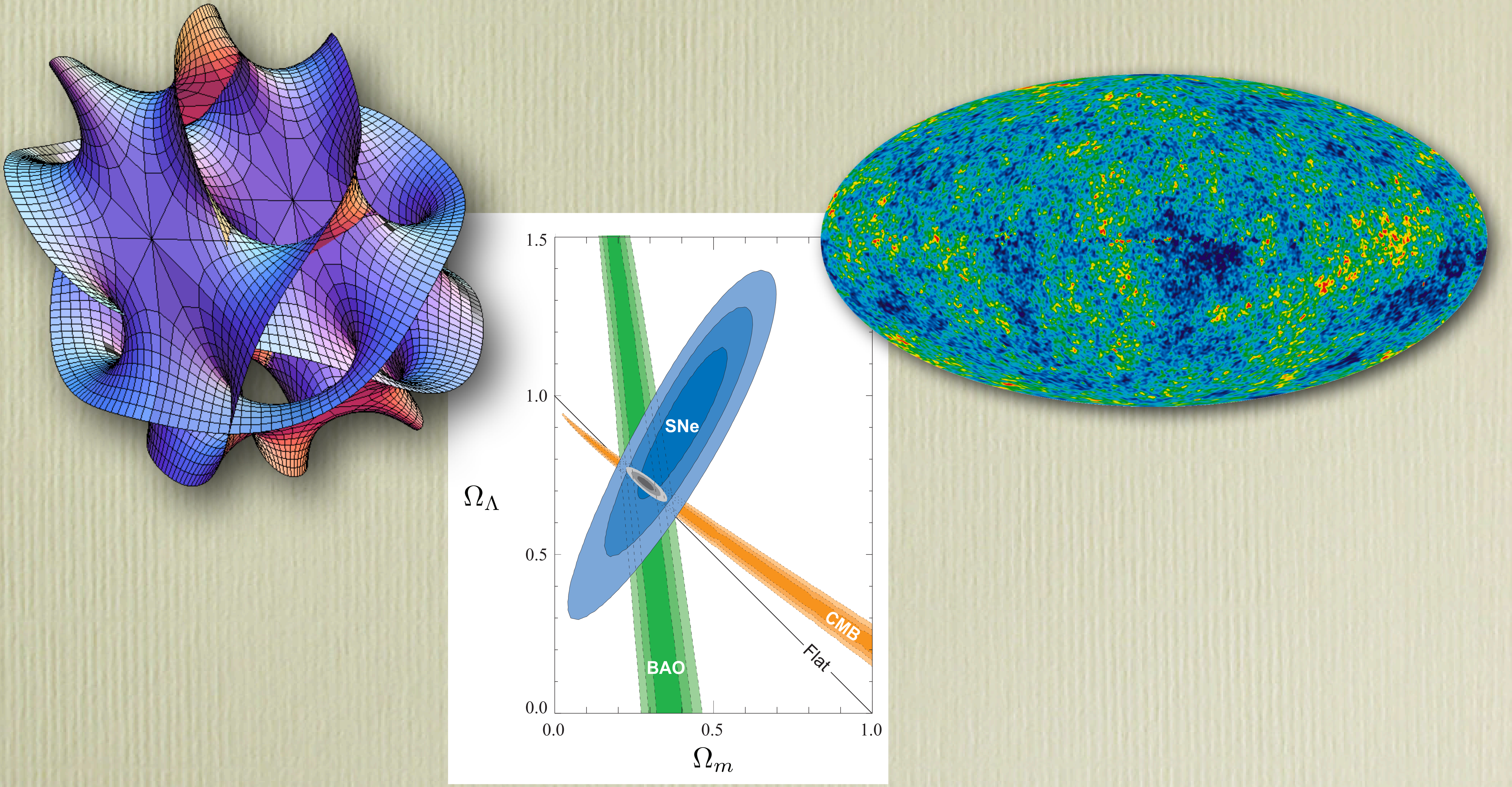
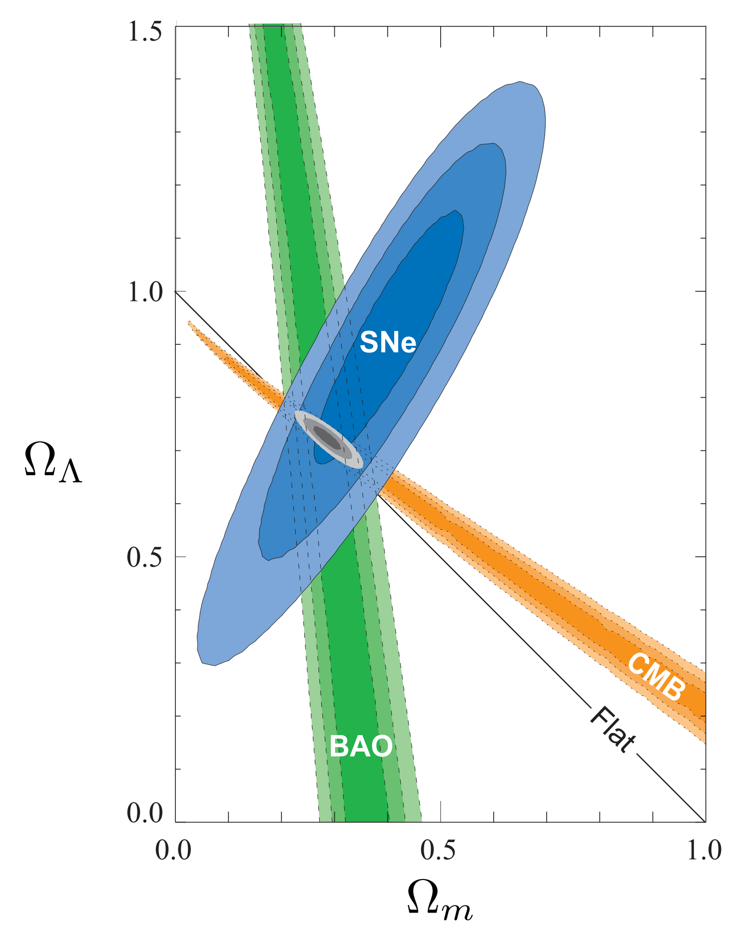
A sufficient condition for IIB/F-theory dS vacua



