kinetic term

$$\frac{v^2}{4}\langle D^\mu U^\dagger D_\mu U\rangle = e\left(\varphi^+ i \overleftrightarrow{\partial}_\mu \varphi^-\right)\left(\frac{c_Z^2 - s_Z^2}{2c_Z s_Z}Z^\mu + A^\mu\right) + \ldots \tag{68}$$

In the large-s limit, the leading new physics contributions to $e^+e^- \to \varphi^+\varphi^-$ can be shown to come only from the gauge-fermion operators $\mathcal{O}_{\psi Vi}$: The operator $\mathcal{O}_{\beta_1}$ contains

[2]Obviously, such divergences are actually cut off at the scale of new physics, where new degrees of freedom regulate them. Therefore, such divergences never violate unitarity, but rather signal the point where the EFT ceases to be valid.

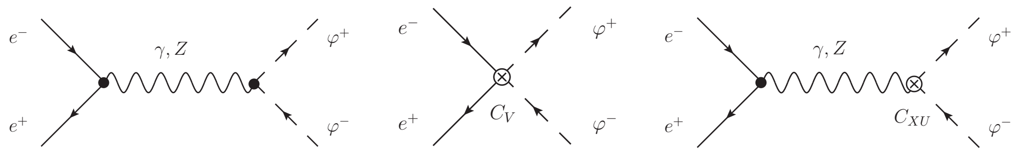

Figure 5: *Different contributions to* $e^+e^- \to \varphi^+\varphi^-$*. The left-hand diagram is the standard model piece while the central and right-hand diagrams are the same contribution from new physics, expressed in terms of gauge-fermion (central) or triple-gauge operators (right).* C_V *and* C_{XU} *are short-hand notations for* C_{V7-10} *and* C_{X7-9}*, respectively.*

a $Z\varphi^+\varphi^-$ coupling proportional to the standard model expression in (68). This contribution is not enhanced in the large-s limit and therefore subleading. No $(\gamma, Z)\varphi^+\varphi^-$ coupling arises from $\mathcal{O}_{XU1}$, $\mathcal{O}_{XU2}$, $\mathcal{O}_{XU4}$ and $\mathcal{O}_{XU5}$, which are bilinear in the gauge fields. Finally, $\mathcal{O}_{XU3}$ and $\mathcal{O}_{XU6}$ produce $(\gamma, Z)\varphi^+\varphi^-$ only together with at least one additional Goldstone particle and therefore do not contribute to the process of interest here.

The gauge-fermion operators give rise to the central diagram in Fig. 5. They read explicitly

$$
\begin{aligned}
\mathcal{O}_{\psi V7} = -2\mathcal{O}_{\psi V8} = \frac{1}{2}(\mathcal{O}_{\psi V9} + \mathcal{O}^\dagger_{\psi V9}) = (\bar{e}_L\gamma^\mu e_L)\frac{1}{v^2}\left(\varphi^+ i \overleftrightarrow{\partial}_\mu \varphi^-\right) + \ldots \\
\mathcal{O}_{\psi V10} = (\bar{e}_R\gamma^\mu e_R)\frac{1}{v^2}\left(\varphi^+ i \overleftrightarrow{\partial}_\mu \varphi^-\right) + \ldots \qquad (69)
\end{aligned}
$$

Notice the difference between the unitary and Landau gauge: the gauge-fermion operators, which in unitary gauge corrected the s and t-channel vertices, now take the form of $e^+e^-\varphi^+\varphi^-$ local terms.

The interference between the standard model and the new physics (NP) contribution can be easily computed and results in

$$
\begin{aligned}
\frac{d\sigma(e^-_R e^+_L \to W^-W^+)_{NP}}{d\cos\theta} &= \frac{\pi\alpha^2\sin^2\theta}{8s_Z^2c_Z^2M_W^2}C_{V10} \\
\frac{d\sigma(e^-_L e^+_R \to W^-W^+)_{NP}}{d\cos\theta} &= \frac{\pi\alpha^2\sin^2\theta}{16s_Z^4c_Z^2M_W^2}\left(C_{V-} + 2C_{V9}\right) \qquad (70)
\end{aligned}
$$

which agrees with the results in section 4 (assuming C_{V9} to be real).

As discussed in section 5, the equations of motion imply relations between gauge-fermion, oblique and triple-gauge operators. We have already discussed the convenience of working with gauge-fermion operators while eliminating triple-gauge operators. However, it is still instructive to rederive the large-energy limit in the basis where gauge-fermion operators are absent. In this basis, the central diagram in Fig. 5 gets replaced by the rightmost one, where the $(\gamma, Z)\varphi^+\varphi^-$ vertices come from the triple-gauge operators

$$\begin{aligned}
\mathcal{O}_{XU7} &= -\frac{4ig'}{v^2} B_{\mu\nu}\partial^\mu\varphi^+\partial^\nu\varphi^- \\
\mathcal{O}_{XU8} &= 2\mathcal{O}_{XU9} = -\frac{4ig}{v^2} W^3_{\mu\nu}\partial^\mu\varphi^+\partial^\nu\varphi^-
\end{aligned} \tag{71}$$

The results for the cross-sections now take the form

$$\begin{aligned}
\frac{d\sigma(e^-_R e^+_L \to W^- W^+)_{NP}}{d\cos\theta} &= -\frac{\pi^2\alpha^3\sin^2\theta}{s^2_Z c^4_Z M^2_W} C_{X7} \\
\frac{d\sigma(e^-_L e^+_R \to W^- W^+)_{NP}}{d\cos\theta} &= -\frac{\pi^2\alpha^3\sin^2\theta}{4s^6_Z c^4_Z M^2_W}\left(s^2_Z C_{X7} + c^2_Z\left(C_{X8} + \frac{1}{2}C_{X9}\right)\right)
\end{aligned} \tag{72}$$

The equivalence of (70) and (72) can be checked using the high-energy version of the equations relating $\mathcal{O}_{XU7,8,9}$ and $\mathcal{O}_{V7,8,9,10}$ given in section 5. In terms of the corresponding coefficients these relations read

$$\begin{aligned}
C_{X7} &= -\frac{c^2_Z}{8\pi\alpha} C_{V10} \\
C_{X8} &= \frac{s^2_Z}{4\pi\alpha}\left(C_{V-} - \frac{1}{2}C_{V10}\right) \\
C_{X9} &= -\frac{s^2_Z}{\pi\alpha}\left(C_{V-} + C_{V9} - \frac{1}{2}C_{V10}\right)
\end{aligned} \tag{73}$$

In the $\mathcal{O}_{XUi}$ basis (71), the enhancement $\sim s$ of the relative corrections is obvious since the $(\gamma, Z)\varphi^+\varphi^-$ vertices carry three derivatives, instead of one in the standard model case (68). The same enhancement comes about differently in the $\mathcal{O}_{\psi Vi}$ basis (69). These operators give local $e^+e^-\varphi^+\varphi^-$ vertices, which are similar to the standard model amplitudes, but without the gauge-boson propagator $\sim 1/s$. This then leads to the relative enhancement $\sim s$ of the corrections when they are computed from the $\mathcal{O}_{\psi Vi}$.

7 NLO Lagrangian for linearly transforming Higgs

In the case of a linearly transforming Higgs field, the next-to-leading order Lagrangian consists of the operators of dimension 5 and 6 listed in [29,30]. The terms of the NLO Lagrangian relevant for $e^+e^- \to W^+W^-$ can be written as

$$\mathcal{L}_{\rm NLO} = \frac{1}{\Lambda^2}\sum_{i=1}^{9} z_i Q_i \tag{74}$$

with real dimensionless coefficients z_i and the dimension-6 operators

$$\begin{aligned}
Q_1 &= (D_\mu \phi^\dagger \phi)(\phi^\dagger D^\mu \phi) \\
Q_2 &= g g' B_{\mu\nu}\, \phi^\dagger W^{\mu\nu} \phi \\
Q_3 &= g g' \varepsilon^{\mu\nu\lambda\rho} B_{\mu\nu}\, \phi^\dagger W_{\lambda\rho} \phi \\
Q_4 &= \bar{l}\gamma^\mu l\, (\phi^\dagger i D_\mu \phi - i D_\mu \phi^\dagger \phi) \\
Q_5 &= \bar{l}\gamma^\mu T^a l\, (\phi^\dagger T^a i D_\mu \phi - i D_\mu \phi^\dagger T^a \phi) \\
Q_6 &= \bar{e}\gamma^\mu e\, (\phi^\dagger i D_\mu \phi - i D_\mu \phi^\dagger \phi) \\
Q_7 &= \mathcal{O}_{4f}\ , \quad Q_8 = \mathcal{O}_{W1}\ , \quad Q_9 = \mathcal{O}_{W2}
\end{aligned} \tag{75}$$

We take the Higgs doublet ϕ to be normalized such that its vev is $\langle\phi\rangle = (0, v)^T$ with $v = 246\,\text{GeV}$.

The operators

$$\begin{aligned}
Q_{10} &= \phi^\dagger \phi\, B_{\mu\nu} B^{\mu\nu}\ , \qquad & Q_{11} &= \phi^\dagger \phi\, W^a_{\mu\nu} W^{a\mu\nu} \\
Q_{12} &= \phi^\dagger \phi\, \varepsilon^{\mu\nu\lambda\rho} B_{\mu\nu} B_{\lambda\rho}\ , \qquad & Q_{13} &= \phi^\dagger \phi\, \varepsilon^{\mu\nu\lambda\rho} W^a_{\mu\nu} W^a_{\lambda\rho}
\end{aligned} \tag{76}$$

have been omitted from (74) since they have no impact on the $e^+e^- \to W^+W^-$ amplitude. This becomes clear in the unitary gauge and after dropping contributions with the physical Higgs field h, which are of no interest in the present case. We may thus replace $\phi^\dagger\phi \to v^2$. The operators Q_{12} and Q_{13} then reduce to total derivatives, whereas Q_{10} and Q_{11} take the form of the usual gauge kinetic terms. The impact of Q_{10} and Q_{11} can be eliminated by a simultaneous rescaling of the gauge field and the corresponding gauge coupling [24]. Explicitly, the contribution from Q_{11} is eliminated, to first order, through the transformations $W^a_\mu \to (1 + \delta_W) W^a_\mu$ and $g \to (1 - \delta_W) g$ with $\delta_W = 2 z_{11} v^2/\Lambda^2$ in the leading-order Lagrangian. This holds because the field W^a_μ enters interaction terms in this Lagrangian only in the combination $g W^a_\mu$. In particular, the above transformation leaves $g W^a_\mu$ invariant and the non-abelian field strength transforms homogeneously as $W^a_{\mu\nu} \to (1 + \delta_W) W^a_{\mu\nu}$. A similar transformation removes the impact of Q_{10}.

Comparing with the NLO Lagrangian in the nonlinear realization of the Higgs sector, in unitary gauge and for $h \to 0$, one finds that the coefficients in (6) are related to $z_1, \ldots, z_9$ as

$$\begin{array}{lll}
\beta_1 = -z_1\, v^2/\Lambda^2 & C_{V7} = -2 z_4\, v^2/\Lambda^2 & C_{4f} = z_7 \\
C_{X1} = -z_2\, v^2/\Lambda^2 & C_{V8} = z_5\, v^2/\Lambda^2 & C_{W1} = z_8 \\
C_{X4} = -z_3\, v^2/\Lambda^2 & C_{V9} = \frac{1}{2} z_5\, v^2/\Lambda^2 = C^*_{V9} & C_{W2} = z_9 \\
 & C_{V10} = -2 z_6\, v^2/\Lambda^2 &
\end{array} \tag{77}$$

In addition, since the operators $\mathcal{O}_{XU2}$, $\mathcal{O}_{XU3}$, $\mathcal{O}_{XU5}$, $\mathcal{O}_{XU6}$ correspond to operators of dimension 8 in the linear-Higgs basis [28], at NLO in this basis we may put

$$C_{X2} = C_{X3} = C_{X5} = C_{X6} = 0 \tag{78}$$

The 15 real parameters β_1. C_{X1}, ..., C_{X6}, C_{V7}, C_{V8}, Re C_{V9}, Im C_{V9}, C_{V10}, C_{4f}, C_{W1} and C_{W2} from the nonlinear Lagrangian thus reduce to the nine real coefficients z_1, ..., z_9 in the linear-Higgs basis.