Jan Louis, Markus Rummel, Roberto Valandro & AW:
(arXiv:1107.2115) & work in progress (arXiv:1205.xxxx)
University of Hamburg & DESY Hamburg

- the observation:

➡the Universe expands speeding up
 - $\Lambda > 0$ ... we live in de Sitter space!



- a task:

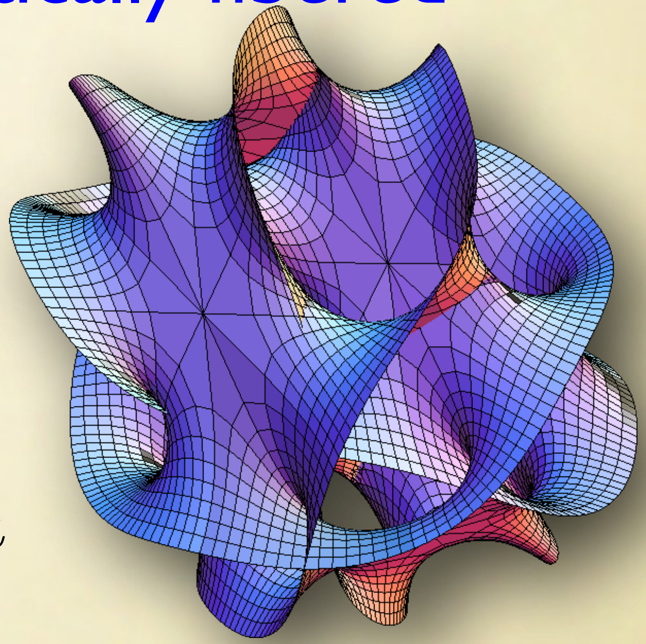
➡there is a <u>necessary dS condition</u> in supergravity - positive sectional curvature of the Kahler potential
 [Covi, Gomez-Reino, Gross, Louis, Palma, Scrucca]

➡there's nothing wrong using e.g. anti-D3s - but it maybe worthwhile to get dS from a 'clean' system:

try to get a *<u>sufficient dS condition</u>* for whole class of vacua purely in terms of the topological data determining the 4d supergravity of F-theory/IIB

dS

- Try to use just the closed string moduli sector of a 4d N=1 F-theory compactification on an elliptically fibered CY 4-fold



$$\text{dilaton } S$$

$$h^{1,1} \text{ volume moduli } T_i$$

$$h^{2,1} \text{ complex structure moduli } U_a$$

to get a dS vacuum from spontaneous F-term breaking

- key ingredient: the leading perturbative *O*(α'^3) correction

- leads to 'Kahler uplifted' dS vacua - checked for one volume modulus + dilaton *S* and one complex structure *U*

[Balasubramian, Berglund] [AW]

the general setup ...

- The Kahler potential for the volume moduli: α'- & loop corrections

Becker-Becker-Haack-Louis ; Berg-Haack-Körs ; Llana-Rocek-Saueressig-Theis-Vandoren
v. Gersdorff-Hebecker

$$K = -2\ln\left[\mathcal{V} + \alpha'^3\left(\xi + \text{1-loop}\right)\right] + \mathcal{O}(\alpha'^4)$$

$$\xi \sim -\chi \cdot (S+\bar{S})^{3/2} \quad ; \quad \text{if } h^{1,1}=1: \ \mathcal{V} \sim \frac{1}{\sqrt{\kappa}}(T+\bar{T})^{3/2}$$

- Fluxes fix S and U. Stabilization of T by an interplay of the leading $\mathcal{O}(\alpha'^3)$-correction above and non-perturbative effects:

$$W = W_0 + e^{-a\,T}\,, \quad W_0 = \int_{CY} G \wedge \Omega$$

some relationships ...

- 4d N=1 supergravity specified by K and W just given, so far have 3 known branches of vacua:

i) small W_0:

- α'-corrections are irrelevant
- vacua are SUSY AdS prior to uplifting

→ the ‘KKLT‘ branch [KKLT]

ii) arbitrary W_0:

- leading α'-correction is relevant
- blow-up volume moduli scale as the log of the total volume
- vacua are non-SUSY AdS prior to uplifting

→ the ‘LVS‘ branch [Balasubramian, Berglund, Conlon Quevedo]

iii) $|W_0| = O(1...50)$:

- leading α'-correction is relevant
- largish rank condensing gauge group → non-SUSY dS vacua (this talk)

[Balasubramian, Berglund; AW; Markus Rummel, AW]

de Sitter vacua from 'Kahler uplifting' at large volume

- 4d *N*=1 supergravity - scalar potential:

$$V = e^K \left\{ K^{-1}_{T\bar{T}} [a^2 e^{-2aT_r} + (-a e^{-aT_r} \overline{W K_T} + c.c)] + 3\xi \frac{\xi^2 + 7\xi\mathcal{V} + \mathcal{V}^2}{(\mathcal{V} - \xi)(\xi + 2\mathcal{V})^2} |W|^2 \right\}$$

- expand to leading order in ξ/*V* and e^{-aT}:

$$V \simeq 4AW_0 \frac{ate^{-at}}{\mathcal{V}^2} \cos(a\tau) + \frac{3W_0^2}{4\mathcal{V}^3} \quad \sim \quad \frac{2C}{9x^{9/2}} - \frac{e^{-x}}{x^2}$$

$$x \equiv at \quad , \quad T = t + i\tau \quad , \quad C \equiv -\frac{27}{64}\sqrt{\frac{2}{3}}\frac{W_0}{A}\frac{\xi a^{3/2}\sqrt{\kappa}}{A}$$

de Sitter vacua from 'Kahler uplifting' at large volume

$$V'(x) = \frac{-W_0 a^3 A}{2\gamma^2}\frac{1}{x^{11/2}}\left(C - x^{5/2}(x+2)e^{-x}\right) \ ,$$

$$V''(x) = \frac{-W_0 a^3 A}{2\gamma^2}\frac{1}{x^{13/2}}\left(\frac{11}{2}C - x^{5/2}(x^2+4x+6)e^{-x}\right)$$

C is bounded - satisfying the bound guarantees a dS vacuum ('sufficient' condition)