8 Examples of new physics scenarios

In previous sections we already commented on the fact that a global electroweak fit of the effective theory coefficients does not seem very informative, given the strong correlations between them [5]. In order to obtain an estimate of the size of the coefficients beyond naive dimensional analysis, it is then useful to resort to different UV completions. In this section we will discuss two such scenarios, which affect $e^+e^- \to W^+W^-$ in a complementary way, namely UV completions with heavy fermions (constituent technicolor) or with heavy vectors (Z' models). Models with heavy scalars can be shown to affect $e^+e^- \to W^+W^-$ only at the loop level and will therefore not be considered.

8.1 Constituent technicolor

Constituent technicolor is a very simple model of strongly coupled dynamics first introduced in [36]. The model consists of a flavour doublet of chiral heavy fermions $\mathcal{Q} = (\mathcal{U}, \mathcal{D})^T$ with electric charges $\pm 1/2$ to preserve anomaly cancellation. Since the strong interaction between techniquarks is neglected, except for their dynamical mass, it can be considered a model for a fourth quark generation. The full Lagrangian can then be written as

$$\mathcal{L} = \mathcal{L}_{SM} + i\bar{\mathcal{Q}}_L \not{D}\, \mathcal{Q}_L + i\bar{\mathcal{U}}_R \not{D} \mathcal{U}_R + i\bar{\mathcal{D}}_R \not{D}\, \mathcal{D}_R - (m_U \bar{\mathcal{Q}}_L U P_+ \mathcal{U}_R + m_D \bar{\mathcal{Q}}_L U P_- \mathcal{D}_R + h.c.) \tag{79}$$

Integrating out the heavy fermions to one loop induces a direct correction to the ZWW and γWW vertices but also to gauge boson bilinears. One finds [36]

$$\begin{aligned}
&\tilde{\beta}_1 = \frac{4}{v^2}(m_U + m_D)^2 \delta^2 \xi && \\
&\tilde{C}_{X1} = -\xi; && \tilde{C}_{X7} = -\xi \\
&\tilde{C}_{X2} = -\frac{16}{5}\delta^2 \xi; && \tilde{C}_{X8} = -\left(1 - \frac{2}{5}\delta^2\right)\xi \\
&\tilde{C}_{X3} = -2\delta\xi; && \tilde{C}_{X9} = -\frac{28}{5}\delta^2 \xi
\end{aligned} \tag{80}$$

where

$$\xi = \frac{N_{TC}}{96\pi^2}; \qquad \delta = \frac{m_U - m_D}{m_U + m_D} \tag{81}$$

Choosing for illustration $N_{TC} = 4$, $\delta = 1/60$ and $m_U + m_D = 3\,\text{TeV}$, one finds that $\tilde{\beta}_1 \approx 7 \cdot 10^{-4}$, $\tilde{C}_{X1} \approx \tilde{C}_{X7} \approx \tilde{C}_{X8} \approx -4 \cdot 10^{-3}$, $\tilde{C}_{X3} \approx -1 \cdot 10^{-4}$, and $2\tilde{C}_{X2} \approx \tilde{C}_{X9} \approx -7 \cdot 10^{-6}$, which comply with the naive dimensional estimate $C_i \sim 1/(16\pi^2)$. Using (67) one can trade the triple-gauge operators for gauge-fermion vertices. In the basis we have been using in this paper we find

$$
\begin{aligned}
\beta_1 &= \left[\frac{4}{v^2}(m_U + m_D)^2\delta^2 + e^2\left(\frac{7\delta^2}{5s_Z^2} - \frac{1}{c_Z^2}\right)\right]\xi \\
C_{X1} &= \left(1 - \frac{2}{5}\delta^2\right)\xi; & C_{V-} &= e^2\left[\frac{1}{c_Z^2} - \frac{1}{s_Z^2}\left(1 - \frac{2}{5}\delta^2\right)\right]\xi \\
C_{X2} &= -\frac{2}{5}\delta^2\xi; & C_{V9} &= \frac{e^2}{s_Z^2}(1+\delta^2)\xi \\
C_{X3} &= -2\delta\xi; & C_{V10} &= 2\frac{e^2}{c_Z^2}\xi
\end{aligned}
\tag{82}
$$

Doing the same numerical exercise, $\beta_1 \approx 1.6 \cdot 10^{-4}$, $C_{X1} \approx 4 \cdot 10^{-3}$, $C_{X2} \approx -5 \cdot 10^{-7}$, $C_{X3} \approx -1 \cdot 10^{-4}$, and $C_{V9} \approx -1.4 C_{V-} \approx 1.7 C_{V10} \approx 1.7 \cdot 10^{-3}$. Two things are worth noticing: (i) the size of the triple gauge operators is big enough to invert the sign of C_{X1} in this change of basis, while $|C_{X1}|$ remains the same; (ii) $C_{X4} = C_{X5} = C_{X6} = 0$ because constituent technicolor is CP-conserving.

8.2 Z' models

We next consider models with a Z' [55,56,57], following the approach developed in [58]. The Z' is the gauge boson of a local $U(1)'$ symmetry and will be assumed to have a mass generated through a dynamical mechanism not necessarily related to electroweak symmetry breaking. Since we are interested in an EFT approach we will not be concerned with the dynamical details. Within these assumptions, we will set to zero a bare $Z - Z'$ mass-mixing term, implying that the Higgs sector of the standard model is charged under $U(1)_Y$, but not under $U(1)'$, and *vice versa* for the Higgs sector of Z'. In contrast, a kinetic mixing is in general allowed and will be included.

In formulating the Z' model we will use the chiral Lagrangian description of the standard-model part, as given in (1). The results can then be interpreted in two different ways. Either, electroweak symmetry is dynamically broken and the nonlinear chiral Lagrangian is non-renormalizable with a cutoff Λ at about a few TeV. In this case the Z' mass should be below that scale. The limit of interest is $v \ll M_{Z'} < \Lambda$, in which case Z' is a light degree of freedom in the chiral Lagrangian, but still heavy enough in order to be integrated out at the weak scale v. Alternatively, we may consider the conventional renormalizable standard model with the Higgs field written in polar coordinates, $H \equiv (\tilde{\phi}, \phi) = (v + h)U$, and with the physical Higgs scalar h disregarded, since it does not enter in the applications of interest here. In this case the Z' mass could be taken to be (much) larger than a few TeV.