- Lower bound on C: $V(x_{min}) = V'(x_{min}) = 0$

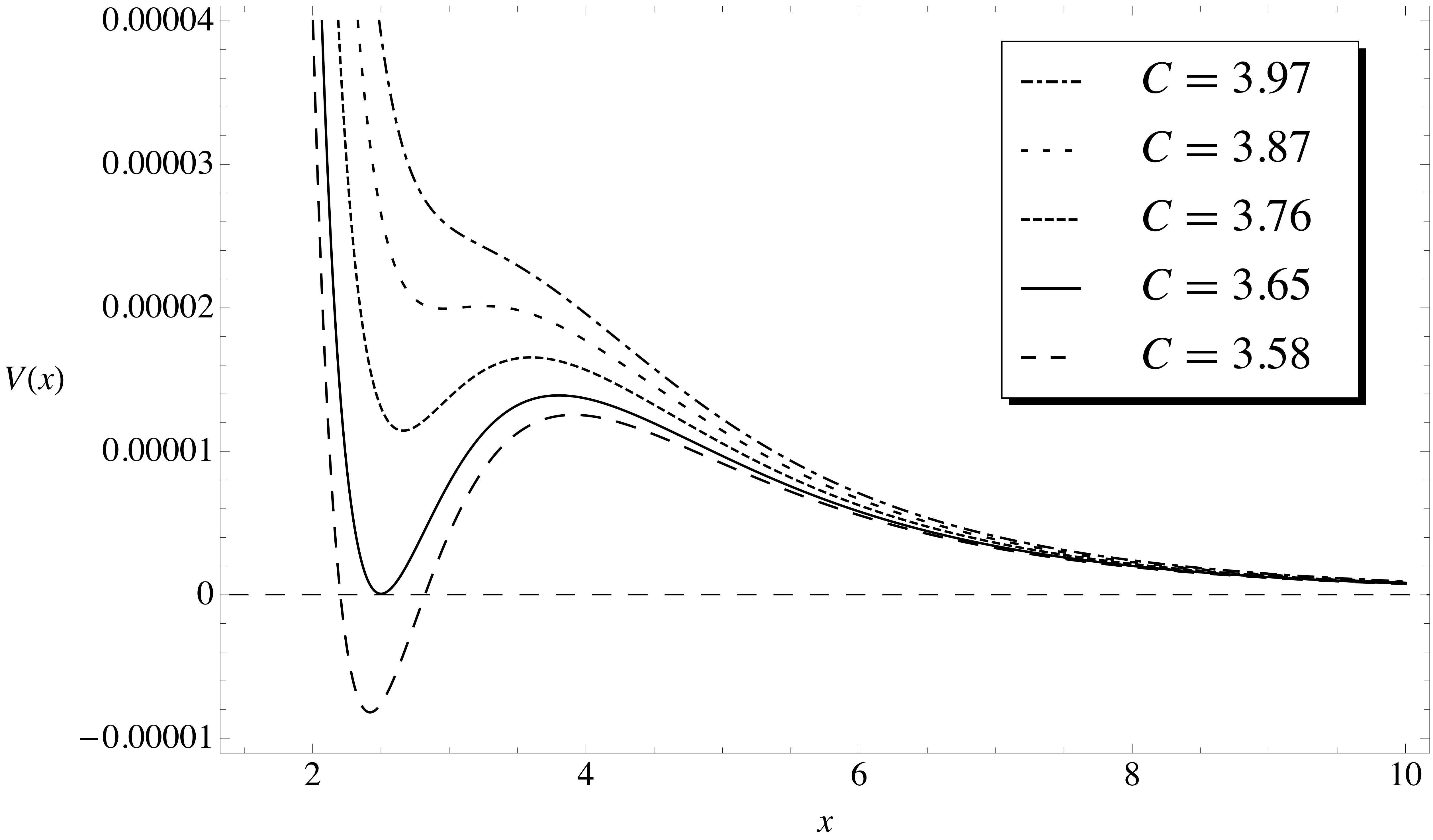
$$\{x_{min}, C\} = \{\tfrac{5}{2}, \tfrac{225}{8}\sqrt{\tfrac{5}{2}}e^{-\frac{5}{2}}\} \simeq \{2.5, 3.65\}$$

- Upper bound on C: $V'(x_{min}) = V''(x_{min}) = 0$

$$\{x_{min}, C\} \simeq \{\tfrac{3+\sqrt{89}}{4}, 3.89\} \simeq \{3.11, 3.89\}$$

note: these last two slides were found independly by [deAlwis, Givens '11]

de Sitter vacua from 'Kahler uplifting' at large volume



this works for arbitrary h^{1,1} on Swiss-cheese CYs!

- resulting scalar potential

$$V = \frac{4W_0}{\mathcal{V}^2}\left(atAe^{-at}\cos(a\tau) + \sum_{i=2}^{h^{1,1}} a_i t_i A_i e^{-a_i t_i}\cos(a_i\tau_i)\right) + \frac{3\xi W_0^2}{4\mathcal{V}^3}$$

$$+\sum_{i=2}^{h^{1,1}} \frac{2\sqrt{2}}{3}\frac{a_i^2 A_i^2}{\mathcal{V}^2}\frac{\sqrt{t_i}}{\gamma_i}e^{-2a_i t_i}\,\mathcal{V}$$

of $O(\xi^2/V^2)$ at the minimum

all $V_{\tau_I\tau_I} > 0$ if $W_0 < 0$ at $\tau_I = 0$;

or if $W_0 > 0$ at $\tau_I = \pi/a_I$

$V_{t_I\tau_J} = 0$ at these points, thus all axions are massive

what about the other moduli?

- Starting point: *S* and *U*$_a$ at SUSY locus, T stabilization breaks SUSY - expand *S* and *U*$_a$ in $\xi/\mathcal{V}$ & check that shifts are small:

$$K = -2\ln(\mathcal{V}+\xi/2) - \ln(S+\bar{S})\ ,\ \xi \equiv \tilde{\xi}\,(S+\bar{S})^{3/2} \sim -\chi\,(S+\bar{S})^{3/2}$$

$$W = W_0 + Ae^{-aT} = C_1 - C_2\,S + Ae^{-aT}$$

$$\Rightarrow\quad V \sim e^K\Big(\underbrace{K^{S\bar{S}}|D_S W_0|^2}_{V_0} + K^{T\bar{T}}|D_T W|^2 + K^{T\bar{S}} D_T W\,\overline{D_S W_0}\Big)$$

δV perturbs *S* away from S_0 ...

$$S_0 = -\frac{C_1}{C_2}\ :\ D_S W_0\big|_{S_0} = 0\ ,\qquad \frac{\partial V}{\partial S} = 0 \quad \Rightarrow \frac{\delta S}{S_0} \sim \frac{\xi}{\mathcal{V}}$$

what about the other moduli?

- mass scales:

$$m_t^2 \sim \frac{\hat{\xi}}{\mathcal{V}^3} \qquad m_{\mathrm{Re}\,S}^2 \sim \frac{1}{\mathcal{V}^2}$$

$$m_\tau^2 \sim \frac{\hat{\xi}}{\mathcal{V}^3} \qquad m_{\mathrm{Im}\,S}^2 \sim \frac{1}{\mathcal{V}^2}$$

$$m_{3/2}^2 = e^K |W|^2 \sim \frac{1}{\mathcal{V}^2}$$

$$m_{KK} = \frac{1}{L\sqrt{\alpha'}} \sim \frac{1}{\hat{\mathcal{V}}^{2/3}}$$

➡ SUSY is broken at the GUT scale, but below KK-scale!

what about the other moduli?

- similarly for the complex structures U_a ...

$$K = \ldots - \ln\left(\int_{CY_3} \Omega(U_a) \wedge \bar{\Omega}(\overline{U_a})\right)$$

C_1, C_2 become functions of U_a

$$\Rightarrow \quad V \sim e^K \Big(\underbrace{K^{a\bar{b}} D_{U_a} W_0 \overline{D_{U_b} W_0}}_{V_0} + \underbrace{K^{T\bar{T}} |D_T W|^2 + K^{T\bar{S}} D_T W \overline{D_S W_0}}_{\delta V \text{ perturbs } U \text{ away from } U_0 \ldots} \Big)$$

need $(\partial_a \partial_{\bar{b}} V_0) > 0$

$$U_{a,0} \;:\; D_{U_a} W_0\big|_{U_{a,0}} = 0 \;, \qquad \frac{\partial V}{\partial U_a} = 0 \quad \Rightarrow \frac{\delta U_a}{U_{a,0}} \sim \frac{\xi}{\mathcal{V}}$$

[c.f. also: Gallego, Serone]

Moduli space of $\mathbb{CP}_{1,1,1,6,9}$ [Denef, Douglas, Florea '04]

Consider Calabi-Yau 3-fold defined as degree 18 hypersurface in

$$\mathbb{CP}_{1,1,1,6,9}: \ (x_1, x_2, x_3, x_4, x_5) \sim (\lambda x_1, \lambda x_2, \lambda x_3, \lambda^6 x_4, \lambda^9 x_5)$$

$$\text{e.g.} \quad x_1^{18} + x_2^{18} + x_3^{18} + x_4^3 + x_5^2 = 0 \qquad (h^{1,1} = 2 \text{ and } h^{2,1} = 272)\,.$$

Kähler moduli:

- Non-perturbative effects: $W_{\text{n.p.}} = \mathcal{O}(1)\, e^{-2\pi/30\ T_1} + \mathcal{O}(1)\, e^{-2\pi\ T_2}$.