The Lagrangian for the Z' model then reads

$$\mathcal{L} = \mathcal{L}_{SM,U}(\hat{B}) - \frac{1}{4}\hat{Z}'_{\mu\nu}\hat{Z}'^{\mu\nu} - \frac{\sin\chi}{2}\hat{Z}'_{\mu\nu}\hat{B}^{\mu\nu} + \frac{\cos^2\chi}{2}M^2_{Z'}\hat{Z}'_\mu\hat{Z}'^\mu - \hat{g}\sum_j \hat{Y}_j\bar{f}_j\gamma_\mu f_j\hat{Z}'^\mu \quad (83)$$

$\mathcal{L}_{SM,U}(\hat{B})$ is the lowest-order standard model Lagrangian (1) where the hypercharge gauge field is identified with $\hat{B}$. It is convenient to eliminate the kinetic mixing using

$$\begin{pmatrix} \hat{B}_\mu \\ \hat{Z}'_\mu \end{pmatrix} = \begin{pmatrix} 1 & -\tan\chi \\ 0 & 1/\cos\chi \end{pmatrix}\begin{pmatrix} B_\mu \\ Z'_\mu \end{pmatrix} \quad (84)$$

This field redefinition modifies the Z' coupling to fermions and generates a coupling between Z' and the Goldstone fields. The Lagrangian becomes

$$\begin{aligned}\mathcal{L} = \mathcal{L}_{SM,U}(B) &- \frac{1}{4}Z'_{\mu\nu}Z'^{\mu\nu} + \frac{M^2_{Z'}}{2}Z'_\mu Z'^\mu + \frac{v^2}{8}g'^2\tan^2\chi Z'_\mu Z'^\mu \\ &- \left[\frac{v^2}{2}g'\tan\chi\langle U^\dagger iD_\mu UT_3\rangle + \sum_j \tilde{g}_j\bar{f}_j\gamma_\mu f_j\right]Z'^\mu \end{aligned} \quad (85)$$

where

$$\tilde{g}_j = \hat{g}\frac{\hat{Y}_j}{\cos\chi} - g'Y_j\tan\chi \quad (86)$$

Integrating out the Z' at tree level, and expanding to first order in $1/M^2_{Z'}$, gives the effective Lagrangian

$$\begin{aligned}\mathcal{L}_{eff} = \mathcal{L}_{SM} &+ \frac{v^4}{8M^2_{Z'}}g'^2\tan^2\chi\langle U^\dagger D_\mu UT_3\rangle^2 - \sum_{i,j}\frac{\tilde{g}_i\tilde{g}_j}{2M^2_{Z'}}(\bar{f}_i\gamma_\mu f_i)(\bar{f}_j\gamma^\mu f_j) \\ &- \frac{g'v^2\tan\chi}{2M^2_{Z'}}\sum_j \tilde{g}_j\bar{f}_j\gamma_\mu f_j\langle U^\dagger iD^\mu UT_3\rangle \end{aligned} \quad (87)$$

For $e^+e^- \to W^+W^-$ the only relevant operators that receive contributions are $\mathcal{O}_{\beta_1}$, $\mathcal{O}_{\psi V7}$ and $\mathcal{O}_{\psi V10}$. (Here we will not discuss further the renormalization of G_F due to the 4-fermion operators, which is a subleading effect at large s.) The coefficients read

$$\begin{aligned}\beta_1 &= \frac{v^2}{8M^2_{Z'}}g'^2\tan^2\chi \\ C_{V7} &= -\frac{g'v^2\tan\chi}{2M^2_{Z'}}\tilde{g}_l \\ C_{V10} &= -\frac{g'v^2\tan\chi}{2M^2_{Z'}}\tilde{g}_e \end{aligned} \quad (88)$$

For illustration we choose $M_Z' = 1$ TeV, $\sin\chi = 0.3$, $\hat{g} = g'$ and $\hat{Y}_l = \hat{Y}_e = -1$. With this choice of parameters one finds that $\beta_1 \approx 0.9\cdot 10^{-4}$, $C_{V7} \approx 1.1\cdot 10^{-3}$ and $C_{V10} \approx 0.9\cdot 10^{-3}$.

The numerical values in this example are similar to those of sec. 8.1. However, whereas the signs of the relevant couplings in 8.1 are essentially fixed, the signs could be flipped in the Z' scenario. This would lead to a clear discrimination between the two models.

For completeness we will also comment on the linear case. Within the same assumptions, one can proceed in an analogous way, replacing the kinetic U field term by the corresponding term for the linear Higgs model. The Lagrangian now takes the form

$$\mathcal{L} = \mathcal{L}_{SM,\phi}(B) - \frac{1}{4} Z'_{\mu\nu} Z'^{\mu\nu} + \frac{M_{Z'}^2}{2} Z'_\mu Z'^\mu + \frac{g'^2}{8} \tan^2\chi\, \phi^\dagger \phi\, Z'_\mu Z'^\mu$$
$$+ \left[\frac{g'}{4} \tan\chi (\phi^\dagger i \overleftrightarrow{D}_\mu \phi) - \sum_j \tilde{g}_j \bar{f}_j \gamma_\mu f_j \right] Z'^\mu \qquad (89)$$

Upon integrating out the Z' boson and matching to the linear basis of [30] one obtains the coefficients z_1, z_4, z_6, in the notation of section 7. Their expression in terms of (88) can be inferred from (77).

9 Comments on W^+W^- production at the LHC

It is interesting at this point to discuss how the conclusions we have reached in our analysis for linear colliders extend to hadron colliders. After LEP [8], both Tevatron [59, 60,61] and LHC [62,63] have also studied W^+W^- production and, more generally, bounds on triple gauge couplings. The main advantage of a hadron collider over a linear one is that one can disentangle the anomalous WWZ and $WW\gamma$ contributions by looking at $W\gamma$ production [64] and WZ production [65]. W^+W^- is afflicted with a larger background and, at least in principle, bounds are expected to be less stringent.

A full-fledged analysis of W^+W^- production at the LHC deserves a separate paper.

Here we will content ourselves with commenting on the qualitative features one would expect when an effective field theory point of view is adopted. For the qualitative approach we are pursuing it will suffice to work at the partonic level. The inclusion of parton distribution functions (PDFs), which are required in a complete analysis, will not affect our conclusions. A recent analysis of W^+W^- production at the LHC, based on a subset of the NLO operators in the linear-Higgs scenario, can be found in [3,4].