Complex structure moduli: [Greene,Plesser'89], [Candelas,Font,Katz,Morrison'94]

- $\Gamma = \mathbb{Z}_6 \times \mathbb{Z}_{18}$ modding fixes a $2 \subset h^{2,1} = 272$ parameter subspace.
- No flux and $D_i W = 0$ on all non-invariant cycles
  $\Rightarrow$ Effectively all 272 complex structure moduli stabilized.
- $G(z)$ via mirror symmetry in the large complex structure limit:

$$G(z_1, z_2) = \sum_{i+j\leq 3} c_{ij} z_1^i z_2^j + \xi + G_{\text{instanton}}(e^{-2\pi z_1}, e^{-2\pi z_2})$$

Finding flux vacua

- The 3-fold fixes all free parameters except of the VEV's $\langle T_1\rangle$, $\langle T_2\rangle$, $\langle S\rangle$, $\langle z_1\rangle$ and $\langle z_2\rangle$ and flux vectors $f = \{f_i\}$, $h = \{h_i\}$, $i = 1, .., 6$.
- D3-Tadpole constraint:

$$L = \int_X F_3 \wedge H_3 = L_{\mathrm{max}} - N_{D3}, \quad L_{\mathrm{max}} = \frac{\chi(\text{4-fold})}{24}.$$

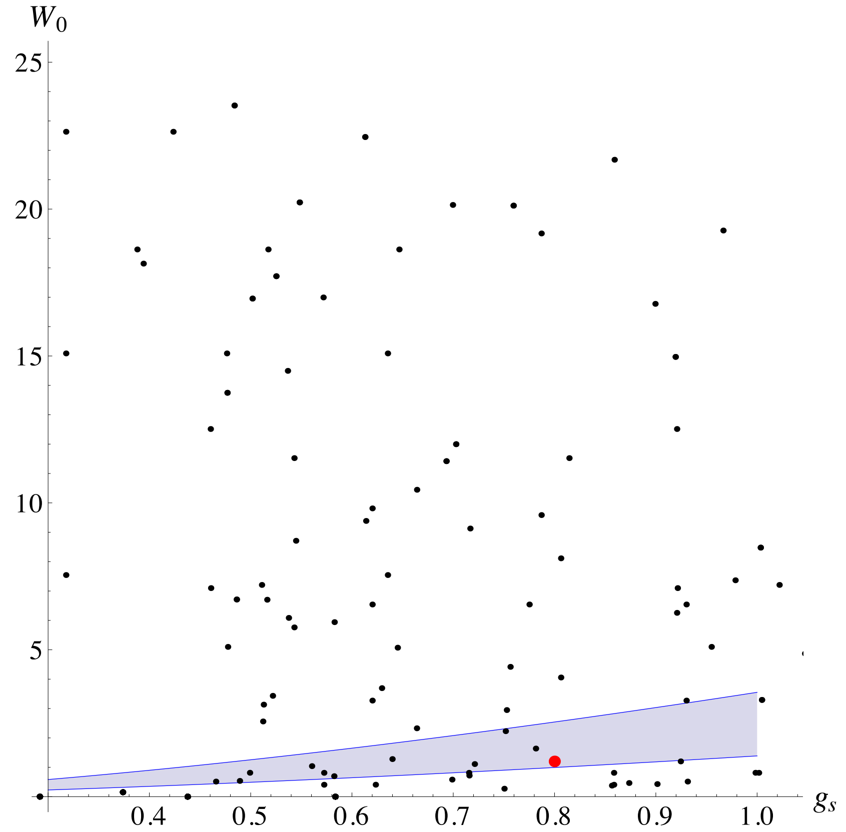
Strategy:

- Fix $\langle S\rangle$, $\langle z_1\rangle$ and $\langle z_2\rangle$ to rational value.
- Neglect $G_{\mathrm{instanton}}$ and set ξ to a rational value, such that:

$$0 = \{W_0, D_S W_0, D_{z_1} W_0, D_{z_2} W_0\} = \mathbf{A}\cdot\{f_1, .., f_6, h_1, .., h_6\}, \text{ with } \mathbf{A} \in \mathbb{Q}^{8\times 12}.$$

- Find basis of solutions $\{f_i\}$, $\{h_i\}$ with minimal tadpole L.
- Generate shift in W_0, $\langle S\rangle$, $\langle z_1\rangle$ and $\langle z_2\rangle$ for $\xi \in i\mathbb{R}$, $G_{\mathrm{instanton}} \neq 0$.

Solutions and Kähler moduli stabilization



De Sitter vacuum can be constructed if:

- $L < L_{\max}$,
- $\{W_0, g_s = \langle \mathrm{Re}\, S \rangle^{-1}, A\}$ fullfills:

  $$1.25 < |W_0|\, A\, g_s^{-3/2} < 1.34,$$

- $V^{(c.s.)}_{z^a z^b} > 0$.

Explicit example:

- $\{f, h\} = \{0, -16, 56, -28, -12; 4, 0, 0, 0, -9, 10\}, \quad L = 408$
-

g_s	A	$\langle T_1 \rangle$	$\langle T_2 \rangle$	$\langle z_1 \rangle$	$\langle z_2 \rangle$	W_0	$\hat{\mathcal{V}}$
0.8	1.5	14.3	0.8	0.98	0.99	-1.06	4.2

Finite number of solutions? Statistics?

$\mathbb{CP}^4_{11169}$ part of n-twisted fibrations

$$\begin{tikzcd} \mathcal{M}_n : \quad \mathbb{P}^1 \arrow[r] & \mathcal{M}_n \arrow[d] \\ & \mathbb{P}^2 \end{tikzcd}$$

they have a GLSM description:

	u_1	u_2	u_3	u_4	u_5
Q^1	1	1	1	$-n$	0
Q^2	0	0	0	1	1

$\underline{\mathcal{M}_n}$

↝ toric variety

toric divisors: D_i : $x_i = 0$

basis: (D_5, D_4) $\quad D_4 = D_5 - n \cdot D_1$

or (D_1, D_5)

4-fold as Weierstrass model over $\mathcal{M}_n$:

$$Y^2 = X^3 + f(\vec{u}) \cdot X \cdot Z^4 + g(\bar{u}) \cdot Z^6 = 0$$

	u_1	u_2	u_3	u_4	u_5	X	Y	Z
Q^1	1	1	1	$-n$	0	0	0	$-(3-n)$
Q^2	0	0	0	1	1	0	0	-2
Q^3	0	0	0	0	0	2	3	1

homology classes:

$$[\mu_n] = K_{\mu_n} = (3-n)[D_1] + 2[D_3]$$

$$[Z] = -K_{\mu_n} = \overline{K}_{\mu_n}$$

Can enforce ADE singularities!

Tate decomposition of f and g into functions: a_1, a_2, a_3, a_4, a_6

$$[a_k] = k \cdot K_{\mu_n}$$

example : $S_p(N)$ from Kodaira class. on divisor

$$D_i : u_i = 0$$

$$a_k = u_i^{d_k} \boxed{a_{k,d_k}} \longrightarrow \overset{!}{\sim} u_i^{\text{pos.}\#}$$

	d_1	d_2	d_3	d_4	d_6
$S_p(N)$	0	0	N	N	$2N$

example D_5, take a_3:

$$a_3 = u_5^N \cdot a_{3,N}$$

and $[a_3] = 3 \cdot K_{u_n}\Big|_{D_5} = 6[D_5]$

$\Rightarrow N \leq 6$

at max $Sp(6)$ or $SU(12)$

or E_8 on D_5.

[Denef, Douglas, Florea]
[Denef '08]

and: $\nu = \frac{\sqrt{2}}{3n}\left(D_5^{3/2} - D_4^{3/2}\right)$

↷ bound on rank on D_5 and $n=18$ for $\mathbb{CP}^4_{11169}$ base

$\rightarrow \tilde{\nu} \sim 5$

↷ choose (D_1, D_5):

$$\nu = \frac{\sqrt{-2n}}{3} D_1^{3/2} + \mathcal{O}(D_5)$$

$n < 0$, $Sp(N)$ on D_1 :

$$a_3 = u_1^N \cdot a_{3,N}$$

$$[a_3] = 3 \cdot K_{\mu_n}\Big|_{D_1} = 3(3-n)[D_1]$$

$$\Rightarrow N \leq 3(3-n)$$

looks unbounded, <u>but</u>:

check factorization of a_i !

$\Rightarrow \quad n > -18$

for: $n = -18$

d_1	d_2	d_3	d_4	d_6
1	2	3	4	5

$\Rightarrow E_8$

for $n < -18$: off Kodaira!

so rank bound, but for

e.g.:

$$n = -6$$

$$\Rightarrow Sp(27) \text{ on } D_1$$
$$Sp(6) \text{ on } D_5$$

and due to: $V = \frac{\sqrt{-2n}}{3} D_1^{3/2}$

$\Rightarrow \boxed{V \approx 70}$ dS vacuum can show:

- blow-up of $Sp(N)$ on D_1, , D_5 always possible
- intersection #'s from SR-ideal give always 1 exceptional divisor with

contributes in W ⟵ $\chi_0(D) = 1$

work in progress:

base $\mathcal{B}$ as : $P_{m,2} = 0$

in

$$\tilde{\mathcal{M}}_n \quad \begin{array}{c|cccccc} & u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \\ \hline Q^1 & 1 & 1 & 1 & -n & 0 & 0 \\ \hline Q^2 & 0 & 0 & 0 & 1 & 1 & 1 \end{array}$$

$$K_{\tilde{\mathcal{M}}_n} = (3-n)[D_1] + 3[D_5]$$

$$K_B = (3-n-m)[D_1] + [D_5]$$

- B nonsingular for all $n < 0$,
  $m < -n$
- for $Sp(N)$ on D_1:

$$\boxed{N \leq 3(3-n-m)}$$

and a_i never extra-factorize!

$\Rightarrow$ unbounded-rank $Sp(N)$

in compact elliptic

Calabi-Yau 4-fold

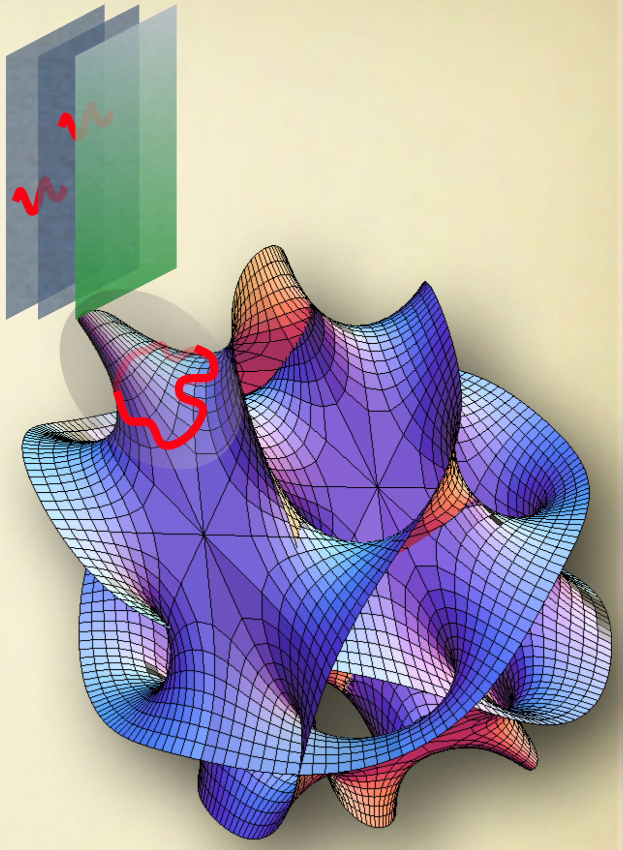
$\Downarrow$

very large volumes

in principle reachable

...

Conclusions



- there is a sufficient condition in terms of

  - the Euler characteristic
  - choice of ADE-type singularities,
  - and choice of fluxes / W_0

  which if satisfied guarantees the existence of classically stable dS vacua within the 4d $N=1$ supergravity descending from a IIB/F-theory compactification on an elliptically fibered 4-fold.
  They break SUSY at the GUT scale.

- Compactifications similar to P_{11169} of IIB provide fully explicit examples - ongoing work (to appear)