At the operator level the only difference between W^+W^- at linear and hadron colliders arises in the initial state vertex (both in s and t channels), where the hadronic initial state has twice the number of operators as the leptonic one. To be more precise, while in e^+e^- colliders one finds the 3 combinations

$$\frac{1}{2}\mathcal{O}_{\psi V7} - \mathcal{O}_{\psi V8}, \quad \mathcal{O}_{\psi V9} + \mathcal{O}_{\psi V9}^\dagger, \quad \mathcal{O}_{\psi V10}, \qquad (90)$$

in a pp collider 6 operators contribute, namely

$$\frac{1}{2}\mathcal{O}_{\psi V1} \pm \mathcal{O}_{\psi V2}, \quad \mathcal{O}_{\psi V3} + \mathcal{O}^{\dagger}_{\psi V3}, \quad \mathcal{O}_{\psi V6} + \mathcal{O}^{\dagger}_{\psi V6}, \quad \mathcal{O}_{\psi V4}, \quad \mathcal{O}_{\psi V5} \tag{91}$$

The first thing to notice is that while in $e^+e^- \to W^+W^-$ one can trade the gauge-fermion operators for triple gauge operators, therefore eliminating them altogether, in $pp \to W^+W^-$ this is no longer possible: gauge-fermion operators cannot be omitted in general. Obviously one can still work in a basis where 3 of the gauge-fermion operators are removed. This is however a rather arbitrary choice, which might be sensible for a specific process but not for a global electroweak fit. When one is interested in fitting more than one process, given the larger number of fermions compared to gauge bosons, it seems more natural to remove the triple gauge operators instead.

Even without a detailed analysis one can anticipate the structure of the dominant new physics contribution to $pp \to W^+W^-$. Since at $\sqrt{s} = 7$ GeV the invariant mass of the W pair $\hat{s}$ satisfies $M_W^2 \ll \hat{s} \ll \Lambda^2$, a large-$\hat{s}$ expansion is warranted. Using the equivalence theorem as in section 6, one can easily conclude that 5 out of the 6 gauge-fermion operators contribute at leading-$\hat{s}$, whose precise coefficients can be determined once PDFs are included. Therefore, W^+W^- production, somewhat against the common lore, can actually be used both at linear and hadron colliders as an excellent probe of new physics in the gauge-fermion sector.

10 Conclusions

In this paper we have analyzed new physics contributions to the process $e^+e^- \to W^+W^-$, consistently using an effective field theory treatment. The essential aspects and results can be summarized as follows:

- The analysis employs the most general basis of next-to-leading order operators in the electroweak chiral Lagrangian.
- Complete relations between the anomalous couplings and the NLO coefficients in the effective Lagrangian have been derived. The anomalous couplings include those that modify gauge-fermion interactions.
- Equations-of-motion constraints have been discussed and used to eliminate redundant operators in order to work with a minimal basis of NLO terms. The redundancy relations imply consistency checks of the relations described in the previous item.
- Polarized cross sections have been computed for $e^+e^- \to W^+W^-$ with both W's on-shell, and with an emphasis on relative corrections to first order in the new-physics coefficients. Specifically, both right- and left-handed electrons, and W's with longitudinal (L) or transverse (T) polarization (LL, LT, TT) have been considered, as well as the case of an unpolarized W pair.

- CP-odd operators do not contribute to the considered observables.
- Of particular interest for colliders in the TeV range is the high-energy, or large-s limit, $M_W^2 \ll s \ll \Lambda^2$. The relative corrections to the cross sections were quoted explicitly through $\mathcal{O}(s/M_W^2)$ and $\mathcal{O}(1)$ in an M_W^2/s expansion, emphasizing the terms that grow with s.
- The relative corrections growing with s have been discussed and explained with the help of the Goldstone-boson equivalence theorem.
- The choice of a basis for the NLO operators is arbitrary in principle and cannot affect the physics. For illustration we have discussed two possible bases and the relation between them. The basis without redundant triple-gauge boson operators but with all gauge-fermion terms appears as a convenient choice.
- Our results, obtained within the chiral Lagrangian framework, have also been expressed in terms of the basis of dimension-six operators in the standard model with a linearly realized Higgs sector. The translation is straightforward in the case of $e^+e^- \to W^+W^-$.
- The potential size of the new physics coefficients has been estimated using naive dimensional counting ($C_i \sim 1/16\pi^2$) and explicit models (constituent technicolor, Z').

The framework discussed here should be useful to identify and to interpret new physics effects from the dynamics of electroweak symmetry breaking in studies of $e^+e^- \to W^+W^-$ at a TeV-scale linear collider in a systematic way. A similar approach can be pursued for many other collider observables with other final states as well. Of interest will also be the application to W pair production at the LHC. Recent measurements [62,63] show somewhat enhanced cross sections for this process. Although the deviation from the standard model is not significant at present, such effects could well be the signature of new physics as described by NLO terms in the electroweak effective Lagrangian. The rise with energy of these effects provides an exciting opportunity, both for the future running of the LHC at 14 TeV and for $e^+e^- \to W^+W^-$ at a linear collider.

A Relative corrections to cross sections not enhanced by s/M_Z^2

In this appendix we list the constant terms in the relative NLO corrections to the various standard-model cross sections, denoted by f_i^{LH}, f_i^{RH} ($i = LL$, LT, TT, Σ) in section 4. These terms are of $\mathcal{O}(1)$ in the expansion for large s/M_Z^2 and therefore not enhanced in the large-s limit. We use the definition

$$C_G \equiv 2(\beta_1 - \delta_G) \tag{92}$$

$$
\begin{aligned}
f_{TT}^{LH} &= -\frac{C_{W1}}{\Lambda^2}\frac{6e^2M_Z^2}{s_Z^2}\frac{(1-\cos\theta)\left[1-\cos\theta\,(1+2c_Z^2)\right]}{1+\cos^2\theta}+\frac{4e^2C_{X1}}{c_Z^2-s_Z^2}-\frac{2e^2C_{X2}}{s_Z^2}\\
&\quad+4\,\mathrm{Re}\,C_{V9}^e+\frac{2c_Z^2C_G}{c_Z^2-s_Z^2}\\
f_{TT}^{RH} &= 0
\end{aligned}
\tag{93}
$$

$$
\begin{aligned}
f_{LL}^{LH} &= \frac{4c_Z^2C_{V-}}{\cos\theta-1}\Big[c_Z^2-s_Z^2+\left(1-6c_Z^2\right)\cos\theta\Big]+2c_Z^2C_G-4e^2C_{X1}\\
&\quad-2e^2\frac{1+2c_Z^2}{s_Z^2}C_{X2}+4\,\mathrm{Re}\,C_{V9}^e\left[8c_Z^4\frac{\cos\theta}{1-\cos\theta}-\left(s_Z^2-c_Z^2\right)^2\right]\\
f_{LL}^{RH} &= -\frac{6e^2C_{X1}}{s_Z^2}-\frac{2e^2C_{X2}}{s_Z^2}
\end{aligned}
$$

$$
f_{LT}^{RH} = -\frac{e^2C_{X1}}{s_Z^2}-\frac{e^2C_{X2}}{s_Z^2}+\frac{2e^2C_{X3}\cos\theta}{s_Z^2\,(1+\cos^2\theta)}+\frac{C_{W1}}{\Lambda^2}\frac{6e^2c_Z^2M_Z^2}{s_Z^2}
\tag{94}
$$

$$
\begin{aligned}
f_{LT}^{LH} &= -\frac{4c_Z^2C_{V-}}{(\cos\theta-1)\chi^2}\bigg[\cos^5\theta\left(32c_Z^6+24c_Z^4-2\right)+\cos^4\theta\left(56c_Z^6+12c_Z^4-6c_Z^2+1\right)\\
&\quad+2\cos^3\theta\left(-8c_Z^6+8c_Z^4+4c_Z^2-1\right)+2\cos^2\theta\left(-32c_Z^6+12c_Z^4-2c_Z^2+1\right)\\
&\quad-8c_Z^2\cos\theta\left(2c_Z^4+3c_Z^2-1\right)+8c_Z^6-20c_Z^4+2c_Z^2+1\bigg]\\
&\quad+\frac{2e^2C_{X1}}{(\cos\theta-1)\,(c_Z^2-s_Z^2)\,\chi^2}\bigg[\cos^5\theta\left(112c_Z^8+80c_Z^6-4c_Z^2+1\right)\\
&\quad+6c_Z^2\cos^4\theta\left(40c_Z^6+12c_Z^4-2c_Z^2+1\right)+8c_Z^2\cos^3\theta\left(4c_Z^6+4c_Z^4-c_Z^2-1\right)\\
&\quad+4c_Z^2\cos^2\theta\left(-56c_Z^6+4c_Z^4-10c_Z^2+1\right)\\
&\quad+\cos\theta\left(-144c_Z^8-80c_Z^6+24c_Z^4-4c_Z^2-1\right)-2c_Z^2\left(8c_Z^6+28c_Z^4-10c_Z^2+1\right)\bigg]
\end{aligned}
$$

$$
\begin{aligned}
&+ \frac{2c_Z^2 e^2 C_{X2}}{(\cos\theta - 1)s_Z^2\chi^2}\Big[\cos^5\theta\left(8c_Z^6 - 12c_Z^4 - 18c_Z^2 - 5\right) \\
&\quad - 2\cos^4\theta\left(4c_Z^6 + 24c_Z^4 + 9c_Z^2 - 1\right) + 4\cos^3\theta\left(-12c_Z^6 + 6c_Z^4 + 4c_Z^2 - 1\right) \\
&\quad + 4\cos^2\theta\left(-4c_Z^6 + 20c_Z^4 + c_Z^2 + 1\right) + \cos\theta\left(40c_Z^6 + 4c_Z^4 + 18c_Z^2 + 1\right) \\
&\quad + 2\left(12c_Z^6 - 8c_Z^4 + 3c_Z^2 + 1\right)\Big] \\
&\qquad + \frac{\left(c_Z^2 - s_Z^2\right) e^2 C_{X3}}{(\cos\theta - 1)s_Z^2\chi^2}\Big[3\cos^4\theta\left(8c_Z^6 + 20c_Z^4 + 6c_Z^2 - 1\right) \\
&\qquad + 12c_Z^2\cos^3\theta\left(4c_Z^4 + 4c_Z^2 - 1\right) - 4\cos^2\theta\left(10c_Z^4 + 2c_Z^2 - 1\right) \\
&\qquad - 4c_Z^2\cos\theta\left(12c_Z^4 + 8c_Z^2 - 1\right) - 24c_Z^6 - 4c_Z^4 + 6c_Z^2 - 1\Big] \\
&- \frac{4\,\mathrm{Re}\,C_{V9}^e}{(\cos\theta - 1)\chi^2}\Big[\cos^5\theta\left(32c_Z^8 + 8c_Z^6 - 12c_Z^4 - 2c_Z^2 + 1\right) \\
&\quad + \cos^4\theta\left(32c_Z^8 - 32c_Z^6 - 12c_Z^4 + 4c_Z^2 - 1\right) \\
&\quad + 2\cos^3\theta\left(-32c_Z^8 + 24c_Z^6 + 8c_Z^4 - 4c_Z^2 + 1\right) \\
&\quad + 2\cos^2\theta\left(-32c_Z^8 + 48c_Z^6 - 4c_Z^4 + 4c_Z^2 - 1\right) \\
&\quad + \cos\theta\left(32c_Z^8 - 24c_Z^6 + 28c_Z^4 - 6c_Z^2 + 1\right) + 32c_Z^8 - 32c_Z^6 + 4c_Z^4 + 4c_Z^2 - 1\Big] \\
&- \frac{2c_Z^2 C_G}{\left(c_Z^2 - s_Z^2\right)\chi}\Big[\cos^2\theta\left(-8c_Z^4 - 2c_Z^2 + 1\right) - 12c_Z^4\cos\theta - 4c_Z^4 - 2c_Z^2 + 1\Big] \\
&+ \frac{C_{W1}}{\Lambda^2}\frac{12e^2c_Z^2M_Z^2}{s_Z^2\left(\cos\theta - 1\right)\chi^2}\Big[\cos^5\theta\left(24c_Z^8 + 20c_Z^6 + 6c_Z^4 + 3c_Z^2 + 1\right) \\
&\quad + \cos^4\theta\left(40c_Z^8 + 16c_Z^6 + 6c_Z^4 + 2c_Z^2 - 1\right) + 2\cos^3\theta\left(-8c_Z^8 + 12c_Z^6 - 2c_Z^2 + 1\right) \\
&\quad + 2\cos^2\theta\left(-24c_Z^8 + 8c_Z^6 - 6c_Z^4 + 2c_Z^2 - 1\right) \\
&\quad - \cos\theta\left(8c_Z^8 + 28c_Z^6 - 10c_Z^4 + 7c_Z^2 - 1\right) + 8c_Z^8 - 16c_Z^6 - 2c_Z^4 + 2c_Z^2 - 1\Big]
\end{aligned}
\qquad (95)
$$

where

$$\chi = 1 + \left(2c_Z^2\left(1+\cos\theta\right)+\cos\theta\right)^2 \tag{96}$$

$$\begin{aligned}
f_\Sigma^{LH} = &-\frac{4c_Z^2 C_{V-}}{\eta^2}\frac{(\cos\theta-1)}{(\cos\theta+1)}\Big[\cos^4\theta\left(16c_Z^6+40c_Z^4-10c_Z^2-1\right)+2\cos^3\theta\left(16c_Z^6+1\right) \\
&+4c_Z^2\cos^2\theta\left(8c_Z^4+24c_Z^2+1\right)+2\cos\theta\left(16c_Z^6-1\right)+16c_Z^6-8c_Z^4+6c_Z^2+1\Big] \\
&+\frac{4e^2C_{X1}}{(\cos\theta+1)\left(c_Z^2-s_Z^2\right)\eta}\Big[\cos^3\theta\left(16c_Z^4-2c_Z^2-1\right) \\
&+\cos^2\theta\left(8c_Z^4-2c_Z^2+1\right)+\cos\theta\left(10c_Z^2-3\right)+8c_Z^4-6c_Z^2+3\Big] \\
&-\frac{2e^2C_{X2}}{(\cos\theta+1)s_Z^2\eta}\Big[-\cos^3\theta\left(c_Z^2-s_Z^2\right)+\cos^2\theta\left(8c_Z^4+2c_Z^2-1\right) \\
&+\cos\theta\left(16c_Z^4-6c_Z^2-1\right)+8c_Z^4+6c_Z^2+1\Big] \\
&+\frac{8e^2C_{X3}}{\eta}\frac{\left(c_Z^2-s_Z^2\right)}{s_Z^2}\frac{(\cos\theta-1)}{(\cos\theta+1)}\Big[\cos\theta\left(c_Z^2+1\right)+c_Z^2\Big] \\
&+\frac{4\,\mathrm{Re}\,C_{V9}^e}{(\cos\theta+1)\eta^2}\Big[\cos^5\theta\left(32c_Z^8-64c_Z^6+20c_Z^4+4c_Z^2-1\right) \\
&+\cos^4\theta\left(32c_Z^8+64c_Z^6-20c_Z^4-12c_Z^2+3\right) \\
&+2\cos^3\theta\left(64c_Z^8-96c_Z^6-4c_Z^4+4c_Z^2-1\right) \\
&+2\cos^2\theta\left(64c_Z^8+96c_Z^6+4c_Z^4+4c_Z^2-1\right)+3\cos\theta\left(32c_Z^8-4c_Z^4-4c_Z^2+1\right) \\
&+96c_Z^8+12c_Z^4+4c_Z^2-1\Big] \\
&+\frac{2c_Z^2C_G}{\left(c_Z^2-s_Z^2\right)\eta}\Big[\left(\cos^2\theta+1\right)\left(9c_Z^4-s_Z^4\right)-2\cos\theta\left(c_Z^2-s_Z^2\right)\Big] \\
&+\frac{C_{W1}}{\Lambda^2}\frac{96e^2c_Z^4M_Z^2}{s_Z^2\eta}\frac{\cos\theta-1}{\cos\theta+1}
\end{aligned} \tag{97}$$

where

$$\eta = \left(1+\cos^2\theta\right)\left(1+8c_Z^4\right)-2\cos\theta \tag{98}$$

$$
f_{\Sigma}^{RH} = -\frac{2e^2 C_{X1} \left(7 + \cos^2\theta\right)}{s_Z^2 \sin^2\theta} - \frac{2e^2 C_{X2}}{s_Z^2} - \frac{8e^2 C_{X3} \cos\theta}{s_Z^2 \sin^2\theta} \tag{99}
$$

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References

[1] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B **716**, 1 (2012) [arXiv:1207.7214 [hep-ex]].

[2] S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **716**, 30 (2012) [arXiv:1207.7235 [hep-ex]].

[3] C. Degrande, N. Greiner, W. Kilian, O. Mattelaer, H. Mebane, T. Stelzer, S. Willenbrock and C. Zhang, arXiv:1205.4231 [hep-ph].

[4] C. Degrande, arXiv:1302.1112 [hep-ph].

[5] Z. Han and W. Skiba, Phys. Rev. D **71**, 075009 (2005) [hep-ph/0412166].

[6] G. Cacciapaglia, C. Csaki, G. Marandella and A. Strumia, Phys. Rev. D **74**, 033011 (2006) [hep-ph/0604111].

[7] M. E. Peskin and T. Takeuchi, Phys. Rev. D **46**, 381 (1992).

[8] J. Alcaraz *et al.* [ALEPH and DELPHI and L3 and OPAL and LEP Electroweak Working Group Collaborations], hep-ex/0612034.

[9] K. J. F. Gaemers and G. J. Gounaris, Z. Phys. C **1**, 259 (1979).

[10] U. Baur and D. Zeppenfeld, Phys. Lett. B **201**, 383 (1988).

[11] M. S. Bilenky, J. L. Kneur, F. M. Renard and D. Schildknecht, Nucl. Phys. B **409**, 22 (1993).

[12] M. S. Bilenky, J. L. Kneur, F. M. Renard and D. Schildknecht, Nucl. Phys. B **419**, 240 (1994) [hep-ph/9312202].

[13] G. Gounaris, J. L. Kneur, D. Zeppenfeld, Z. Ajaltouni, A. Arhrib, G. Bella, F. A. Berends and M. S. Bilenky *et al.*, In *Geneva 1995, Physics at LEP2, vol. 1* 525-576, and Preprint - Gounaris, G. (rec.Jan.96) 52 p [hep-ph/9601233].

[14] K. Hagiwara, T. Hatsukano, S. Ishihara and R. Szalapski, Nucl. Phys. B **496**, 66 (1997) [hep-ph/9612268].

[15] J. A. Aguilar-Saavedra *et al.* [ECFA/DESY LC Physics Working Group Collaboration], hep-ph/0106315.

[16] W. Menges, in '2nd ECFA/DESY Study 1998-2001', 1635-1669

[17] G. Weiglein *et al.* [LHC/LC Study Group Collaboration], Phys. Rept. **426**, 47 (2006) [hep-ph/0410364].

[18] L. Linssen, A. Miyamoto, M. Stanitzki and H. Weerts, arXiv:1202.5940 [physics.ins-det].

[19] V. V. Andreev, G. Moortgat-Pick, P. Osland, A. A. Pankov and N. Paver, arXiv:1205.0866 [hep-ph].

[20] U. Baur and D. Zeppenfeld, Nucl. Phys. B **308**, 127 (1988).

[21] "ATLAS: Detector and physics performance technical design report. Volume 2," CERN-LHCC-99-15.

[22] R. M. Thurman-Keup, A. V. Kotwal, M. Tecchio and A. Byon-Wagner, Rev. Mod. Phys. **73**, 267 (2001).

[23] A. F. Falk, M. E. Luke and E. H. Simmons, Nucl. Phys. B **365**, 523 (1991);

[24] A. De Rujula, M. B. Gavela, P. Hernandez and E. Masso, Nucl. Phys. B **384**, 3 (1992).

[25] J. Bagger, S. Dawson and G. Valencia, Nucl. Phys. B **399**, 364 (1993) [hep-ph/9204211].

[26] K. Hagiwara, S. Ishihara, R. Szalapski and D. Zeppenfeld, Phys. Rev. D **48**, 2182 (1993).

[27] H. -J. He, Y. -P. Kuang and C. P. Yuan, hep-ph/9704276.

[28] G. Buchalla and O. Cata, JHEP **1207**, 101 (2012) [arXiv:1203.6510 [hep-ph]].

[29] W. Buchmuller and D. Wyler, Nucl. Phys. B **268**, 621 (1986).

[30] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, JHEP **1010**, 085 (2010) [arXiv:1008.4884 [hep-ph]].

[31] R. Contino, arXiv:1005.4269 [hep-ph].

[32] A. C. Longhitano, Phys. Rev. D **22**, 1166 (1980).

[33] A. C. Longhitano, Nucl. Phys. B **188**, 118 (1981).

[34] T. Appelquist, M. J. Bowick, E. Cohler and A. I. Hauser, Phys. Rev. D **31**, 1676 (1985).

[35] A. I. Hauser, Ph. D. Thesis, University of Yale, 1985.

[36] T. Appelquist and G. H. Wu, Phys. Rev. D **48**, 3235 (1993) [arXiv:hep-ph/9304240].

[37] T. Appelquist and G. H. Wu, Phys. Rev. D **51**, 240 (1995) [arXiv:hep-ph/9406416].

[38] C. Arzt, M. B. Einhorn and J. Wudka, Nucl. Phys. B **433**, 41 (1995) [hep-ph/9405214].

[39] A. Nyffeler and A. Schenk, Phys. Rev. D **62**, 113006 (2000) [hep-ph/9907294].

[40] C. Grojean, W. Skiba and J. Terning, Phys. Rev. D **73**, 075008 (2006) [hep-ph/0602154].

[41] K. Hagiwara, R. D. Peccei, D. Zeppenfeld and K. Hikasa, Nucl. Phys. B **282**, 253 (1987).

[42] B. Holdom, Phys. Lett. B **258**, 156 (1991).

[43] G. Passarino, Nucl. Phys. B **868**, 416 (2013) [arXiv:1209.5538 [hep-ph]].

[44] C. R. Ahn, M. E. Peskin, B. W. Lynn and S. B. Selipsky, Nucl. Phys. B **309**, 221 (1988).

[45] A. Denner, S. Dittmaier, M. Roth and D. Wackeroth, Nucl. Phys. B **587**, 67 (2000) [hep-ph/0006307].

[46] A. Denner, S. Dittmaier, M. Roth and L. H. Wieders, Nucl. Phys. B **724**, 247 (2005) [Erratum-ibid. B **854**, 504 (2012)] [hep-ph/0505042].

[47] A. Bierweiler, T. Kasprzik, H. Kühn and S. Uccirati, JHEP **1211**, 093 (2012) [arXiv:1208.3147 [hep-ph]].

[48] S. Dawson and G. Valencia, Phys. Rev. D **49**, 2188 (1994) [hep-ph/9308248].

[49] M. Diehl and O. Nachtmann, Z. Phys. C **62**, 397 (1994).

[50] M. Diehl, O. Nachtmann and F. Nagel, Eur. Phys. J. C **27**, 375 (2003) [hep-ph/0209229].

[51] J. M. Cornwall, D. N. Levin and G. Tiktopoulos, Phys. Rev. D **10**, 1145 (1974) [Erratum-ibid. D **11**, 972 (1975)].

[52] M. S. Chanowitz and M. K. Gaillard, Nucl. Phys. B **261**, 379 (1985).

[53] C. Grosse-Knetter and I. Kuss, Z. Phys. C **66**, 95 (1995) [hep-ph/9403291].

[54] A. Dobado and J. R. Pelaez, Phys. Lett. B **329**, 469 (1994) [Addendum-ibid. B **335**, 554 (1994)] [hep-ph/9404239].

[55] P. Galison and A. Manohar, Phys. Lett. B **136**, 279 (1984).

[56] P. Langacker, Rev. Mod. Phys. **81**, 1199 (2009) [arXiv:0801.1345 [hep-ph]].

[57] P. Langacker, arXiv:0911.4294 [hep-ph].

[58] K. S. Babu, C. F. Kolda and J. March-Russell, Phys. Rev. D **57**, 6788 (1998) [hep-ph/9710441].

[59] T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **104**, 201801 (2010) [arXiv:0912.4500 [hep-ex]].

[60] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **80**, 053012 (2009) [arXiv:0907.4398 [hep-ex]].

[61] V. M. Abazov *et al.* [D0 Collaboration], arXiv:0907.4952 [hep-ex].

[62] G. Aad *et al.* [ATLAS Collaboration], arXiv:1210.2979 [hep-ex].

[63] S. Chatrchyan *et al.* [CMS Collaboration], arXiv:1301.4698 [hep-ex].

[64] M. Dobbs and L. Lefebvre, ATL-PHYS-2002-022.

[65] M. Dobbs and L. Lefebvre, ATL-PHYS-2002-023.